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WOOLWICH MATHEMATICAL PAPERS

FOR ADMISSION INTO

Woolwich, Eng. —

THE ROYAL MILITARY ACADEMY

FOR THE YEARS

1880—1888

EDITED BY

Eldred John
E. J. BROOKSMITH, B.A., LL.M.,
ST JOHN'S COLLEGE, CAMBRIDGE; INSTRUCTOR OF MATHEMATICS
AT THE ROYAL MILITARY ACADEMY, WOOLWICH.

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MATHEMATICAL EXAMINATION PAPERS

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JULY, 1880.

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Shew also that the two triangles are equal in all respects.

3. The straight lines which join the extremities of two equal and parallel straight lines towards the same parts are themselves equal and parallel.

Shew also that the straight lines which join the extremities towards opposite parts, bisect each other.

4. Describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle.

Shew that if a parallelogram be a rhombus, each of the parallelograms described about its diameter will also be a rhombus.

5. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by the side on which, when produced, the perpendicular falls and the straight line inter-

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6. If a straight line touch a circle, the straight line drawn from the centre to the point of contact shall be perpendicular to the line touching the circle.

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11. If from the vertical angle of a triangle a straight line be drawn perpendicular to the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the perpendicular and the diameter of the circle described about the triangle.

[*Name your authority in all cases where you do not follow Euclid.*]

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II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy in numerical results.*]

1. Add together $8\frac{7}{11}$ of $\frac{3}{5}$, $9\frac{1}{2}$ of $\frac{13}{76}$, $3\frac{3}{4}$ of $\frac{8}{15}$, and $6\frac{8}{9}$ of $\frac{18}{23}$.

2. Subtract $3\frac{7}{17}$ of $4\frac{10}{29}$ from $\frac{21\frac{9}{22}}{1\frac{13}{60}}$.

3. Multiply together $\frac{5}{18}$ of $\frac{18}{35}$, by $2\frac{9}{20}$ of $6\frac{3}{4}$ of $10\frac{16}{23}$.

4. Divide $25\frac{25}{26}$ by $3\frac{23}{39}$.

5. Add together ·023 of a £, ·946 of a shilling, and 3·48 pence, and subtract the sum from ·26 of a guinea. Give the answer in pence and the decimal of a penny.

6. Multiply 36·2894 by ·00893.

7. Divide 99·994 by 2890.

8. Divide 42·547 by ·00542.

9. Express 3 ozs. 7 dwts. 12 grs. as the decimal of a lb. troy.

10. What is the difference between ·038 of a mile and $\frac{11}{24}$ of a furlong? Express the answer as the fraction (vulgar) of a furlong.

11. Find, by Practice, the value of 3 tons 5 cwt. 2 qrs. 21 lbs. at £12 per ton.

12. Divide £56 between *A*, *B*, *C*, and *D* in the ratio of the numbers 3, 5, 7, and 9.

13. In what time will £345 amount to £454. 2*s*. $1\frac{1}{2}$*d*. at $2\frac{3}{4}$ per cent. per annum simple interest?

14. Two horses can plough in a given time as much as 3 oxen, but the cost of 4 oxen is only equal to that of 3 horses, the daily cost of a horse being 3*s*.

A certain field can be ploughed by 3 horses in 8 days. What would be the cost of ploughing it by oxen in 6 days?

15. The value of a certain house in 1880 has increased 35 per cent. since 1877. The house was rated in 1877 at two-thirds of its value, and in 1880 it is rated at three-fifths of its value, the rate in the £ remaining the same. Compare the rate paid in 1877 with that paid in 1880.

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16. When is one number prime to another?

Find the greatest common measure and the least common multiple of 7560, 27720, and 108108.

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20. Find the true discount on £142. 1*s*. 9*d*. due 18 months hence at $3\frac{1}{2}$ per cent. per annum.

21. Find, correct to a farthing, the present value of £10,000 due 8 years hence at 5 per cent. per annum compound interest.

22. Find log $10\frac{2}{25}$, and calculate to six decimal places the value of

$$\sqrt[3]{\left(\frac{294 \times 125}{42 \times 32}\right)^2}.$$

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log 2 = ·3010300	log 67683 = 4·8304796
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1. Find the Greatest Common Measure of

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$$(a^2-3a+2)x^2+(2a^2-4a+1)x+a(a-1).$$

2. Find the value of

$$x^3-x^2+3x+5, \text{ when } x=1+2\sqrt{-1};$$

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2. According to the ratio definition, point out which of the elementary trigonometrical functions are never less than unity, and which may be either less or greater than unity.

Prove $(\sin A)^2+(\cos A)^2=1$, and express the numerical values of $\sin 135°$ and $\tan 150°$ with their proper signs.

3. If $(A+B)$ is an angle in the first quadrant, obtain by means of a geometrical figure the formula $\cos(A+B)=\cos A\cos B-\sin A\sin B$, and deduce from it the expression for $\cos(A-B)$.

Find $\sec(A+B)$ in terms of $\sec A$ and $\sec B$, and prove

$$\sec 105°=-\sqrt{2}(1+\sqrt{3}).$$

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4. Prove:

(1) $\sin 3A = 3 \sin A - 4 (\sin A)^3$.

(2) $\sin 54^\circ = \frac{\sqrt{5}+1}{4}$.

(3) $\frac{\sin(A-C)}{\cos A \cos C} + \frac{\sin(B-A)}{\cos B \cos A} + \frac{\sin(C-B)}{\cos C \cos B} = 0$.

5. Prove that in every circle the angle subtended by an arc equal to the radius is an invariable angle. Express that angle in degrees and decimals of degrees, and find the circular measure of one minute. On a circle 10 feet in radius it was found that an angle of $22^\circ\ 30'$ was subtended by an arc 3 feet $11\frac{1}{8}$ inches in length; hence calculate, to four decimal places, the numerical ratio of the circumference of a circle to its diameter.

6. If a, b, c be the sides subtending respectively the angles A, B, C of a plane triangle,

Prove:

(1) $\frac{\sin A}{\sin(A+B)} = \frac{a}{c}$.

(2) $a = (b-c) \sec \theta$, if $\tan \theta = \frac{2\sqrt{bc}}{b-c} \sin \frac{A}{2}$.

(3) $\tan^{-1} \frac{a}{b+c} + \tan^{-1} \frac{b}{a+c} = 45^\circ$ when C is a right angle.

7. If (r) be the radius of the circle inscribed in the triangle ABC, prove

(1) $r = \frac{2 \text{ area}}{a+b+c}$. (2) $r = \frac{b \cdot \sin \frac{A}{2} \cdot \sin \frac{C}{2}}{\cos \frac{B}{2}}$.

8. In a right-angled triangle ABC, C being the right angle, find AB, if $B = 30^\circ$ and $BC = 100$ feet.

$A'B'C'$ is also a right-angled triangle, C' the right angle and $B' = 30^\circ$, find $A'C'$ when the area of the triangle $A'B'C'$ is three times the area of ABC.

9. When two sides and an included angle of a triangle are given, investigate a formula for determining the other two angles. Shew that for this determination it is not necessary to know the absolute lengths of the two sides, provided their ratio is given.

Ex. The included angle is $70^\circ\ 30'$, the ratio of the containing sides is $5 : 3$, find the other angles.

Given $\log_{10} 2 = \cdot 3010300$,

$\text{Log} \cot 35^\circ\ 15' = 10 \cdot 1507464$,

$\text{Log} \tan 19^\circ\ 28'\ 50'' = 10 \cdot 5486864$.

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10. A tower stood at the foot of an inclined plane whose inclination to the horizon was 9°; a line was measured straight up the incline from the foot of the tower of 100 feet in length, and at the upper extremity of this line the tower subtended an angle of 54°; find the height of the tower.

Note.—For sin 54° see question 4.

11. Shew that the logarithms of proper fractions are negative. Express the true value of the logarithm of $\frac{1}{300}$ to the base 10. How would it be expressed with a negative characteristic?—Since sin 30° = $\frac{1}{2}$, explain why the logarithm of sin 30 is tabulated 9·6989700. Given that log tan 38° 16′ is tabulated 9·8969714, determine log cotan 38° 16′.

Given Log tan 38° 16′ 10″ = 9·8970147,
Log tan 38° 16′ 20″ = 9·8970580,
find the angle whose logarithmic tangent is 9·8970365.

Note.—$\text{Log}_{10}2$ is given in question 9.

FURTHER EXAMINATION.

V. PURE MATHEMATICS. (1.)

1. How do you measure the inclination of a straight line to a plane, and of a plane to a plane?

2. If two planes cut one another, their common section is a straight line.

3. Name and define the five regular solid figures; and shew that there cannot be more than five.

4. If two straight lines be cut by three parallel planes, they shall be cut in the same ratio.

5. Find the roots of the equation

$$x^3 + 3x + 36 = 0,$$

and shew that every equation of the form $x^3 + qx + r = 0$ has two impossible roots and one negative root.

6. Draw the straight lines

$$2x + 3y = 4,$$

and

$$3x - 2y = 5;$$

and prove that they are at right angles to each other.

7. Prove that the subnormal in the parabola is constant; and shew how to draw a normal to the curve at any given point.

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8. A quadrilateral is inscribed in a circle, one of its diagonals coinciding with the diameter of the circle; find in terms of its sides (1) the radius of the circle, (2) the other diagonal, (3) the area of the quadrilateral.

9. Draw the circle represented by the equation

$$x^2 = 2ay - y^2,$$

and transform the equation into polar co-ordinates.

10. Prove (1) geometrically, (2) analytically, that the perpendiculars dropped from the foci upon the tangent to any point of an ellipse intersect the tangent on the circumference of the circle described on the axis major as diameter.

11. Find the equation to the ellipse referred to a pair of conjugate diameters as axes, and shew that equal conjugate diameters are parallel to the lines joining the extremities of the major and minor axes.

12. Find the equation to the tangent to an hyperbola, and the locus of its intersection with the perpendicular upon it from the centre.

13. Prove the following trigonometrical formulæ (n being a positive integer):

$$(1)\quad \cos na = \cos^n a - \frac{n \cdot \overline{n-1}}{1 \cdot 2} \cos^{n-2} a \sin^2 a$$

$$+ \frac{n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3}}{1 \cdot 2 \cdot 3 \cdot 4} \cos^{n-4} a \sin^4 a - \&c.$$

$$(2)\quad \sin a + \sin 2a + \&c. + \sin na = \frac{\sin \frac{n+1}{2} a \sin \frac{na}{2}}{\sin \frac{a}{2}}.$$

14. Explain the method of computing the value of $\pi = 3{\cdot}1415926\ldots$, and calculate $e = 2{\cdot}7182818\ldots$

VI. PURE MATHEMATICS. (2.)

[N.B.—*Great importance will be attached to accuracy in numerical results.*]

1. Find the algebraical expression which, when divided by $x^2 + x - 1$, gives $x^3 - 3x^2 + 4x - 7$ for the quotient and $11x - 7$ for the remainder.

2. Prove that

$$x(y+z)^2 + y(z+x)^2 + z(x+y)^2 - 4xyz = (y+z)(z+x)(x+y),$$

and

$$\frac{1}{(y-z)^2} + \frac{1}{(z-x)^2} + \frac{1}{(x-y)^2} = \left(\frac{1}{y-z} + \frac{1}{z-x} + \frac{1}{x-y}\right)^2.$$

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3. Having given $\log_{10}2, = \cdot 301030$ and $\log_{10}7 = \cdot 845098$, find $\log_{10}14$, $\log_{10}98$, and $\log_{1000}\sqrt{\frac{8}{343}}$.

4. Find the sums of the series:

$$1+\frac{7}{6}+\frac{4}{3}+\frac{3}{2}+\frac{5}{3}+\frac{11}{6}+\ldots \text{ to } 25 \text{ terms},$$

and

$$1+\frac{1}{6}+\frac{1}{36}+\frac{1}{216}+\ldots \text{ to infinity}.$$

Prove that

$$1+\frac{1}{6}+\frac{1\cdot 3}{1\cdot 2}\cdot\frac{1}{6^2}+\frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\cdot\frac{1}{6^3}+\frac{1\cdot 3\cdot 5\cdot 7}{1\cdot 2\cdot 3\cdot 4}\cdot\frac{1}{6^4}+\ldots=\sqrt{\frac{3}{2}}.$$

5. On a certain road the number of telegraph posts per mile is such that if there were one less in each mile the interval between the posts would be increased by $2\frac{14}{15}$ yards. Find the number of the posts per mile.

6. Prove that

$$\cos 30^\circ+\cos 60^\circ+\cos 210^\circ+\cos 270^\circ=\tfrac{1}{2},$$

and that

$$\frac{\sin 3\theta+2\sin 5\theta+\sin 7\theta}{\sin 5\theta+2\sin 7\theta+\sin 9\theta}=\frac{\sin 5\theta}{\sin 7\theta}.$$

7. If $A+B+C=90^\circ$, prove that

$$\tan B\tan C+\tan C\tan A+\tan A\tan B=1.$$

Hence shew that

$$2(\tan^2 A+\tan^2 B+\tan^2 C)-2=(\tan B-\tan C)^2$$
$$+(\tan C-\tan A)^2+(\tan A-\tan B)^2,$$

and that the expression $\tan^2 A+\tan^2 B+\tan^2 C$ is never less than unity.

8. Expand each of the fractions

$$\frac{4x-1}{(1-x)(1+2x)}, \text{ and } \frac{1+x}{(1-x)(1+x^2)},$$

in a series of ascending powers of x, and find in each case the coefficient of x^n.

9. A length of 300 yards of paper, the thickness of which is the hundred and fiftieth part of an inch, is rolled up into a solid cylinder; find approximately the diameter of the cylinder.

10. Find the centre of a circle cutting off three equal chords from the sides of a triangle.

11. In any triangle the straight line bisecting an angle, and the straight line passing through the middle point of the opposite side, perpendicular to it, meet on the circumscribing circle.

12. Having given the base of a triangle, the vertical angle, and the ratio of the sides, construct the triangle.

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13. A man, walking along a straight road at the rate of three miles an hour, sees in front of him, at an elevation of 60°, in the vertical plane through his path, a balloon which is travelling horizontally in the same direction at the rate of six miles an hour; ten minutes after he observes that the elevation is 30°; prove that the height of the balloon above the road is $440\sqrt{3}$ yards.

14. A quadrilateral $ABCD$ can be inscribed in a circle; if E, F, G, H be the centres of the circles which circumscribe the triangles ABC, BCD, CDA, DAB, prove that the quadrilateral $EFGH$ is also inscribable in a circle.

[$\pi = 3{\cdot}14159$.]

VII. PURE MATHEMATICS. (3.)

1. Find the relation between the coefficients and roots of an equation.

One root of the equation

$$x^3 - 13x^2 + 15x + 189 = 0$$

exceeds another root by 2; solve the equation.

2. Prove that incommensurable roots enter by pairs into an equation with rational coefficients.

If one root of such a biquadratic equation be $\sqrt{a}+\sqrt{b}$, where $\sqrt{a}$ and $\sqrt{b}$ are dissimilar quadratic surds, prove that the other roots are

$$-\sqrt{a}+\sqrt{b},\ \sqrt{a}-\sqrt{b},\ -\sqrt{a}-\sqrt{b}.$$

3. Prove that an odd number of roots of the equation $f'(x)=0$ lies between every two adjacent real roots of the equation $f(x)=0$.

Prove that the equation $ay^2+y^3-bx^2=0$ has always one positive value of y for every value of x, and that it has two negative or two impossible values of y according as x is numerically greater than $\frac{2a}{3}\sqrt{\frac{a}{3b}}$.

4. Find the differential coefficient of x^n for all values of n. Differentiate

(1) $\dfrac{1-x}{\sqrt{1+x^2}}$. (2) $x^x \log_a x$. (3) $\sec^{-1}\sqrt{1-e^{ax}}$.

5. If $y = x \log \dfrac{x}{a+bx}$, prove that $x^3 \dfrac{d^2y}{dx^2} = \left(y - x\dfrac{dy}{dx}\right)^2$.

6. State, without proving, the method of finding the limiting value of $\dfrac{f(x)}{F(x)}$ for values of x which make $f(x)$ and $F(x)$ zero.

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Find $$lt_{\theta=0} \frac{\cot\theta \tan^{-1} m \tan\theta - m\cos^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}.$$

7. What is meant by the statement that a function of x is capable of expansion in positive integral powers of x? Prove that Maclaurin's theorem holds in all such cases. Find the first 5 terms in the expansion of $y = \log_e(1 + x\sin x)$.

8. Prove that if a is such a value of x as to make $f(x)$ a maximum or minimum, then $f'(x)$ must change sign when $x = a$.

Hence deduce a method for determining all such values of x.

Prove that $\sqrt[e]{e}$ is a maximum value of $\left(\frac{1}{x}\right)^x$.

If SP and SQ be two focal distances in an ellipse inclined to each other at the angle α, find the greatest and least values of the area of the triangle PSQ.

9. Find expressions for the perpendicular on the tangent and for the radius of curvature in polar curves.

In the curve $$r = 2a\cos^2\frac{\theta}{2},$$

prove that $$\rho = \frac{2r}{3}\sec\frac{\theta}{2}.$$

10. Find the asymptote of, and trace, the curve whose equation is

$$ay^2 - x^2y - x^3 = 0.$$

Also trace the curve $$r = a\frac{\theta - \alpha}{\theta + \alpha}.$$

11. In the curve

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}},$$

prove that if $x = a\cos^3\theta$, then $y = a\sin^3\theta$, and that the equation of the tangent at the point determined by θ is

$$y\cos\theta + x\sin\theta = a\sin\theta.$$

Find the locus of intersection of tangents at right angles to one another.

12. Find the following integrals:

$$(1)\quad \int\frac{dx}{x\sqrt{1+x^2}}. \qquad (2)\quad \int\cos^{-1}\theta\, d\theta.$$

$$(3)\quad \int\frac{dx}{(1+x)(1+x^2)}.$$

13. Find the area of the loop of the first of the two curves in Question 10.

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VIII. STATICS.

1. When any number of forces act upon a particle in one plane, explain generally how their resultant may be found by the application of the parallelogram of forces to these different forces in succession. If the forces are in equilibrium, what does the resultant become? Assuming the parallelogram of forces, prove the equilibrium property of the triangle of forces.

A picture of given weight hanging vertically against a smooth wall is supported by a string passing over a smooth peg driven into the wall; the ends of the string are fastened to two points in the upper rim of the frame which are equidistant from the centre of the rim, and the angle at the peg is 60^0; compare the tension in this case with what it will be when the string is shortened to two-thirds of its length.

2. *State* (without proof) the condition of equilibrium when two forces act on a straight lever in one plane to turn it round a fulcrum.

Assuming this property, prove the condition of equilibrium of a lever acted on by any number of forces at different arms in one plane to turn it round a fulcrum.

If any number of forces represented by the sides of a regular hexagon taken in order act along the sides to turn the hexagon round an axis perpendicular to its plane, shew that the moment of the forces is the same through whatever point within the hexagon the axis passes. Is this true if the hexagon is not regular?

3. If any system of forces acting in one plane on a rigid body have a single resultant, the moment of the resultant round any point in the plane is equal to the algebraical sum of the moments of the forces. If the forces not in equilibrium cannot be reduced to a single resultant, they will be equivalent to a single couple.

ABCD is a rectangle; *AB*, *BC*, adjacent sides, are 3 and 4 feet. Along *AB*, *BC*, *CD*, taken in order, forces of 30, 40, 30 lbs. act respectively; find their resultant.

4. Define the centre of gravity of a body. In what sense can a plane area be said to have a centre of gravity?

Given the centre of gravity of a given area, and also of a portion of it, find the centre of gravity of the remainder.

ABC is a triangle, *DE* a line drawn within the triangle, parallel to the base *BC* intersecting the other sides in *D* and *E*, *DE* and *BC* are equal to (b) and (a) respectively; if (h) be the line drawn from *A* bisecting *BC*,

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prove that the distance of the centre of gravity of the trapezoid $BCED$ from A is

$$\frac{2}{3}h \cdot \frac{a^2+ab+b^2}{a(a+b)}.$$

5. Describe any arrangement of three equal moveable pulleys by which a mechanical advantage is obtained, and in the arrangement described find the relation of the power to the weight when there is equilibrium, neglecting the weight of the pulleys. Find also the whole pressure sustained by the upper block.

6. Describe the common balance, and find its position of equilibrium when loaded with unequal weights. State the requisites of a good balance, and shew how its sensibility may be secured.

7. State the principle of virtual velocities as applicable to the simple mechanical powers. Shew that the principle holds good on a straight lever loaded with unequal weights. If the weights hang from the ends of the lever by strings, examine how the position of the centre of gravity of the weights is affected as the lever is turned round the fulcrum in a vertical plane.

8. Explain generally how a rough surface affects the direction of the resistance of a pressure applied to it. What is meant by the limiting angle of resistance of such a surface?

A weight (P) hangs over the top of a rough inclined plane by means of a string attached to a weight (W) on the plane, find the relation of (P) to (W) when (P) is just on the point of descending, and shew that if the inclined plane make an angle (2ϕ) with the *vertical*, (W) will just be moved when it is equal to (P), (ϕ) being the limiting angle of resistance.

9. A homogeneous sphere of given radius rests at the bottom of a hemispherical bowl of larger radius, examine the conditions of stable or unstable equilibrium, the surfaces being such as to prevent sliding.

If the sphere is so loaded that the height of its centre of gravity above the lowest point is $\frac{5}{4}$ of its radius, determine the radius of the hemisphere when the equilibrium is neutral.

10. How is work mathematically estimated? Distinguish between the useful work done by a machine and the work applied to it. How is the modulus of a machine expressed with reference to the useful work and the work applied?

When weights are raised through different heights, prove that the whole work expended is equal to the work that would be expended in lifting a weight equal to the sum of the weights through the same height as that to which the centre of gravity of the weights has been raised.

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A tank 24 feet long, 12 feet broad, 16 feet deep, is to be filled by water in a well, the surface of which is always at a depth of 80 feet below the bottom of the tank; determine the work done in filling the tank, and find the horse-power of a steam-engine whose modulus is ·5, that will fill the tank in 4 hours, the weight of a cubic foot of water being 62·5 pounds.

IX. DYNAMICS.

Candidates are not expected to answer the whole of this paper.

1. Define linear and angular velocity.

If a railway carriage be moving at the rate of 30 miles an hour, and the radius of one of its wheels be 2 feet, what is the angular velocity of the wheel if there be no sliding? What also is the relative velocity of the centre and highest point of the wheel?

Why are garden rollers generally divided into two parts?

2. Two particles are started simultaneously from the points A and B, 5 feet apart, one from A towards B with a velocity which would cause it to reach B in 3 seconds, and the other at right angles to the former and with $\frac{3}{4}$ths of its velocity.

Find their relative velocity in magnitude and direction, the shortest distance between them, and the time at which they are nearest to one another.

3. When a body is projected vertically upwards prove that the velocity is the same at the same points in the ascent and descent.

If the time of a body's fall from a certain height at one place on the earth's surface be m seconds less than that at another place, and the velocity acquired in the fall be a feet per second greater, prove that $\frac{a}{m}$ is the geometric mean of the accelerations of gravity at the two places.

4. The path of a projectile is a parabola.

If a gun be fired at the elevation θ, where θ is very small, so as to hit a mark at a distance a, and b be the range when fired at an elevation of $45°$, prove that θ is equal to $\frac{a}{2b}$ approximately.

5. Find the acceleration of a particle sliding down a smooth inclined plane. What is the vertical component of this acceleration?

If two particles P and Q start simultaneously from A, one sliding down the plane AB at the angle α to the horizon, and the other falling freely, prove that their relative vertical acceleration is $(g \cos^2 \alpha)$.

Hence prove that the line PQ is always perpendicular to AB.

6. Two unequal weights W and W' are connected by a string passing over a smooth pulley. Find the acceleration.

Hence explain the principle and use of Attwood's machine.

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7. If the weights in the last question be contained in scale-pans attached to the ends of the string, prove that if the weights of the pans be neglected, the pressure between each pan and the contained weight is equal to the tension of the string.

Find the pressures when the weight of each pan is w, and prove that they are to each other as $W : W'$.

8. A ball A impinges directly upon an exactly equal and similar ball B lying upon a smooth horizontal plane; if e be the coefficient of elasticity prove that after the impact A's velocity will be to B's velocity as $1+e : 1-e$.

9. If, after the impact described in the foregoing question, A moves on and impinges directly upon a perfectly elastic vertical wall from which it rebounds and meets B a second time, prove that after the second impact A will be reduced to rest.

Find the total loss of vis viva. What becomes of the kinetic energy which has thus disappeared?

10. Find the acceleration of a particle moving in a circle with uniform velocity. Two weights W and W' are placed on a smooth table, and connected together by a string passing through a small fixed ring on the table. If they are projected with the velocities v and v' at right angles to the string, find the ratio in which the string must be divided by the ring in order that both weights may describe circles round the ring as centre.

11. A string AP of length a is fastened to a point A and carries a weight P. If P be projected vertically upwards from the position in which AP is horizontal, find the least velocity of projection with which it may describe a circle round A, and state what happens if the velocity be less than this.

If P be projected vertically upwards when AP is below the horizontal line, and at an angle α to it, find the least value of v that P may ultimately describe a circle in this case.

12. If a particle move under the action of gravity in a smooth cycloidal groove whose axis is vertical and vertex downwards, prove that the acceleration varies as the distance from the vertex.

13. If the particle in the last question start from rest from an extremity of the base of the cycloid, and if T be the time in which it reaches the vertex, prove that at any time t, the height of the particle above the vertex is equal to its depth below the base at the time $T-t$.

14. A pendulum oscillating seconds at one place is carried to another place at which it loses 2 minutes a day. Compare the accelerations of gravity at the two places.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1880.

PRELIMINARY EXAMINATION.

I. EUCLID (Books I.—IV. and VI.). (Obligatory.)

1. Distinguish between *a postulate* and *an axiom*. In which propositions are the *2nd postulate* and the *12th axiom* of Euclid respectively used for the first time? What axioms are used in *Prop. IV., Book I.?*

2. The angles at the base of an isosceles triangle are equal to one another; and if the equal sides be produced, the angles on the other side of the base shall be equal to one another.

State *fully* the converse of this proposition.

3. If one side of a triangle be produced, the exterior angle shall be greater than either of the interior opposite angles.

Complete the figure which is necessary for the proof of the second part of this proposition.

How many of the exterior angles of any triangle *must* be obtuse?

4. Parallelograms upon the same base, and between the same parallels, are equal to one another.

Of all triangles that can be drawn upon a given base and between the same parallels, shew that an isosceles triangle has the least perimeter.

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5. Describe a square that shall be equal to a given rectilineal figure.

Divide a given straight line into two parts, such that the rectangle contained by them shall be equal to the square of their difference.

6. If two circles touch one another internally, the straight line which joins their centres, being produced, shall pass through the point of contact.

If two circles touch externally at E, and AB, CD be any two parallel diameters of the circles, shew that the straight lines AD, BC will pass through E.

7. If a straight line touch a circle, the straight line drawn from the centre to the point of contact shall be perpendicular to the line touching the circle.

8. The angles in the same segment of a circle are equal to one another.

If PQ, RS, two chords of a circle, intersect within the circle, shew that their inclination to one another is equal to one half of an angle at the centre of the circle standing upon an arc equal to the sum of the arcs PR and QS.

9. Inscribe an equilateral and equiangular pentagon in a given circle.

Give a *construction* (*without proof*) for the inscription of a regular twenty-sided figure in a given circle.

10. If the vertical angle of a triangle be bisected by a straight line which also cuts the base, the segments of the base shall have the same ratio which the other sides of the triangle have to one another.

Find a point C in ACB, a given segment of a circle, such that the straight line AC is double of the straight line CB.

11. Describe a rectilineal figure which shall be similar to one given rectilineal figure, and equal to another given rectilineal figure.

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II. ARITHMETIC. (Obligatory.)

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy in numerical results.*]

1. Add together $2\frac{3}{5}$ of $\frac{25}{28}$, $3\frac{3}{4}$ of $\frac{3}{5}$, $\frac{7}{11}$ of $3\frac{3}{10}$, and $\frac{1}{24}$ of $5\frac{1}{3}$.

2. Subtract $\frac{5\frac{1}{36}}{2\frac{1}{5}}$ from $4\frac{1}{8}$ of $2\frac{18}{121}$.

3. Multiply $3\frac{9}{11}$ of $\frac{3}{20}$ of $6\frac{1}{2}$ by $2\frac{3}{5}$ of $\frac{23}{36}$.

4. Divide $3\frac{5}{12}$ by $4\frac{11}{28}$.

5. Add together ·0021 of a cwt., ·045 of a quarter, ·37 of a lb., and subtract the sum from 35·263 ounces. Give the answer in ounces and the decimal of an ounce.

6. Multiply ·0057632 by 2·873.

7. Divide 64·01 by ·0346.

8. Divide 1·6023 by 294.

9. Express 1 furlong 4 poles as the decimal of a mile.

10. What is the difference between ·0057 of a lb. troy and $\frac{23}{25}$ of a dwt.? Express the answer as the fraction (vulgar) of a dwt.

11. Find, by Practice, the dividend on £3,245. 15*s*. at 13*s*. 9*d*. in the £.

12. Find the amount of £2,060 in 3 years at $4\frac{1}{2}$ per cent. simple interest, neglecting fractions of a penny.

13. Multiply by duodecimals 3 ft. 5 in. by 4 ft. 9 in., and the product by 8 ft. 7 in. What does the answer become when expressed in cubic feet, cubic inches, and the fraction of a cubic inch ?

14. *A*, *B*, and *C* working together can do a piece of work in 6 days. *A* could do it alone in 24 days. After working together for 2 days *A* is taken ill. How long will *B* and *C* take to finish it ?

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15. Find the value of $\frac{\cdot 384615}{2\cdot 142857}$ of £2. 2*s*. 3*d*.

16. Extract the square root of 1·07101801 and the cube root of 160103007.

17. Find the difference between the interest and the true discount on £264. 10*s*. for 3 years at 5 per cent. per annum simple interest.

18. *A* starts from London for Epsom, distant 14 miles, walking at the rate of 3 miles an hour. *B* starts from London on the same road $1\frac{3}{4}$ hours later, driving 8 miles an hour. At what distance from Epsom will *B* overtake *A* ?

19. 40 lbs. troy of standard gold can be coined into 1,869 sovereigns, the proportion of pure gold to alloy in standard gold being 22 to 2. What weight of pure gold is there in a sovereign?

20. The average temperature for Monday, Tuesday, and Wednesday was 53°. The average for Tuesday, Wednesday, and Thursday was 56°, that for Thursday being 60°. What was the temperature on Monday?

21. Find by logarithms, a fourth proportional to the 5th power of 11, the 4th power of 7, and the 5th power of 5.

22. Find log ·001155, and calculate to 6 places of decimals the value of

$$\frac{(330 \times \frac{1}{45})^4}{\sqrt{22 \times 70}}.$$

Logarithms required for Questions 21 *and* 22.

log 2 = ·3010300	log 17814 = 4·2507614
log 3 = ·4771213	log 17815 = 4·2507858
log 7 = ·8450980	log 46588 = 4·6682741
log 11 = 1·0413927	log 46589 = 4·6682834

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III. ALGEBRA. (Obligatory.)

[N.B.—*Great importance will be attached to accuracy in numerical results.*]

1. State and prove the rules for the removal of brackets from algebraical expressions.

Find the value of

$$\frac{4y}{5}(y-x)-35\left[\frac{3x-4y}{5}-\frac{1}{10}\left\{3x-\frac{5}{7}(7x-4y)\right\}\right],$$

when $x=-\frac{1}{2}$ and $y=2$.

2. Divide $2a^2x^2-2(3b-4c)(b-c)y^2+abxy$ by $ax+2(b-c)y$, and find the coefficients of x^2, x^3, and x^4 in $(x+a)^3\times(x-a)^5$.

3. Shew that the product of the greatest common measure and the least common multiple of two quantities is equal to the product of the quantities themselves.

Find the greatest common measure and least common multiple of

$$(2a^2+3y^2)x+(2x^2+3a^2)y, \text{ and}$$
$$(2x^2-3a^2)y+(2a^2-3y^2)x.$$

4. Simplify

$$\frac{1}{6m-2n}+\frac{1}{3m+2n}-\frac{3}{6m+2n}.$$

5. Divide

$$\frac{\left(\frac{3x+x^3}{1+3x^2}\right)^2-1}{\frac{3x^2-1}{x^3-3x}+1} \text{ by } \frac{\frac{9}{x^2}-\frac{33-x^2}{3x^2+1}}{\frac{3}{x^2}-\frac{2(x^2+3)}{(x^2-x)^2}}.$$

6. Multiply $\sqrt{2x}+\sqrt{2(2x-1)}-\frac{1}{\sqrt{2x}}$

by $\frac{1}{\sqrt{2x}}+\sqrt{2(2x-1)}-\sqrt{2x}$,

and find the expression whose square is

$$3x-1+2\sqrt{2x^2+x-6}.$$

7. Find the condition that the equation $ax^2+bx+c=0$ may have equal roots.

Also find the condition that the roots of $a^2x^2+b^2x+c^2=0$ may be the squares of the roots of $ax^2+bx+c=0$.

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8. Solve the equations:

(1) $16x^2+3p^2=16px.$

(2) $\left.\begin{array}{l}\dfrac{5x+6}{10}-\dfrac{11y-5}{21}=11\\[2ex] \dfrac{1}{25}(55y-12)=\dfrac{7x}{5}-37\end{array}\right\}.$

(3) $\dfrac{y}{x}-\dfrac{x}{y}=\dfrac{x+3}{y+3}=\dfrac{x+y}{xy}.$

9. Shew that each of the equal fractions

$$\frac{a_1}{b_1},\ \frac{a_2}{b_2},\ \frac{a_3}{b_3},\ldots\ldots\ldots\frac{a_n}{b_n}$$

is equal to

$$\frac{p_1a_1+p_2a_2+p_3a_3+\ldots\ldots\ldots+p_na_n}{p_1b_1+p_2b_2+p_3b_3+\ldots\ldots\ldots+p_nb_n}.$$

If $$\frac{bx-ay}{cy-az}=\frac{cx-az}{by-ax}=\frac{z+y}{x+z},$$

then will each of these fractions $=\dfrac{x}{y}$, unless $b+c=0$.

10. A carrier charges 3*d.* each for all parcels not exceeding a certain weight; and on heavier parcels he makes an additional charge for every 7 lbs. above that weight. The charge for half a cwt. is 1*s.* 3*d.*, and the charge for 9 stones is five times that for 1 qr. What is the scale of charges?

11. Investigate a formula for the sum of any number of quantities in Arithmetical progression.

Find the sum of

$$8,\ 7\tfrac{1}{5},\ 6\tfrac{2}{5}\ \ldots\ldots\ldots \text{ to } 19 \text{ terms.}$$

12. To each of three consecutive terms of a geometrical series the second of the three is added. Shew that the three resulting quantities are in Harmonical progression.

13. Find the number of permutations of n things taken all together which are not all different.

How many different permutations can be made of the letters of the word *essences*, and how many of these will begin with n and end with s?

14. Expand $(1-3x)^{\frac{2}{3}}$ to 5 terms, and write down the general term in its simplest form.

Also if a_r denote the coefficient of x^r in the expansion of $(1-x)^{2m-1}$, shew that $a_{r-1}+a_{2m-r}=0$.

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IV. PLANE TRIGONOMETRY AND LOGARITHMS.

(Obligatory.)

[N.B.—*Great importance will be attached to accuracy in working.*]

1. State briefly the advantages or disadvantages of the English or French methods, respectively, of measuring angles. If an angle be expressed in French minutes, shew that it will be transferred to English minutes when multiplied by ·54. Verify this by first expressing an angle of an equilateral triangle in French measure. Divide the English angle $77°$ into two parts, so that the number of English seconds in one part may equal the number of French seconds in the other part.

2. Define the sine and cosine of an angle. Prove that the sine of any angle is the cosine of its complement. If A be greater than two and less than three right angles, represent geometrically the complement of $(A - 180°)$. If (n) be a positive whole number, shew that the angles $(2n \,.\, 180° + A)$ and $\{(2n+1)\,180° - A\}$ have the same sine as A. Given the sine of an angle, find the tangent. Prove $\tan 60° = \sqrt{3}$.

3. Without assuming the formula for $\sin(A+B)$, prove geometrically $\sin 2A = 2 \sin A \,.\, \cos A$, A being less than $45°$. Obtain the equation

$$\tan 2A = \frac{2 \tan A}{1 - (\tan A)^2},$$

and hence prove
$$\tan\left(\frac{45°}{2}\right) = \sqrt{2} - 1,$$

4. Express $\sin A$ in terms of $\sin 2A$. In the general expression how many values of $\sin A$ are thus obtained? Shew in any particular case how the correct value is to be selected. Assuming the equation

$$\cos 3A = 4(\cos A)^3 - 3 \cos A,$$

find $\sin 18°$, and hence exhibit the true value of $\sin 9°$ without reducing the surds.

5. Prove

(1) $(\cos A)^4 - (\sin A)^4 = \cos 2A$.

(2) $\sin A + \sin(72° + A) + \sin(36° - A) = \sin(72° - A) + \sin(36° + A)$.

(3) $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2} = 120°$.

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6. Given the three sides of a triangle, find the cosine of one of the angles, and hence express the sine of half that angle in terms of the sides and in a form adapted to logarithmic computation. If a, b, c be the sides subtending the angles A, B, C respectively of the triangle ABC,

prove
$$c = \frac{(a-b)\cos\frac{C}{2}}{\sin\frac{A-B}{2}}.$$

7. Find the radius of the escribed circle of the triangle ABC, when the circle touches the side BC and the sides AB and AC produced.

8. In a triangle ABC, given $BC = 2AC$, $A = 3B$, find the angles of the triangle and the ratio of AB to BC and AC.

9. State the property which furnishes the principle for constructing the proportional parts in the ordinary tables of logarithms.

Given $\log_{10} 6{\cdot}0389 = {\cdot}7809578$, $\log_{10} 6{\cdot}0390 = {\cdot}7809650$, calculate the corresponding proportional parts, and find the number of which ${\cdot}7809601$ is the logarithm.

Given $\log_{10} 2 = {\cdot}3010300$, find $\log_{10} {\cdot}250$ and the tabulated logarithms of $\sin 30^\circ$ and $\operatorname{cosec} 30^\circ$.

10. Explain when the solution of a triangle is said to be ambiguous.

Supposing the data for the solution of a triangle to be as in the two following cases (α), (β), point out in each case whether the solution will be ambiguous or unambiguous.

Find the angle C in the case not ambiguous, and the third side of the obtuse angle in the ambiguous case.

(α) $A = 30^\circ$, $AB = 250$ feet, $BC = 125$ feet.

(β) $A = 30^\circ$, $AB = 250$ feet, $BC = 200$ feet.

Log $\sin 38^\circ\ 41' = 9{\cdot}7958800$.

Log $\sin\ 8^\circ\ 41' = 9{\cdot}1789001$.

Note.—The logarithms of Question 9 to be used, when required.

11. A church tower BCD with a spire above it stands on a horizontal plane, B being a point in its base, and BC being 9 feet vertically above B. The height of the tower is 289 feet and of the spire 35 feet; from the extremity A of a horizontal line BA, it is found that the angle subtended by the spire is equal to the angle subtended by BC; prove that $BA = 180$ feet nearly.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

($\pi = 3{\cdot}1416$.)

1. Draw a straight line perpendicular to a given plane from a given point without it.

2. The semi-vertical angle of a conical box is $30°$, and the length of its slant side is 10 inches. Find the volume of the largest sphere which can be placed inside it.

3. If a solid angle be formed by three plane angles, any two of them are together greater than the third.

4. Shew how to find the area of the portion of a spherical surface included between two small parallel circles.

5. Find the length of the perpendicular from the point (4, 5) on the straight line $3x+4y=10$.

6. Find the area of the triangle included between the three straight lines

$$x+y=5, \qquad 2x-3y+5=0, \qquad x+6y=5$$

and find the radius of the circle inscribed in the triangle.

7. Find the radius and co-ordinates of the centre of the circle represented by the equation.

$$x^2+y^2+6x-4y=12.$$

and find the length of a tangent drawn to this circle from the point $(-3, -5)$.

8. Shew that if two tangents be drawn to a parabola from an external point they will subtend equal angles at the focus.

9. Shew that the locus of points from which the tangents drawn to an ellipse are at right angles to each other is a fixed circle.

10. Shew that the tangent at any point of a hyperbola makes with the asymptotes a triangle of constant area, and that the part intercepted on the tangent is equal to the diameter conjugate to that passing through the point of contact.

11. Find the equation to the normal at any point of an ellipse in terms of the tangent of the angle which it makes with the major axis.

12. Assuming Demoivre's Theorem, prove that

$$\cos\alpha = 1 - \frac{\alpha^2}{1\,.\,2} + \frac{\alpha^4}{\underline{|4}} - \frac{\alpha^6}{\underline{|6}} + \ldots\ldots$$

$$\sin\alpha = \alpha - \frac{\alpha^2}{\underline{|3}} + \frac{\alpha^5}{\underline{|5}} - \frac{\alpha^7}{\underline{|7}} + \ldots\ldots$$

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13. Find an expression for $\tan^{-1}x$ in a series of ascending powers of x; and hence shew how to approximate to a value of π.

VI. PURE MATHEMATICS (2).

[*Full marks may be obtained without doing the whole of this paper. Great importance will be attached to accuracy in numerical results.*]

1. If
$$x=a+b+\frac{(a-b)^2}{4(a+b)},$$
and
$$y=\frac{a+b}{4}+\frac{ab}{a+b},$$
shew that
$$(x-a)^2-(y-b)^2=b^2.$$

2. Shew how to rationalize an expression of the form
$$\sqrt[p]{a}+\sqrt[q]{b}.$$
Simplify
$$\frac{(\sqrt{2}+\sqrt{3})(\sqrt{3}+\sqrt{5})(\sqrt{5}+\sqrt{2})}{(\sqrt{2}+\sqrt{3}+\sqrt{5})^2}.$$

3. Between two quantities A and B a harmonic mean H is inserted. Between A and H and between H and B geometric means G_1 and G_2 are inserted, and it is found that G_1, H, G_2 are in A.P. Find the ratio of A to B.

4. The duration of a railway journey varies directly as the distance and inversely as the velocity. The velocity varies directly as the square root of the quantity of coal used per mile and inversely as the number of carriages in the train. In a journey of 25 miles in half an hour with 18 carriages 10 cwt. of coal is required. How much coal will be consumed in a journey of 15 miles in 20 minutes with 20 carriages?

5. The difference of two numbers that are expressed by the same digits is 35453221; in what scale are the numbers expressed?

6. The base of a right circular cone is 6 inches in diameter and its height is 8 inches. Find the area of the conical surface. Find also the weight of the cone (to 5 places of decimals), supposing it to be constructed of cast iron, of which one cubic foot weighs 440 lbs.

7. If the radius of the circumscribed circle of a triangle be equal to the diameter of the inscribed circle, shew that the triangle is equilateral.

8. If any hexagon, inscribed in a circle, has two opposite angles equal, shew that it will have two opposite sides parallel.

9. A sphere whose radius is 2 inches rests on three wires in the form of a plane triangle, the sides of which are 3, 4, and 5 inches respectively. What will be the height of the top of the sphere above the plane of the wire?

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10. Find the sine, cosine, and tangent of the dihedral angle between two faces of a regular tetrahedron; also of half the angle between two adjacent faces of a regular octohedron.

11. A circle is described touching the side BC of a triangle ABC, and the two other sides AB and AC, produced. Shew that the same circle will be the inscribed circle of a triangle $AB'C'$ similar to ABC, formed by producing the sides AB and AC, and that the sides of the triangle $AB'C'$ will be

$$\frac{sa}{s-a}, \quad \frac{sb}{s-a}, \quad \frac{sc}{s-a}$$

respectively, where $$s=\frac{a+b+c}{2}.$$

12. In a triangle the least angle is $45°$, and the tangents of the angles are in A.P. If the area be 3 square yards, find the lengths of the sides in feet, and express the other angles in the form $\tan^{-1} x$.

13. Shew that

$$\frac{n}{1}\cdot 2\cdot 1+\frac{n(n-1)}{\lfloor 2}\cdot 4\cdot 3+\frac{n(n-1)(n-2)}{\lfloor 3}\cdot 6\cdot 5+\ldots$$
$$+\frac{n}{1}(2n-2)(2n-3)+2n(2n-1)=2^n n^2.$$

(N.B. $\pi=3{\cdot}14159$.)

VII. PURE MATHEMATICS (3).

1. If two values of x substituted in $f(x)$ give results with opposite signs, prove that at least one root of the equation $f(x)=0$ must lie between them. Prove that if a and b be positive numbers the equation

$$(x-a)^2(x+b)-x^2=0$$

has one negative and two positive roots.

2. Prove that impossible roots enter an equation with possible coefficients in pairs.

If one root of an equation be $\dfrac{1}{a+b\sqrt{-1}}$, prove that $\dfrac{1}{a-b\sqrt{-1}}$ will also be a root, if a and b be possible.

3. A real root of the equation $f'(x)=0$ lies between every adjacent two real roots of $f(x)=0$.

Prove that the equation

$$2x^3+3ax^2+6a^2x+b=0$$

has only one real root.

4. Find from first principles the differential coefficient of a^x and of $\operatorname{cosec}^{-1} x$.

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5. Find $\frac{dy}{dx}$ in the following cases:

$$(1)\quad y=\frac{1-x^2}{\sqrt{1+x^2}}. \quad (2)\quad y=\cos^{-1}(1-2x^2).$$

$$(3)\quad x=e^{\tan^{-1}\frac{y-x^2}{x^2}}.$$

6. In case (3) of last question find $\frac{d^2y}{dx^2}$, and prove that

$$x^4\frac{d^2y}{dx^2}-(x^3+2xy)\frac{dy}{dx}+4y^2=0.$$

7. State the different indeterminate forms which functions may assume. Find the value of $lt_{x=0}\frac{\log(1-x^2)}{\log\cos x}$.

8. State fully, *without proof*, the conditions that $f(x)$ shall be a maximum or minimum when $x=a$.

Find the point P on the parabola $y^2=4ax$, such that the perpendicular on the tangent at P from the point on the axis distant h from the vertex may be the least possible. What is the geometrical meaning of the result?

9. If ψ be the angle between the radius vector and tangent at any point of a curve, prove that

$$\cos\psi=\frac{dr}{ds},\quad \sin\psi=r\frac{d\theta}{ds},$$

the symbols having their ordinary meaning.

Prove that $\sin^2\psi\frac{d\psi}{d\theta}+r\frac{d^2r}{ds^2}=0.$

10. Trace the curve $\quad ay^2-2axy+x^3=0,$
and find the asymptotes of the curve

$$xy^3-x^3y-ay^3-2ax^3=0.$$

11. Shew how to find the envelope of a series of curves having one arbitrary parameter.

A series of circles have their centres on a given straight line, and their radii proportional to the distances of the corresponding centres from a given point in that line. Find the envelope.

12. Integrate the following differentials:

$$(1)\quad \sqrt{ax-x^2}dx. \qquad (2)\quad \sec^3 x dx.$$

$$(3)\quad \frac{dx}{(x^2-2ax+a^2+c^2)(x-b)}.$$

13. Find the whole area between the axis of y, the positive axis of x, and the arc of the curve whose equation is

$$y=\frac{a^4}{(a^2+x^2)^{\frac{3}{2}}}.$$

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VIII. STATICS.

1. The resultant of two equal forces acting at the same point of a body bisects the angle between them. Assuming this, without proving it, shew that if two unequal forces acting at a point are represented in magnitude and direction by two straight lines drawn from that point, their resultant is represented in direction by the diagonal of the parallelogram described on those two straight lines as sides.

If the two forces do not meet at a point, shew how their effect on the body may be determined.

2. If a body is free to rotate about a certain fixed axis, what will be the effect of a force acting on that body, but not intersecting the axis? Shew in what case the effect is nil.

An uniform rod, 4 inches in length, is placed with one end inside a smooth hemispherical bowl of which the axis is vertical and the radius $\sqrt{3}$ inches long. Shew that one fourth of the rod will project over the rim of the bowl.

3. Shew that if a body be suspended by a single point, its centre of gravity must always lie in the same vertical line with that point. Distinguish the cases when the centre of gravity is above and below the point of suspension.

4. Having given the position of the centre of gravity of a body and that of a part of it, find the position of that of the remainder of the body.

From an uniform right cone whose vertical angle is 60° is cut out the largest possible sphere. Find the centre of gravity of the remaining portion of the cone.

5. Find the magnitude and direction of the resultant of two parallel and unequal forces, whose directions are opposite.

A heavy uniform beam, whose weight is 40 lbs., is suspended in a horizontal position by two vertical strings, each of which can sustain a tension of 35 lbs. How far from the centre of the beam must a weight of 20 lbs. be placed, so that one of the strings may just break?

6. Explain what is meant by friction, the coefficient of friction, and the limiting angle of friction.

An uniform beam rests with one end on the ground, and the other against a vertical wall. If μ, the coefficient of friction of the ground, be the same as that of the wall, shew that $2\tan^{-1}\mu$ is the inclination of the ladder to the vertical, when it is on the point of slipping.

7. Explain how the use of the inclined plane may facilitate mechanical operations.

A number of loaded trucks, each containing one ton, on one part of a tramway, whose inclination to the horizon is α, supports an equal number

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of empty trucks, on another part whose inclination is β. Find the weight of a truck.

8. Determine the relation between the power and the weight in that system of pulleys in which each of the strings, all of which are supposed parallel, is attached to the weight, taking into account the weight of the pulleys.

If the pulleys be equal, and the power equal to the weight of one of them, and the number of pulleys be five, prove that the ratio of the weight to the power is 57 : 1.

9. Determine the relation between the power applied and the weight supported in the case of the screw.

Find the *mechanical advantage* of a screw whose diameter is 6·78 inches, and the distance between whose successive threads is ·71 inch

$$\left(\pi = \frac{355}{113}\right).$$

10. What are the three different forms of levers? Shew how they may be reduced to one general case.

Two equal uniform beams AB, BC are freely jointed at B, and A is fixed to a hinge at a point in a wall about which AB can revolve in a vertical plane. At what point in BC must you apply a vertical force to keep the two beams in one horizontal line? Find the value of the force.

11. If a force moves its point of application through a certain space, find how much work is done. What is the difference between work done *by* a force and *against* a force, and how is it expressed mathematically?

A capstan, whose diameter is 20 inches, is worked by a lever which measures 5 feet from the axis of the capstan. Find in foot-pounds the amount of work done in drawing up by a rope one ton over 35 feet of the surface of a smooth plane inclined to the horizon at an angle $\cos^{-1}\frac{4}{5}$. The rope may be supposed to be always parallel to the surface of the plane. Find also the force applied to the end of the lever, and the distance through which its point of application moves.

IX. DYNAMICS.

(Note.—*When needed the measure of gravity may be taken as* 32 *feet.*)

1. Obtain a formula that expresses the relation between space, time, and velocity when the motion is uniform.

A train moving uniformly describes 88 yards in 3 seconds; find its rate per hour in miles. In what time will it travel 600 miles with a stoppage of 5 minutes after every 100 miles?

Supposing the circumference of the earth at the equator to be 25,000 miles, and the time of the earth's rotation to be 24 hours, find the velocity

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in space, in miles per hour, of a cannon ball when it is fired at the equator with a velocity of 1,650 feet per second (1) in the direction of the earth's rotation, (2) in the opposite direction.

2. If a body's velocity is observed to increase or decrease, what must be the condition of increase or decrease when the force acting on the body is uniform? A body is set in motion in a right line; at the end of 3 seconds it is moving at the rate of 224 feet per second, at the end of 4 seconds at the rate of 256 feet per second, at the end of 5 seconds at the rate of 288 feet per second; what is the measure of the force acting on the body?

3. If a body acted on by gravity is projected upwards with a velocity V, find its velocity at the end of (t) seconds, and shew that it will just ascend to a height h, when $h=\frac{V^2}{2g}$.

A body so projected from the ground descends to the ground again and rebounds continually, losing at each rebound half the velocity with which it strikes the ground; what is the whole space described by the body?

4. Shew that velocities may be resolved and compounded analogously to the resolution and composition of forces. If two sides AC, CB of a triangle represent in magnitude and direction the component velocities of a particle, prove that the third side AB will represent the actual velocity of the particle, in magnitude and direction.

The velocity of a ship in a straight course on an even keel is $8\frac{1}{2}$ miles an hour; a ball is bowled across the deck, perpendicular to the ship's length, with a uniform velocity of 3 yards in a second; describe the true path of the ball in space, and shew that it will pass over 45 feet in 3 seconds nearly.

5. Assuming that the path of a projectile in vacuo is a parabola, shew that the velocity at any point is that which would be acquired in falling freely from the directrix to that point. If y be the vertical height of the projectile above the horizontal plane when it is moving with a velocity (v), and V be the velocity of projection, prove $v^2=V^2-2gy$.

6. When a projectile strikes a given mark, find the direction in which it is moving at the instant of impact.

If (A) be the point of projection, (α) the angle of projection, PN and P_1N_1 equal perpendiculars from the curve to the horizontal plane passing through A, (β) and (β_1) the angles which PN and P_1N_1 respectively subtend at A, prove $\tan\beta+\tan\beta_1=\tan\alpha$.

7. A weight (W) is drawn up a smooth inclined plane by a weight P hanging over the top of the plane and attached to (W) by an inextensible string; find the accelerating force and the tension of the string.

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Three weights (w) are fastened to a string whose length (l) is equal to that of an inclined plane; one weight is attached to each end, and the other weight to the middle of the string; when one weight hangs over the top of the plane the weights are in equilibrium; if the second weight also is just made to hang vertically, find the velocity with which the third weight reaches the top of the plane.

8. Prove that the times of descent down all the chords drawn from the highest point of a given vertical circle are the same; express the time of descent for such a circle in an invariable form.

A particle P descends from the highest point down the chord which is the side of a regular hexagon inscribed in the circle, and Q down the vertical diameter; if $P=2Q$, shew that their common centre of gravity will descend along the chord which is the side of an equilateral triangle inscribed in the circle, assuming that the path of the centre of gravity is a straight line.

9. If a smooth sphere impinge directly upon another sphere, explain the mechanical action that takes place during the impact.

When an elastic sphere (m) impinges on another elastic sphere (m_1) at rest, find generally the velocity of each after impact; if, after the impact, (m) remain at rest and (m_1) move on with one eighth of the velocity with which it is struck, find the elasticity and the ratio of the radii of the two spheres, supposed of the same material.

10. How is the resistance of an inclined plane modified by friction? If (ϕ) be the limiting angle of resistance on a rough inclined plane whose inclination is (α), shew that the accelerating force down the plane is

$$\frac{g \sin(\alpha - \phi)}{\cos \phi}.$$

11. When a given mass which is made fast to the end of an inextensible string revolves uniformly round the other end of the string, which is fixed, find the tension of the string.

If the weight of the body is given in pounds, how is the mass expressed?

A string 5 feet long can just sustain a weight of 20 pounds; if the revolving weight be 5 pounds, determine the greatest number of complete revolutions that can be made in one minute by the string without breaking.

12. Find the time of an oscillation of a heavy particle moving down the arc of a cycloid. Derive from this the time of an oscillation of a pendulum in a *small* circular arc.

A pendulum whose length is l makes (m) oscillations in 24 hours; when its length is slightly changed it makes ($m+n$) oscillations in 24 hours; shew that the pendulum has been diminished in length by a part equal to $\frac{2n}{m} l$ nearly.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1885.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

1. Define a plane angle, a scalene triangle, a rectangle, the angle of a segment of a circle. What is meant by a parallelogram being "applied" to a straight line? Give an instance from Euclid.

2. When is one straight line said to be at right angles to another straight line? Draw a straight line perpendicular to a given straight line of unlimited length from a given point without it. Why is the given straight line described as of unlimited length?

3. The straight lines which join the extremities of two equal and parallel straight lines towards the same parts are themselves equal and parallel.

Prove that a quadrilateral, which has two opposite sides and two opposite obtuse angles equal, is a parallelogram.

4. In any right-angled triangle the square which is described on the side subtending the right angle is equal to the squares described on the sides which contain the right angle.

If BC is the hypotenuse of a right-angled triangle ABC, and BE is the square described on BC, and AK on AC, shew that AE and BK intersect at right angles to one another.

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5. If a straight line be divided into two equal and also two unequal parts, the rectangle contained by the unequal parts together with the square on the line between the points of section is equal to the square on half the line.

If of the two unequal parts one is three times the other, shew by means of the above proposition that the rectangle contained by the unequal parts is three-fourths of the square on half the line.

6. Prove that the straight line drawn at right angles to the diameter of a circle from the extremity of it falls without the circle.

If the centre of a circle be joined to the vertices of a circumscribing quadrilateral, then any one of the four angles at the centre is supplementary to that one which is not adjacent to it.

7. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

If from any point on the circumference of a circumscribing circle perpendiculars be drawn to the sides of the inscribed triangle (produced if necessary), prove that their feet will lie in the same straight line.

8. About a given circle describe a triangle equiangular to a given triangle.

Having given the vertical angle of a triangle, and the segments into which the base is divided by the inscribed circle, construct the triangle.

9. Describe an isosceles triangle, having each of the angles at the base double of the third angle.

Shew how it is possible to describe within the smaller circle used in the construction a triangle equal in all respects to the required triangle.

10. Define similar rectilineal figures; and shew the necessity for each part of the definition.

If two similar triangles ABC, $A'B'C'$ are so placed that their homologous sides are parallel, shew that AA', BB', CC' will meet in a point, unless the triangles are equal, in which case these lines will be parallel.

11. If the vertical angle of a triangle be bisected by a straight line, which likewise cuts the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square on the straight line which bisects the angle.

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II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy in numerical results.*]

1. Find the value of

$$\frac{3\frac{7}{16} \text{ of } 2\frac{10}{11}}{\frac{4}{13} \text{ of } 8\frac{2}{3}} + \left(1\tfrac{3}{13} \text{ of } \tfrac{5}{12}\right) + \tfrac{63}{104} + \tfrac{1}{8}\left(\tfrac{3}{4} + \tfrac{1}{26}\right).$$

2. Subtract $1\frac{1}{8}$ of $\frac{6\frac{2}{3}}{1\frac{7}{9}}$ from $\frac{1}{8}$ of $13\frac{1}{2}$ of $\frac{8}{9}$ of 4.

3. Multiply together $\frac{3\frac{3}{20}}{\frac{11}{14}}$, $\frac{55}{78}$ of $2\frac{1}{7}$, $1\frac{1}{8}$ of $\frac{39}{70}$, and $\frac{10}{27}$.

4. Divide $4\frac{4}{5}$ by $\frac{4}{9}$ of $2\frac{2}{5}$ and the result by $5\frac{2}{5}$.

5. Add 3·26 quarts to the difference between ·302 of a bushel and ·9273 of a peck, and give the answer in pints and the decimals of a pint.

6. Multiply 73·0214 by ·05031.

7. Divide ·03081 by 237.

8. Divide 51·2705 by ·0205.

9. Express 3 ft. $5\frac{1}{4}$ ins. as the decimal fraction of 5 poles.

10. Reduce ·9043 of 3 cwt. 3 qrs. 9 lbs. 2 oz. to ozs. and the decimals of an oz.

11. Divide 76 lbs. 8 oz. 1 dwt. 6 grs. by 63.

12. There is a well containing 750 gals. of water; two pumps raising 20 and 30 gals. per minute respectively are employed to empty it; while it is constantly supplied by a spring which can refill it in half an hour. The two pumps work together for 15 minutes, when that of larger capacity ceases work for 10 minutes; the two pumps then work together until the well is empty. How long will each pump have been employed?

13. On £2,340 a profit of 2*s.* $7\frac{1}{2}$*d.* in the £ is made. Find (by Practice) the total profit.

14. What sum will amount to £3,995. 4*s.* in three years at $4\frac{1}{2}$ per cent.?

15. If 4 per cent. be lost by selling silk at 10*s.* per yard, at what price per yard should it be sold in order to gain 5 per cent.?

16. Find the square root of 1532·7225, and the cube root of 34 correct to three places of decimals.

17. Define Greatest Common Measure and Least Common Multiple. Resolve 2310, 6552, and 12165 into their prime factors, and thence deduce their L.C.M.

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18. Calculate (by duodecimals) the cubic content of a rectangular block of marble 7 ft. 5 ins. 7 pts. long, 4 ft. 2 ins. wide, and 3 ft. 4 ins. 7 pts. high.

What is the result when expressed in cubic feet, cubic inches, and the decimal of a cubic inch?

19. If a mile be equivalent to 1,600 metres find the number of square metres in $7\frac{1}{2}$ acres.

20. Three railway tickets, a 1st, a 2nd, and half a third class were purchased for 16*s.* $10\frac{1}{2}$*d.* The 1st class ticket cost $1\frac{2}{3}$ times as much as the 2nd, and the 2nd class $1\frac{1}{2}$ times as much as a whole 3rd class ticket. The distance travelled was 45 miles. Find the cost of each ticket and the rate per mile for each class.

21. Eight bells, which toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively, begin tolling all simultaneously with the clock striking; how many hours must elapse before they all toll simultaneously again with the clock striking?—The clock is supposed to strike at the hour only.

22. Find log 000·77, $\log\left(\frac{192\cdot5}{8\cdot4}\right)^2$, and the number whose log is $\bar{2}\cdot7286403$.

23. Find by logarithms the amount of £5,500 in 15 years at 5 per cent. per annum compound interest, giving the result in pounds and the decimal of a pound.

Logarithms required.

log 2 = ·3010300	log 535·35 = 2·7286378
log 3 = ·4771213	log ·053536 = $\bar{2}$·7286459
log 7 = ·8450980	log 1·1434 = ·0581982
log 11 = 1·0413927	log 1·1435 = ·0582362

III. ALGEBRA.

(Including equations, progressions, permutations and combinations, and the binomial theorem.)

[N.B.—*Great importance will be attached to accuracy in numerical results.*]

1. Shew directly from the meanings of the symbols employed that

$$a+(b-c)=a+b-c,$$

and

$$a-(b-c)=a-b+c.$$

Reduce to the simplest possible form

$$4\left\{a-\tfrac{3}{2}\left(b-\frac{4c}{3}\right)\right\}\ \left\{\tfrac{1}{2}(2a-b)+2(b-c)\right\}.$$

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2. Divide

$$a-x+4a^{\frac{1}{4}}x^{\frac{3}{4}}-4a^{\frac{1}{2}}x^{\frac{1}{2}} \text{ by } a^{\frac{1}{2}}+2a^{\frac{1}{4}}x^{\frac{1}{4}}-x^{\frac{1}{2}},$$

and find the value of $27x+48x^2-8x^4$, when

$$x=\tfrac{1}{4}(\sqrt{21}-3).$$

3. Find the Greatest Common Measure of

$$x^5-x^3-4x^2-3x-2,\ 5x^4-3x^2-8x-3;$$

and the Least Common Multiple of

$$8a^3-18ab^2,\ 8a^3+8a^2b-6ab^2,\ 8a^2-2ab-3b^2.$$

4. Shew that

$$\frac{(a+b)^3-c^3}{a+b-c}+\frac{(b+c)^3-a^3}{b+c-a}+\frac{(c+a)^3-b^3}{c+a-b}$$

is equal to $$2(a+b+c)^2+a^2+b^2+c^2.$$

5. Simplify the expressions

(1) $$\frac{1}{3(3x+2)}+\frac{1}{3x-2}-\frac{4}{3(3x-1)}.$$

(2) $$\frac{x-2}{x-2-\dfrac{x}{x-\dfrac{x-1}{x-2}}}.$$

6. Find the square root of $57-12\sqrt{15}$, and the cube root of

$$1-6a+21a^2-44a^3+63a^4-54a^5+27a^6.$$

7. Solve the equations:

(1) $$\frac{x-b}{x-a}-\frac{x-a}{x-b}=\frac{2(a-b)}{x-(a+b)}.$$

(2) $$\left.\begin{array}{l}2x+3y=1\tfrac{1}{2}\\ 4x^2+9xy+9y^2=11\end{array}\right\}.$$

8. If $a:b::c:d$, shew that

$$2a+3b:2c+3d::3a-4b:3c-4d.$$

9. Shew that p is the sum and q the product of the roots of the equation

$$x^2-px+q=0.$$

Form an equation whose roots will be the Arithmetic and Harmonic means between the roots of this equation.

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10. Insert three Geometrical means between 12 and $\frac{3}{4}$.

Shew that the product of any odd number of consecutive terms of a Geometrical series will be equal to the nth power of the middle term, n being the number of the terms.

11. Explain what is meant by one quantity varying jointly as two others.

The pressure of wind on a plane surface varies jointly as the area of the surface, and the square of the wind's velocity. The pressure on a square foot is 1 lb. when the wind is moving at the rate of 15 miles per hour. Find the velocity of the wind when the pressure on a square yard is 16 lbs.

12. Find the number of permutations of n things taken r together.

In how many different ways can four Englishmen and four Americans be arranged at a round table so that no two of the same nation shall sit together?

13. Shew that, in any expanded binomial, terms equidistant from the beginning and the end will have the same coefficient.

Write down the two middle terms in the expansion of $(a-x)^{15}$, and shew that only two terms in the expansion of $(1-x)^{\frac{12}{5}}$ have a positive sign.

IV. PLANE TRIGONOMETRY.

[N.B.—*Great importance will be attached to accuracy in working.*]

1. Assuming that the ratio of the circumference of a circle to the diameter is 3·1416, find to four places of decimals the value in degrees of the ordinary unit of circular measure.

If the radius of a circle is 4,000 miles, determine the number of miles in an arc which subtends an angle at its centre whose circular measure is $\frac{\pi}{6}$.

2. Give a definition of the sine of an angle which will apply to angles of any magnitude. Carefully prove that, whatever be the magnitude of A,

$$\sin(90^\circ + A) = \cos A,$$
$$\text{and } \cos(90^\circ + A) = -\sin A.$$

3. If A and B be each less than 90°, but their sum greater, draw the appropriate figure, and give a geometrical proof that

$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

Deduce the expression for $\sin(A+B)$, and shew that

$$\cos 4A = 8\cos^4 A - 8\cos^2 A + 1.$$

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4. Prove that

$$(1)\quad \tan A = \frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A}.$$

$$(2)\quad \frac{\cos(A + 45^\circ)}{\cos(A - 45^\circ)} = \sec 2A - \tan 2A.$$

$$(3)\quad \cot^{-1}\tfrac{1}{3} = \cot^{-1} 3 + \cot^{-1}\tfrac{8}{4}.$$

5. Find $\sin 150^\circ$, $\sin 225^\circ$, $\sin 15^\circ$, $\sin 37^\circ\ 30'$ without reducing quadratic surds.

6. Obtain an expression for all the angles whose cosine is equal to a given value.

If $3\cos^2\theta + 2\sqrt{3}\cos\theta = 5\frac{1}{4}$, find the general value of θ. Explain why it is that exactly the same series of angles are given by the two equations—

$$\theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{6}; \text{ and } \theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{3}.$$

7. Find expressions for the radii of the inscribed and escribed circles of a triangle, each expression involving one side and functions of the halves of the angles of the triangle.

If r, r_1, r_2, r_3 be the radii, and a, b, c the sides, prove that

$$\frac{r_1 - r}{a} + \frac{r_2 - r}{b} = \frac{c}{r_3}.$$

8. Shew that in any triangle $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$.

Obtain an expression for the area in terms of the sides, and if the lengths of the sides be 242, 1212, and 1450 yards, shew that the area is 6 acres.

9. Assuming the formula

$$\log\sin(\theta + h) - \log\sin\theta = \mu h \cot\theta - \frac{\mu h^2}{2}\operatorname{cosec}^2\theta;$$

where h is the circular measure of a small angle, and μ the modulus of the logarithms, and powers of h above the second are neglected; explain when the principle of proportional parts will fail in its application to tables of logarithms of sines.

If $L\sin 15^\circ\ 30'$ be $9{\cdot}426899$ and the difference for $1'$ be $\cdot 000455$, find accurately $L\sin 15^\circ\ 30'\ 36''$; and the angle whose tabular logarithmic cosine is equal to $9{\cdot}427263$.

Given $\log_{10} 2 = {\cdot}301030$ and $\log_{10} 3 = {\cdot}477121$, find $\log_{10} 3\frac{2}{25}$.

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10. If in any triangle b, c, and A be given, prove that

$$\tan \tfrac{1}{2}(B-C)=\frac{b-c}{b+c}\cot\frac{A}{2};$$

hence obtain the formula

$$\tan \tfrac{1}{2}(B-C)=\tan^2\frac{\phi}{2}\cot\frac{A}{2},$$

and explain the advantage of such a formula.

If b be to c as 11 to 10, and $A=35^\circ\ 25'$, apply the above formula to find B and C, having given $\log 1{\cdot}1={\cdot}041393$; $L\cos 24^\circ\ 37'\ 12''=9{\cdot}958607$; $L\tan 12^\circ\ 18'\ 36''=9{\cdot}338891$; $L\cot 17^\circ\ 42'\ 30''=10{\cdot}495800$; $L\tan 8^\circ\ 28'\ 56{\cdot}5''=9{\cdot}173582$.

11. From a house on one side of a street observations are made of the angle subtended by the height of the opposite house: first from the level of the street, in which case the angle is $\tan^{-1}(3)$; and afterwards from two windows one above the other, from each of which the angle is found to be $\tan^{-1}(-3)$. The height of the opposite house being 60 feet, find the height of each of the two windows above the street.

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[*Great importance will be attached to accuracy in results.*]

1. If three straight lines meet all in one point, and a straight line stands at right angles to each of them at that point; these three straight lines are in one and the same plane.

2. If two parallel planes be cut by another plane, their common sections with it are parallels.

3. Find a point, at a given distance from a given plane, equidistant from three given points in that plane.

4. Prove that the section of a cone by a plane is an ellipse, parabola or hyperbola, and state the special forms which each of these may assume in particular cases.

5. Shew that the semi-minor axis of an elliptic section of a cone is a mean proportional between the perpendiculars dropped from the extremities of the major axis upon the axis of the cone.

6. Find the equation to the straight line drawn at right angles to the straight line $\frac{x}{a}-\frac{y}{b}=1$ through the point where it cuts the axis of x.

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7. Find the co-ordinates of the point of intersection of the straight lines $2x-3y=1$, $5y-x=3$, and the angle at which they cut one another.

8. Find the length of the common chord of the circles whose equations are

$$(x-a)^2+y^2=a^2,\ x^2+(y-b)^2=b^2,$$

and the equation to the circle described on it as diameter.

9. Shew how to draw two tangents to a parabola from a given external point.

10. Define an asymptote of a hyperbola; and prove that the part of an asymptote intercepted between the directrices is equal to the transverse axis.

11. Define the eccentricity of a conic section; and find the eccentricity of the ellipse whose equation is $x^2+3y^2=a^2$.

12. Shew that the length of the perpendicular from the centre of an ellipse on a tangent equally inclined to the major and minor axes is

$$\sqrt{\frac{a^2+b^2}{2}}.$$

VI. PURE MATHEMATICS (2).

[*Full marks may be gained by doing about* three-fourths *of this paper. Great importance will be attached to accuracy in results.*]

1. Find the four factors of the expression

$$(1+y)^2-2(1+y^2)x^2+(1-y)^2x^4.$$

2. A man borrows every year £25, upon which he pays interest at the rate of 4 per cent. per annum; in how long a time will the interest that he has paid amount to £91?

3. Two persons are awarded pensions in proportion to the square root of the number of years they have served. One has served 9 years longer than the other, and receives a pension greater by £50. If the length of service of the first had exceeded that of the second by $4\frac{1}{2}$ years only, their pensions would have been in the proportion of 9 : 8. How long had they served, and what were their respective pensions?

4. A starts from P to Q half an hour after B, overtakes him midway between P and Q, and arrives at Q at 2 p.m.; after waiting $7\frac{1}{2}$ minutes at Q, he returns and meets B in ten minutes more. At what time did each start from P?

5. AD, BE, CF are the perpendiculars from the angular points of a triangle ABC on the opposite sides, and O is the centre of the circumscribing circle of the triangle; prove that AO is at right angles to EF.

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6. If two tangents be drawn to a circle, prove that any third tangent is divided harmonically by the two tangents, the chord of contact, and the circle.

7. If A_1, B_1, C_1 be the angles of the triangle formed by the centres of the escribed circles of the triangle ABC, prove that $2A_1$, $2B_1$, and $2C_1$ are the supplements, respectively, of A, B, and C.

If the triangle $A_2B_2C_2$ be similarly formed from $A_1B_1C_1$, $A_3B_3C_3$ from $A_2B_2C_2$, and so on, find A_n, B_n, and C_n, and prove that, as n increases indefinitely, the triangle $A_nB_nC_n$ ultimately becomes equilateral.

8. Find the size of a cube which will completely stop up a tube of uniform bore, the section of which is a regular hexagon whose magnitude is given.

9. Resolve into partial fractions the expressions

$$\frac{4}{(x-1)(x-3)} \text{ and } \frac{x+3}{(x-1)(x^2+3)},$$

and expand each in powers of x, obtaining in each case the coefficient of x^r.

10. Through the angular point C of a triangle ABC a straight line CPQ is drawn, on which are let fall the perpendiculars AP, BQ; prove that

$$PQ = AP \cot B - BQ \cot A.$$

11. Prove that

$$2\tan^{-1}\left(\tan\frac{a}{2}\tan\frac{x}{2}\right) = \cos^{-1}\left(\frac{\cos a + \cos x}{1 + \cos a \cos x}\right).$$

12. Prove De Moivre's Theorem for a positive integral index, and for a negative integral index.

If $$\sin(a + \beta\sqrt{-1}) = x + y\sqrt{-1},$$
prove that $$x^2 \operatorname{cosec}^2 a - y^2 \sec^2 a = 1.$$

13. Prove that, in an equation with real coefficients, imaginary roots occur in pairs.

Having given that $2 + \sqrt{-3}$ is one of the roots of the equation,

$$x^4 - 4x^2 + 8x + 35 = 0,$$

find all its roots.

14. State the relations between the coefficients and the roots of a rational algebraic equation.

If α, β, γ be the roots of the equation,

$$x^3 - px^2 + r = 0,$$

prove that the equation, of which the roots are

$$\frac{\beta+\gamma}{\alpha}, \quad \frac{\gamma+\alpha}{\beta}, \quad \frac{\alpha+\beta}{\gamma},$$

is $$rx^3 + 3rx^2 + (3r - p^3)x + r^3 = 0.$$

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15. Find the sums of the series:

(1) $\sin a + \sin 2a + \sin 3a + \sin 4a + \ldots$ to n terms.

(2) $\sin a + n \sin 2a + \frac{n(n-1)}{1 \cdot 2} \sin 3a$

$$+ \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} \sin 4a + \ldots \text{ to } n+1 \text{ terms.}$$

VII. PURE MATHEMATICS (3).

[*Great importance will be attached to accuracy in results.*]

Full marks may be obtained for four-fifths *of this paper.*

1. Find from first principles the differential coefficient of

$$(1) \quad \tan x + \sec x, \qquad (2) \quad \tan^{-1} \frac{x}{a}.$$

2. Find the differential coefficients of the following:

$$(1) \quad (\cos x)^{\cot^2 x} \qquad (2) \quad (\cot^{-1} x)^{\frac{1}{x}}.$$

$$(3) \quad \left(\frac{x}{n}\right)^{nx} \left(1 + \log \frac{x}{n}\right).$$

3. Prove Leibnitz's theorem for obtaining the nth differential coefficient of the product of two functions.

Find the nth differential coefficient of $e^x \sin x$; and shew that

$$\frac{d^n}{dx^n}(e^{ax}y) = e_{ax}\left(\frac{d}{dx} + a\right)^n y.$$

4. State fully the conditions necessary for the truth of Taylor's theorem. Shew whether $\tan^{-1} \sqrt{\frac{x}{a}}$ can be explained by Maclaurin's theorem or not.

Expand $\cot^{-1} x$ in ascending powers of x.

5. Evaluate

$$(1) \quad \log\left(2 - \frac{x}{a}\right) \cot(x - a) \text{ when } x = a.$$

$$(2) \quad (\cos x)^{\cot x} \text{ when } x = 0.$$

6. Shew how to determine the maxima and minima values of a function of a single variable, of which the nth derived function is the first not to vanish for the critical value of the variable.

Find the area and position of the maximum triangle having a given angle which can be inscribed in a given circle, and prove that the area cannot have a minimum value.

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7. Shew how to determine the number and position of the asymptotes of a plane curve.

Find the asymptotes of the curve

$$y^3 - 7yx^2 - 6x^3 + 4ax^2 = 0.$$

8. What is a point of inflexion of a plane curve? Find the condition that a plane curve should have points of inflexion.

Express the condition also in polar co-ordinates.

9. Shew that the envelop of perpendiculars drawn to tangents of the parabola, $y^2 = 4ax$, at the points where they cut the axis is $27ay^2 + 4x^3 = 0$.

10. Integrate

(1) $\displaystyle\int \frac{dx}{\sqrt{1 - 2x - x^2}}$. (2) $\displaystyle\int \frac{2ax^2}{x^4 - a^4} dx$.

(3) $\displaystyle\int \frac{dx}{\sin 2x}$. (4) $\displaystyle\int \tan^{-1} \sqrt{\frac{x}{a}}\, dx$.

11. Shew that

$$\int u dv = uv - \int v du;$$

and find the value of

$$\int_0^{\frac{\pi}{2}} \sin^n x dx.$$

12. Shew that the area of the curve $y = ae^{\frac{x}{c}}$ included between two ordinates varies as the difference of the ordinates; and find the length of the curve between two points.

VIII. STATICS.

Full marks may be obtained by doing four-fifths *of this paper.*

1. If three commensurable forces in one plane are in equilibrium acting on a point, prove, without assuming the "parallelogram of forces," that either force is in the direction of the diagonal of the parallelogram whose sides represent the other two forces.

Assuming the "parallelogram of forces," shew that a single force acting on a point may be resolved in an unlimited number of ways into *two* forces acting on the same point.

Equal forces act on the centre of a regular pentagon along the lines drawn from the centre to the angles of the pentagon; prove that the

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resultant of any two of these adjacent forces is equal and opposite to the resultant of the other three.

2. When three forces, which do not meet, act on a body in one plane, determine the relation that must exist among them, both as to magnitude and position, in order to preserve equilibrium, the extreme forces being like.

A horizontal rod without weight, 6 feet long, rests on two supports at its extremities; a weight of 6 cwt. is suspended from the rod at a distance of $2\frac{1}{2}$ feet from one end: find the reaction at each point of support. If one support could only bear a pressure of one cwt., what is the greatest distance from the other support at which the weight could be suspended?

3. When are couples said to be "like" or "unlike"? Shew that two unlike couples in the same plane will balance each other if their moments are equal.

The sides of a regular polygon taken in order represent the forces acting in the plane of the polygon; shew that the sum of their moments will be the same round any point within the figure. Find the couple, having one side of the polygon for an arm, that will keep the system in equilibrium.

4. Find the centre of gravity of a solid triangular pyramid whose faces are equilateral triangles.

Shew that the position of the centre of gravity for the four faces considered as plane areas will be the same as it is for the solid pyramid.

5. On a smooth inclined plane a weight is just sustained by a force making a given angle with the plane; find the relation of the power to the weight. Determine also the pressure on the plane.

If the weight, the force, and the pressure be respectively as the numbers 4, 3, and 2, find the direction of the force and the inclination of the plane.

6. State the conditions of equilibrium of any number of forces acting on a body in one plane. Explain also when either of these conditions may, in solving problems, be dispensed with.

A uniform beam rests with a smooth end against the junction of the horizontal ground and a vertical wall; it is supported by a string fastened to the other end of the beam and to a staple in the vertical wall. Find the tension of the string, and shew that it will be half the weight of the beam if the length of the string be equal to the height of the staple above the ground.

7. When a weight is placed on a rough surface, explain what is meant by the coefficient of friction and by the limiting angle of resistance. Express one in terms of the other.

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Shew that the least value of a force (P) to move a weight (W) along a rough horizontal plane is ($W \sin \phi$) where (ϕ) is the limiting angle of resistance.

Two equal heavy particles on two equally rough inclined planes of the same height, and placed back to back, are connected by a string passing over the top of the planes; shew that when the particles are on the point of moving, the limiting angle of resistance will be half the difference of the inclination of the planes.

8. State the principle of virtual velocities. Shew that it holds good in that system of pulleys where each pulley hangs by a separate string (usually called the first system). If the principle be stated $P \times P$'s displacement $= W \times W$'s displacement, will it be true in this case though the displacement should not be small?

In the above system, the distance of the highest pulley from the fixed end which passes round it is 16 feet, and the whole height through which the weight can be raised is one foot; find the number of pulleys, the size of the pulley being neglected.

9. If a right cone be placed with its base on an inclined plane, friction being sufficient to prevent sliding, examine the conditions that the cone may just remain at rest on the plane.

If $\frac{1}{\sqrt{3}}$ be the coefficient of friction, find the angle of the cone when it is on the point both of sliding and falling over.

10. How is "work" measured? How is the efficiency of a working agent estimated? With reference to what units is 33,000 taken as the measure of horse-power?

When weights are raised through different heights, prove that the whole work expended is equal to the work that would be expended in lifting a weight equal to the sum of the weights through the same height as that through which the centre of gravity of the weights has been raised.

Find the horse-power of an engine that would empty a cylindrical shaft full of water in 32 hours, if the diameter of the shaft be 8 feet and its depth 600 feet; the weight of a cubic foot of water being 62·5 pounds.

IX. DYNAMICS.

Full marks may be obtained by doing four-fifths *of this paper.*

(*Note.*—The acceleration of gravity may be taken at 32 feet.)

1. How is velocity estimated (1) when uniform, (2) when variable? Give an instance where, in ordinary language, an estimate of variable velocity is made.

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A particle revolves uniformly in a vertical circle with a velocity (v); find the vertical and horizontal velocity at any point. If the time of one revolution is 8 seconds, and the radius of the circle be 12 inches, and the particle start from the highest point, find the horizontal and vertical velocity after it has revolved for one second.

A body descends uniformly down an inclined plane one mile in length in one hour and twenty minutes; if the plane rises one foot vertical for 100 feet in length, find the vertical velocity of the body in feet per second.

2. How is a uniform force measured? State briefly how it is shewn that the force of gravity at any point on the earth's surface is a uniform force according to the definition.

If a body is projected vertically upwards with a given velocity, find (1) the height that it will ascend in a given time, (2) the greatest height it will ascend.

A body is projected upwards with a certain velocity, and it is found that when in its ascent it is at a point 960 feet from the ground it takes four seconds to return to the same point again; find the velocity of projection, and the whole height ascended.

3. When a heavy particle moves down or up a given smooth inclined plane, find the accelerating or retarding force which acts upon it.

If two vertical circles touch each other at their lowest point, and any straight line be drawn from that point to cut the inner and to meet the outer circle, shew that the time of a heavy particle falling from rest along the part of the line (considered as an inclined plane) intercepted between the circles is constant.

A straight line without a circle and in the same plane with it is parallel to its vertical diameter; find the straight line of quickest descent from the given line to the circle, and determine the angle which the line so drawn makes with the tangent at the lowest point of the circle.

4. State the second law of motion, and shew how it is applicable to the theory of projectiles.

Shew that the curve described by a projectile in vacuo is a parabola. If (L) be the latus rectum of the parabola, (R) the range of the projectile, and α the angle of projection, prove $R = L \tan \alpha$.

5. If three bodies are projected simultaneously in the same vertical plane from the same point, prove that the triangle formed by joining the three bodies at any instant of their motion will vary as the square of the time.

If the angles of projection are also the same, shew that at any instant of their motion the bodies will be in a straight line.

6. Give some reasons why the Statical and Dynamical measures of force are differently estimated.

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State the law of motion which connects these measures of force, and shew how to obtain one from the other.

A horizontal pressure of 9 pounds acts on a weight (W) along a smooth horizontal table, and after moving through a space of 25 feet from rest, generates in it a velocity of 10 feet per second; find (W) in pounds.

7. An elastic ball impinges obliquely with a given velocity on a smooth fixed plane, the elasticity between the ball and the plane being given; find the velocity after impact, and the direction of the rebound.

An elastic ball projected at a given angle from a point in a horizontal plane rebounds from the plane; find the range after the first rebound, and the time of flight, the coefficient of elasticity being $\frac{1}{3}$.

8. When one elastic ball impinges directly on another what kind of mutual action is supposed to take place during the impact, and what ratio resulting therefrom does the coefficient of elasticity express?

When one elastic ball impinges directly on another compare the relative velocities before and after impact. If the impinging ball be two pounds, and the other ball be one pound in weight, find the coefficient of elasticity when the velocity with which the larger ball impinges is equal to the velocity of the smaller ball after impact.

9. Two inclined planes with a common altitude are placed back to back; (P) and (Q) are two weights, one on each plane, connected by a string which passes over the top of the planes, and P descends drawing up Q; find the accelerating force and the tension of the string.

P hangs vertically, and is 9 lbs.; Q is 6 lbs. on a plane whose inclination is 30°; shew that P will draw Q up the whole length of the plane in half the time that Q hanging vertically would draw P up the plane.

10. If a body whose mass is (m) revolve uniformly in a circle, radius (r), with a velocity (v), prove that the body is acted on by a force $\frac{mv^2}{r}$ tending to the centre of the circle.

A locomotive engine weighing 9 tons passes round a curve 600 feet in radius with a velocity of 30 miles an hour; what force tending towards the centre of the curve must be exerted by the rails so that the engine may move on this curve?

11. How is the accumulated work or the kinetic energy of a moving body estimated?

A train runs from rest for a mile down a plane whose descent is one foot vertical for 100 feet in length; if the resistances are eight pounds per ton, how far will the train be carried along the horizontal level at the foot of the incline?

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1881.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

1. Define a plane superficies, a sector of a circle, a rhombus, a rhomboid, a square, a rectangle. What is a rhomboid generally called? Criticise Euclid's definition of a square.

2. If two angles of a triangle be equal to one another, the sides also which subtend, or are opposite to, the equal angles, shall be equal to one another.

What other converse proposition may be obtained from Proposition V., Book I.?

3. If two triangles have two angles of the one equal to two angles of the other, each to each, and one side equal to one side, namely, the sides opposite to the equal angles in each, the triangles shall be equal in all respects.

Three given straight lines meet in a given point; shew how a straight line may be drawn to cut them so that its two segments intercepted between the lines may be equal.

4. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of every triangle are together equal to two right angles.

What fractions of a right angle will the angles of a pentagon be, if they are in the ratios of the numbers 1, 3, 6, 9, 11?

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5. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the two parts.

How must a straight line be divided into two parts, so that the rectangle contained by them may be the greatest possible?

6. Draw a straight line from a given point, either without or in the circumference, which shall touch a given circle.

Draw the common tangents to two circles which cut one another.

7. Define the segment of a circle.

A segment of a circle being given, describe the circle of which it is the segment.

8. Inscribe a circle in a given triangle.

Inscribe also a second circle in the space intercepted at one of the angles, so as to touch the circumference of the circle and each of the sides containing the angle.

9. Describe an isosceles triangle BAC, having each of the angles at the base double of the third angle BAC.

If a point D be taken in AB so that AD is equal to BC, and DE be drawn parallel to AC to meet BC in E, shew that AB touches the circumscribing circle of the triangle CDE.

10. Equal triangles which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional; and triangles which have one angle of the one equal to one angle of the other, and their sides about the equal angles reciprocally proportional, are equal to one another.

D is any point in AC, the base of an isosceles triangle ABC. DE and DF are straight lines making equal angles with AC, and meeting the equal sides BC and AB in E and F respectively. Prove that the triangles AED, CDF are equal in area.

11. Describe a rectilineal figure which shall be similar to one given rectilineal figure and equal to another given rectilineal figure.

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II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy in numerical results.*]

1. Find the value of

$$\tfrac{11}{14} + (3\tfrac{1}{4} \text{ of } 1\tfrac{7}{18}) - \tfrac{1}{2}(1\tfrac{1}{8} - 2\tfrac{2}{6}) + \frac{6\frac{3}{4} \text{ of } 3\frac{1}{9}}{2\frac{5}{8} \text{ of } \frac{7}{10}}.$$

2. Subtract $\frac{4\frac{7}{8}}{3\frac{1}{4}}$ of $1\frac{5}{12}$ from $2\frac{5}{11}$ of $2\frac{1}{8}$ of $\frac{13}{36}$.

3. Multiply together $\frac{2\frac{5}{6}}{1\frac{1}{11}}$, $\frac{5}{16}$, $\frac{2\frac{2}{23}}{2\frac{3}{4}}$, and $1\frac{15}{17}$ of $1\frac{3}{20}$.

4. Divide $3\frac{1}{2}$ of $1\frac{5}{12}$ by $5\frac{1}{8}$, and the result by $1\frac{3}{7}$.

5. Add the difference between ·035 of a ton and ·064 of a cwt. to the difference between ·27 of a qr. and ·78 of a lb., and give the answer in lbs. and the decimal of a lb.

6. Multiply 8·07639 by ·002873.

7. Divide 298·08 by ·00345.

8. Divide 7·3 by 584.

9. Reduce ·03257 of an acre to square yards and the decimal of a square yard.

10. Express 4 ozs. 17 dwts. 12 grs. as the decimal of a lb. troy.

11. Divide 12 miles 2 furlongs 20 poles 4 yards 2 feet 6 inches by 47.

12. Find the dividend of £3,407. 15*s*. at 13*s*. 9*d*. in the £.

13. At what rate per cent. simple interest will £245 amount to £324. 18*s*. 7½*d*. in 7¼ years?

14. A man leaves £32,818 to be divided among his four sons in the proportion of the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$. Find the share of each.

15. By selling goods for a certain sum a man gains 5 per cent. If he had sold them for 3 shillings more he would have gained 6 per cent. Find their cost price.

16. *A* buys a pipe of wine and sells it to *B* at a profit of 5 per cent., *B* sells it to *C* at a profit of 5 per cent., and *C* sells it to *D* for

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£49. 12*s*. 3*d*., making a profit of $12\frac{1}{2}$ per cent. What did the wine cost *A*?

17. Find the square root of $968\frac{2897}{19321}$ and the cube root of

$$182284{\cdot}263.$$

18. *A* commences business with a capital of £4,000, and after 4 months takes *B* into partnership with a capital of £300. Two months later they take *C* into partnership with a capital of £5,000. At the end of the year their net profits amount to 16 per cent. on the whole capital invested. What should each receive of the profits?

19. 1 cwt. 2 qrs. 12 lbs. of lead are rolled into a sheet 18 feet long and 6 feet wide. Find its thickness. (A cubic foot of lead weighs 720 lbs.)

20. By buying 3 per cent. consols at a certain price I find I obtain $3\frac{3}{8}$ per cent. for my money and derive a net income therefrom, after paying an income-tax of 6*d*. in the £, of £421. 4*s*. Find the amount of stock and the price at which I bought it.

21. A train leaves London for Brighton at 9 A.M., travelling at a uniform rate of 15 miles an hour. An express train leaves Brighton for London at 10 A.M. and travels at a rate of 40 miles an hour. At what time will they pass each other and at what distance from London, the distance from London to Brighton being 50 miles?

22. Find log ·0021, log $(147)^2$, and

$$\log \sqrt[5]{\frac{2\sqrt{7}}{3\sqrt{11}}},$$

and the number whose log is 2·8470270.

23. Given $3^x = 7175{\cdot}37$, find the value of x.

Logarithms required.

log 1·1 = ·0413927	log 7·0311 = ·8470233
log 2 = ·3010300	log 7·0312 = ·8470295
log 3 = ·4771213	log 71·753 = 1·8558401
log 7 = ·8450981	log 71·754 = 1·8558461

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III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[*Great importance will be attached to accuracy in results.*]

1. State the rule for affixing the correct sign to the product of two algebraical quantities which are affected with the same or different signs. Verify by reference to arithmetic the correctness of this rule.

Find the value of x^2-6x+7 if $x=3-\sqrt{3}$.

2. Multiply $(a^2+b^2+c^2-ab-ac-bc)$ by $(a+b+c)$.

Divide $(ac+bd)^2-(ad+bc)^2$ by $(a-b).(c-d)$.

3. Reduce to its lowest terms

$$\frac{12x^4-4ax^3-23a^2x^2+9a^3x-9a^4}{8x^4-14a^2x^2-9a^4}.$$

4. If $$x=\frac{ab-cd}{(a-b)-(c-d)},$$

prove $$(x+a)(x-b)=(x+c)(x-d),$$

and for this value of x shew that

$$\frac{x+a}{x-b}=\frac{(a-c)(a+d)}{(b-d)(b+c)}.$$

5. Reduce to their simplest forms

(1) $$\frac{3x^2+2x+4}{x^3-1}-\frac{x+1}{x^2+x+1}-\frac{2}{x-1}.$$

(2) $$\frac{3^{\frac{2}{3}}+3^{\frac{1}{3}}+1}{3^{\frac{1}{3}}+1}+\frac{3^{\frac{2}{3}}-3^{\frac{1}{3}}+1}{3^{\frac{1}{3}}-1}.$$

6. Solve the following equations:

(1) $$\left.\begin{array}{c}(a-b)x+(a+b)y=2(a^2-b^2)\\ ax-by=a^2+b^2\end{array}\right\}.$$

(2) $$(x-4)^3+(x-5)^3=31\{(x-4)^2-(x-5)^2\}.$$

7. Form a quadratic equation whose roots are $-3+\sqrt{2}$ and $-3-\sqrt{2}$.

Prove that the equations

$$x^2-px+q=0,\ x^2-p_1x+q_1=0,$$

will have a common root if

$$(p_1q-pq_1)(p-p_1)=(q-q_1)^2.$$

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8. If (p) and (q) are whole numbers, find generally the factor which will rationalise $a^{\frac{1}{p}}+b^{\frac{1}{q}}$.

Hence rationalise $3^{\frac{1}{2}}+2^{\frac{1}{3}}$, and obtain the numerical value of the result.

9. In any scale of notation whose radix is (r), prove that if the sum of the digits of any whole number be divisible by $(r-1)$ the number itself will be divisible by $(r-1)$.

Given 222·22, in the scale whose radix is 5, reduce it to the denary scale.

10. If (a) and (r) are whole numbers, find the sum to (n) terms of the series $a+\frac{a}{r}+\frac{a}{r^2}+\frac{a}{r^3}+$&c., and explain how the sum of an infinite number of terms of a series may have a finite value.

Express the circulating decimal $\cdot\dot{9}$ as a geometrical series, and find its value.

11. Find the total number of permutations of (n) things, taken all together, when there are (p) things of one sort and (q) of another, and the rest are unlike.

12. Two casks, each containing 20 gallons, are filled, one with water, the other with spirit, (x) gallons are drawn from each cask, mixed, and the casks are again filled up with the mixture; when this is done a second time it is found that the quantity of spirit in one cask is to the quantity in the other as 5 to 3. Find (x).

13. Assuming the form of the expansion by the binomial theorem of $(1+x)^{n-1}$ when (n) is any positive whole number, shew that the coefficient of x^r in $(1+x).(1+x)^{n-1}$ is the number of combinations of (n) things taken (r) together. State briefly how this property is applicable to the proof of the binomial theorem. Express the sum of the coefficients of a binomial whose index is a positive whole number in terms of a power of 2, and verify the property in the expansion of $(1+x)^6$.

The coefficient of the 3rd term in the expansion of $(1-x)^{-n}$ is $\frac{2}{9}$; find (n) and the coefficient of the fifth term.

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IV. PLANE TRIGONOMETRY.

(Including the solution of triangles.)

[N.B.—*Great importance will be attached to accuracy in results.*]

1. Define the "circular measure" of an angle, and shew that, if with centre O any circle be described, cutting the bounding lines of an angle BOC in the points P, Q, the circular measure of the angle will be equal to $\frac{\text{arc } PQ}{OP}$.

Find the number of seconds in the angle subtended at the centre of a circle, whose radius is 1 mile, by an arc $5\frac{1}{2}$ inches long.

2. Give accurate definitions of the sine and cosine of an angle, and prove that

$$\cos A = \sin(90^\circ + A) = -\cos(180^\circ + A).$$

Shew that the sine will be algebraically less than the cosine for any angle between $(8n-3)\,45^\circ$ and $(8n+1)\,45^\circ$, where n is zero or any positive integer.

3. Find, geometrically, expressions for the sine and cosine of the sum of two angles in terms of the sines and cosines of the angles themselves.

The cosines of two angles of a triangle are $\frac{3}{5}$ and $\frac{12}{13}$ respectively. Find all the Trigonometrical ratios of the third angle.

4. Express $\sin A$ and $\cos A$ in terms of $\tan\frac{A}{2}$.

Prove that $\tan\left(45^\circ - \frac{A}{2}\right) + \tan\left(45^\circ + \frac{A}{2}\right) = 2\sec A.$

5. Find the sine of 18°, and deduce the sine of 36°.

Two parallel chords of a circle, lying on the same side of the centre, subtend respectively 72° and 144° at the centre. Shew that the distance between the chords is half the radius of the circle.

6. Find all the angles which satisfy the equation

$$2\sin 2\theta = 3\tan\theta.$$

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7. Prove that:

(1) $\sec^6\theta - \tan^6\theta = 1 + 3\tan^2\theta . \sec^2\theta.$

(2) $\text{vers}(270° + A) . \text{vers}(270° - A) = \cos^2 A.$

(3) $\dfrac{\sin A + 2\sin 3A + \sin 5A}{\cos A - 2\cos 3A + \cos 5A} = \dfrac{4\sin A - 3\,\text{cosec}\, A}{4\cos A - 3\sec A}.$

8. Given $\log 2 = \cdot 30103$, $\log 3 = \cdot 4771213$, find L sin 45°, L sec 30°. Also if L sin 15° = 9·4129962, what will be the value of L cos 15°?

9. Prove that in any triangle

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

If the angles adjacent to the base of a triangle are $22\frac{1}{2}°$ and $112\frac{1}{2}°$, shew that the perpendicular altitude will be half the base.

10. Find an expression for the radius of the circle which touches one side of a triangle and the other two produced.

Shew that the sum of the radii of the two escribed circles of a triangle, which touch the side c produced, is equal to $c \cot \frac{C}{2}$.

11. Shew how to solve a triangle by means of logarithms when the three sides are given.

Find the least angle of the triangle whose sides are 24, 22, 14, having given

L tan 17° 33′ = 9·500042, diff. for 1′ = ·000439.

12. A man walking along a straight road, which runs in a direction 30° East of North, notes when he is due South of a certain house. When he has walked a mile further, he observes that the house lies due West, and that a windmill on the opposite side of the road is N.E. of him. Three miles further on he finds that he is due North of the windmill. Find the distance between the house and windmill, and shew that the line joining them makes with the road an angle

$$\tan^{-1}\left(\frac{48 - 25\sqrt{3}}{11}\right).$$

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FURTHER EXAMINATION.

V. PURE MATHEMATICS. (1.)

[*Full marks may be obtained for about* three-fourths *of this paper.*

Great importance will be attached to accuracy in results.]

1. If two straight lines are parallel, and one of them is at right angles to a plane, the other also shall be at right angles to the same plane.

2. Draw a straight line perpendicular to a given plane from a given point without it.

Prove that the perpendicular from the vertex of a regular tetrahedron upon the opposite face, is three times that dropped from its intersection with that face upon any one of the other faces.

3. If a solid angle be contained by three plane angles, any two of them are greater than the third.

4. Define a right cone.

Shew how a plane must cut a right cone that the section may be a hyperbola, and determine the position of the foci and directrices.

5. Shew how to cut a right cone in a section which shall have a latus rectum of given length; and if the vertical angle of the cone be a right angle, prove that the perpendicular distance from the vertex to the cutting plane is equal to one-half of the latus rectum of the section.

6. If TP, TQ be two tangents to a parabola whose focus is S, prove that the triangles TSP, TSQ are similar; and that if O be the centre of the circumscribing circle of the triangle TPQ, the angle TSO is a right angle.

7. If NPQ, a perpendicular at N to the major axis of an ellipse, meet the curve in P, and the auxiliary circle in Q; prove that the tangents at P and Q meet one another upon the major axis.

If the tangent at Q meets the minor axis produced in K, prove that

$$CK.NP=AC.BC.$$

8. If SY, HZ be the perpendiculars from the foci on a tangent to a hyperbola; prove, either geometrically or analytically, that $SY.HZ=BC^2$.

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9. What loci are respectively represented by

(1) $(x+a)^2+(y+b)^2=0$;

(2) $xy-2y=0$;

(3) $y^2-x^2=0$?

Draw the figure in each case.

10. Find the length of the perpendicular from a point (h, k) on the straight line $y=mx+c$; and the equation to a straight line through the foot of the perpendicular bisecting the angle between it and the given straight line.

11. If $y=mx$ be a chord of a circle whose radius is a, the origin of co-ordinates being one extremity of the chord, and the axis of x being a diameter of the circle; shew that the equation to a circle of which this chord is the diameter is

$$(1+m^2)(x^2+y^2)-2a(x+my)=0.$$

12. Find the equation to a tangent at any point of

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1;$$

and if θ be the acute angle between it and the focal distance of the point, prove that the distance of the point from the centre is

$$\sqrt{a^2-b^2\cot^2\theta}.$$

13. Find the equation to the diameter DCD' of a system of parallel chords inclined at an angle θ to the major axis of an ellipse.

If PCP' be one of the chords, and h, k the co-ordinates of P, find the equation to PD; the ordinates of P and D being both supposed to be positive.

14. Obtain the locus of the intersection of two tangents to a hyperbola which are perpendicular to one another.

15. Determine the position of the vertex and the direction of the axis of the parabola

$$y^2+4xy+4x^2+3ax+a^2=0.$$

What relation must exist among the coefficients of the equation

$$ay^2+bxy+dy+ex=0,$$

in order that it may represent two straight lines?

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VI. PURE MATHEMATICS. (2.)

[*Great importance will be attached to accuracy.*]

1. If $$2s = a + b + c \text{ and } 2\sigma^3 = a^3 + b^3 + c^3,$$
prove that
$$(\sigma^3 - a^3)(s - a) + (\sigma^3 - b^3)(s - b) + (\sigma^3 - c^3)(s - c) = a^4 + b^4 + c^4 - s\sigma^3.$$

2. A bill of £4 is paid with 80 coins, consisting of half-crowns, shillings, and fourpenny pieces. If the number of shillings used be as small as possible, find the number of each coin.

3. Two casks, A and B, contain mixtures of wine and water, A in the ratio of 8 : 3, and B in the ratio of 5 : 1. In what ratio must liquid be drawn from each cask to give a mixture in the ratio of 4 : 1?

4. Prove that if $a, b, c, \ldots k$ be n positive quantities, then
$$\frac{(a + b + c + \ldots + k)^n}{n^n} > abc \ldots k.$$
Shew that $(a - x)(x + \sqrt{x^2 + b^2})$ cannot exceed $\frac{1}{2}(a^2 + b^2)$.

5. Given that the probability of a certain event happening is a, while that of a certain other event happening is b, find the probability of both events happening (1) when the two events are independent, (2) when the second event cannot happen unless the first does, and its probability in that case is b.

6. Expand the following expressions in ascending powers of x, and find the coefficient of x^n in each:

(1) $$\frac{4x - 6}{x^3 - 6x^2 + 11x - 6};$$

(2) $$\frac{x(x^2 + 2)}{x^4 + x^2 + 1}.$$

7. If $u = \cot^{-1}(\cos\alpha)^{\frac{1}{2}} - \tan^{-1}(\cos\alpha)^{\frac{1}{2}}$, prove that
$$\sin u = \left(\tan\frac{\alpha}{2}\right)^2.$$

8. Prove that
$$\cos\alpha = 1 - \frac{\alpha^2}{\underline{|2}} + \frac{\alpha^4}{\underline{|4}} - \&c.$$

If $$\cos(\alpha + \beta\sqrt{-1}) = a + b\sqrt{-1},$$
$$\sin(\alpha + \beta\sqrt{-1}) = c + d\sqrt{-1};$$
find α and β.

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9. If a, b, and c be the sides of a triangle, and C the circular measure of the angle opposite to c, prove that

$$\log c = \log a - \frac{b}{a}\cos C - \frac{b^2}{2a^2}\cos 2C - \frac{b^3}{3a^3}\cos 3C - \&c.$$

10. Sum to n terms:

(1) $\sin\theta\cos 3\theta + \sin 2\theta\cos 6\theta + \sin 4\theta\cos 12\theta + \&c.$

(2) $\cos^3\alpha + \cos^3(\alpha+\beta) + \cos^3(\alpha+2\beta) + \&c.$

11. Transform the equation, $x^3 - px^2 + qx - r = 0$, to one the roots of which are obtained by subtracting one of the roots of the given equation from the sum of the other two.

Hence, or otherwise, solve the equation

$$32x^3 + 16x^2 - 18x - 9 = 0.$$

12. Shew that a root of the equation $f'(x) = 0$ lies between every adjacent two of the real roots of $f(x) = 0$.

VII. PURE MATHEMATICS. (3.)

[*Full marks may be obtained by doing* two-thirds *of this paper. Great importance will be attached to accuracy in results.*]

1. Find the differential coefficients of e^x, x^n, $\tan x$.

2. Find $\frac{dy}{dx}$ in the following cases:

(1) $y = \frac{x\sqrt{x^2-4a^2}}{\sqrt{x^2-a^2}}$. (2) $y = \left(1+\frac{1}{x}\right)^x$.

(3) $y = \log^n(x)$ where $\log^n(x)$ means, log log (repeated n times) x.

(4) $y = b\tan^{-1}\left(\frac{x}{a} + \tan\frac{y}{x}\right)$.

3. Define successive differentiation.

If u, y, and z be functions of x, such that

$$\frac{d}{dx}\left(u\frac{dy}{dx}\right) = y, \text{ and } \frac{d}{dx}\left(u\frac{dz}{dx}\right) = z,$$

prove that
$$\frac{d}{dx}u\left(y\frac{dz}{dx} - z\frac{dy}{dx}\right) = 0.$$

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4. State and prove Maclaurin's theorem for any function of x which can be expanded in positive integral powers of x.

Expand $e^{nx}\cos mx$ to four terms, and find the general term.

5. (1) Find the value of $(\log x)^{\log(1-x)}$ when $x=0$.

(2) Also of $\dfrac{\log_{\sin x}(\cos x)}{\log_{\sin\frac{x}{2}}\left(\cos\frac{x}{2}\right)}$ when $x=0$.

6. How may the differential calculus be applied to find the maxima or minima values of a function of x?

SQ is a focal radius vector in a given ellipse inclined at the given angle (α) to SA, where A is the vertex nearest to the focus S. Find the angle ASP, where SP is another focal radius, such that the area of the triangle PSQ may be a maximum.

7. If ψ be the angle between the tangent and radius vector at the point of a curve whose polar co-ordinates are r and θ, prove that

$$\tan\psi = r\frac{d\theta}{dr}.$$

If p and π be the perpendiculars from the pole upon the tangent and normal respectively, prove that

$$\frac{dp}{d\theta} = \pi\left(1+\frac{d\psi}{d\theta}\right).$$

8. Find the rectilinear asymptote of the curve

$$x^5 - ax^3y - 2ax^2y^2 + y^5 = 0.$$

9. Trace the curve

$$y^3 + axy + bx^2 = 0,$$

taking a and b both positive.

10. Find the following integrals:

(1) $\displaystyle\int\frac{x+a}{x^2-2ax}\,dx.$ (2) $\displaystyle\int\frac{dx}{x\sqrt{1+x}}.$

(3) $\displaystyle\int\frac{dx}{\sqrt{e^{2x}-1}}.$ (4) $\displaystyle\int\frac{\tan x\,dx}{1+\cos^2 x}.$

11. Find the area of the curve

$$r^2 = \frac{a^2\cos\theta}{2-\sin\theta}.$$

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VIII. STATICS.

[N.B.—*Great importance will be attached to accuracy in results. Full marks may be obtained by doing* three-fourths *of this paper.*]

1. If the parallelogram of forces be true for the direction of the resultant of any force and each of two other forces taken separately, then it will be true for the direction of the resultant of that force and the two other forces taken together.

2. Deduce the triangle of forces and the polygon of forces from the parallelogram of forces.

If the resultant R of the two forces P and Q inclined to each other at any given angle make the angle θ with P, prove that the resultant of the forces $(P+R)$ and Q at the same angle will make the angle $\frac{\theta}{2}$ with $P+R$.

3. If any number of forces in one plane act upon a rigid body, prove that they may be always replaced by a single force acting at any point and a couple.

4. Find the centre of gravity of a pyramid.

If $ABCD$ be a tetrahedron, and if the plane CDE passing through the edge CD cuts AB in E, prove that the line joining the centres of gravity of the tetrahedrons $ABCD$ and $AECD$ is parallel to AB.

5. Find the conditions of equilibrium when any number of forces in one plane act on a rigid body.

Four heavy rods, equal in all respects, are freely jointed together at their extremities so as to form the rhombus $ABCD$. If this rhombus be suspended by two strings attached to the middle points of AB and AD, each string being inclined at the angle θ to the vertical, prove that in the position of equilibrium the angles of the rhombus will be 2θ and $\pi - 2\theta$.

6. Find the power necessary to support the weight W in a system of movable weightless pulleys in which each string is attached to the weight.

If in such a system each pulley have the weight w, and the sum of the weights of the pulleys be W', and P and W be the power and weight in this case, prove that the power $P+w$ would support the weight $W+W'$ in the same system, if the pulleys had no weight.

7. Find the relation of the power to the weight in the inclined plane, the power acting at any angle to the plane.

A wedge with angle 60° is placed upon a smooth table, and a weight of 20 lbs. on the slant face is supported by a string lying on that face passing through a smooth ring at the top and supporting a weight W hanging vertically. Find the magnitude of W. Find also the force necessary to keep the wedge at rest (1) when the ring is not attached to the wedge, (2) when it is so attached.

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8. State the principal laws of statical friction, and find the answers in the last case, supposing the slant face of the wedge to be rough, the coefficient of friction being $\frac{1}{\sqrt{3}}$ and the 20 lbs. weight on the point of moving down.

9. Distinguish between stable and unstable equilibrium. Two particles A and B are connected by a rod AB and laid upon a smooth table. The particle A is acted on by a force F parallel to a line Ox in the table and in the direction from O to x, and by an equal force F parallel to another line Oy from O towards y. The particle B is acted on by two exactly equal and opposite forces to those on A. Prove that wherever AB is placed on the table there are two positions of equilibrium for AB, stable when A is further from O than B, and unstable when A is nearer to O.

IX. DYNAMICS.

[N.B.—*When needed the force of gravity may be taken as* 32 *feet. Great importance will be attached to accuracy in results.*

Full marks may be gained by doing eight-ninths *of this paper.*]

1. When a body moves with a uniform velocity, establish the relation $s=tv$ which connects the time, space, and velocity.

The velocity of the extremity of the minute hand of a clock is 48 times the velocity of the extremity of the hour hand, which is 3 inches long; find the length of the minute hand.

2. Define acceleration, and state how it is measured. How does it appear that the accelerating force of gravity is independent of the weight of falling bodies? Give instances in which the force accelerating the motion of a body may be half the force of gravity. Find the space described in a given time by a body starting with a given velocity, and moving with uniform acceleration.

A body projected perpendicularly downwards describes 720 feet in (t) seconds, and 2,240 feet in ($2t$) seconds; find (t) and the velocity of projection.

3. If a body be projected down a smooth inclined plane with a velocity V, prove that the velocity at the foot of the plane will be independent of the length of the plane if the height of the point of projection above the horizontal plane be given.

A body begins to slide down a smooth inclined plane from the top, and at the same instant another body is projected upwards from the foot of the plane with such a velocity that the bodies meet in the middle of the plane; find that velocity of projection, and determine the velocities of each body when they meet.

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4. Enunciate the second law of motion, and refer briefly to any experimental facts which lead to its adoption.

Three velocities, whose ratios are as $\sqrt{3} : \sqrt{6} : 2$, are simultaneously impressed on a particle and the particle does not move; find the angles at which the directions of the velocities are inclined to each other.

5. Find the range on a horizontal plane of a projectile in vacuo.

Determine the angle of projection when the range is equal to the height due to the velocity of projection.

Find also the direction in which the projectile is moving at any point of its curve.

6. A projectile is projected from the foot of an inclined plane whose inclination is (β) with velocity (v); if (θ) be the angle between the direction of projection and the inclined plane, prove that the time of flight when the projectile strikes the plane is $\frac{2v}{g} \cdot \frac{\sin \theta}{\cos \beta}$.

Shew that the projectile will strike the plane at right angles if $\cot \beta = 2 \tan \theta$.

7. Define moving force, and shew how the equation $w = mg$ is obtained, where (w) is the weight and (m) the mass of a body.

Describe Atwood's machine, and shew how the force of gravity (g) might be ascertained by means of this machine.

8. A weight (P) hanging freely descends, raising a weight (W) by means of a string passing over a smooth peg; find the accelerating force and the tension of the string.

When (W) has been in motion from rest for 3 seconds the string is suddenly cut; find (P) so that (W) may ascend through $\frac{16}{9}$ths of a foot before it begins to descend, the weight of (W) being four ounces.

9. A ball (m) impinges directly on a ball (m') at rest, with a given velocity (v); find the velocity of each after impact, the modulus of elasticity being (e).

10. Describe the mathematical assumptions by means of which the time of an oscillation of a simple pendulum describing a small circular arc is found to be $\pi \sqrt{\frac{l}{g}}$.

If a pendulum that oscillates seconds be lengthened by its hundredth part, find the number of oscillations it will lose in 24 hours.

11. How is the kinetic energy of a moving body expressed in terms of its "vis viva"? A train of 20 tons is moving at the rate of 30 miles an hour, what is the measure of its "accumulated work"? Explain the term foot-pounds.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1882.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Great importance will be attached to accuracy.*]

1. What is Euclid's definition of a straight line? Is this definition available for demonstration? What axiom concerning straight lines furnishes the test required? Enunciate and prove the proposition in Euclid in the proof of which this axiom is first referred to.

2. Equal triangles upon equal bases in the same straight line and towards the same parts are between the same parallels.

ABC is a triangle; join D and E, the middle points of AB and AC: prove, *by the use of propositions of the first book only*, that DE is parallel to BC.

3. Describe a parallelogram equal to a given triangle and having one of its angles equal to a given rectilineal angle.

Describe a parallelogram, the area and the perimeter of which shall be each equal to the area and perimeter of a given triangle.

4. Enunciate and prove the proposition from which the corollary is inferred that "the difference of the squares of two unequal straight lines is equal to the rectangle contained by their sum and difference."

Find the straight line the square of which shall be equal to the rectangle contained by the sum and difference of two given straight lines.

5. Divide a straight line into two parts so that the rectangle contained by the whole line and one of the parts shall be equal to the square of the other part.

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If AB be divided in C so that the rectangle AB, BC is equal to the square on AC, and CD be taken equal to BC, shew that AC is divided in D so that the rectangle AC, AD is equal to the square on CD.

6. The diameter is the greatest straight line in a circle, and of all others that which is nearer to the centre is greater than one more remote.

Two circles cut one another, through a point of intersection draw a straight line terminated by the circumferences, so that the chords so intercepted in each circle may be equal.

7. The angle in a semicircle is a right angle, the angle in a segment greater than a semicircle is less than a right angle, and the angle in a segment less than a semicircle is greater than a right angle.

When are segments of circles said to be similar? If two circles touch each other externally, any straight line drawn through the point of contact will cut off similar segments; when will all the four segments cut off be similar?

8. Describe a circle about a given triangle.

Hence shew that the perpendiculars drawn from the middle points of the sides of a triangle meet in the same point.

9. Describe a circle about an equilateral and equiangular pentagon; assuming that the straight lines bisecting each of the angles of the pentagon intersect in the same point.

If A, B, C, D, E be the angles of the pentagon taken in order, prove that the line CE is parallel to AB.

10. Define similar rectilineal figures; if the figures be triangles, is there anything superfluous in the definition?

Similar triangles are to each other in the duplicate ratio of their homologous sides.

ABC is a triangle, AE and BF intersecting in G are drawn to bisect the sides BC, AC in E and F; compare the areas of the triangles AGB, FGE.

11. If an angle of a triangle be bisected by a straight line which cuts the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square of the straight line which bisects the angle.

If ABC be a right-angled triangle, whose right angle B is bisected by BF, cutting the base in F and meeting the circumference described about ABC in D, prove that the rectangle contained by BD and BF is equal to twice the area of ABC.

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II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N. B.—*Great importance will be attached to accuracy in numerical results.*]

1. Find the value of

$$\tfrac{9}{11} + (2\tfrac{5}{6} \text{ of } 1\tfrac{1}{11}) - 1\tfrac{9}{17}(3\tfrac{3}{13} - 2\tfrac{1}{4}) + \frac{4\tfrac{9}{11} \text{ of } 2\tfrac{19}{17}}{1\tfrac{29}{51} \text{ of } 5\tfrac{8}{10}}.$$

2. Subtract $\frac{5\frac{4}{9}}{8\frac{10}{11}}$ of $2\frac{5}{11}$ from $5\frac{3}{7}$ of $3\frac{9}{25}$ of $\frac{49}{111}$.

3. Multiply together $\frac{3\frac{1}{38}}{1\frac{35}{67}}$, $\frac{6}{18}$, $\frac{6\frac{9}{14}}{4\frac{3}{7}}$, and $2\frac{5}{17}$ of $5\frac{3}{8}$.

4. Divide $4\frac{3}{5}$ of $1\frac{7}{8}$ by $5\frac{1}{3}$, and the result by $2\frac{1}{4}$.

5. Add together ·0045 of a mile, ·052 of a furlong, and ·153 of a pole; subtract the result from 35 yards, and express the answer in inches and the decimal of an inch.

6. Multiply 17·63745 by ·004905.

7. Divide 364·353 by ·00671.

8. Divide $3\cdot\dot{7}1428\dot{5}$ by $5\cdot0\dot{5}71\dot{4}2\dot{8}$, and express the answer as a decimal correct to four places.

9. Reduce $\frac{19}{24}$ of 2*s.* 7*d.* to the decimal of 15*s.* 10*d.*

10. Express 15 cwt. 2 qrs. 21 lbs. as the decimal of 5 tons.

11. Divide 8 acres 1 rood 31 poles 27 sq. yards 2 sq. feet and 116 sq. inches by 53.

12. What is the rateable value of a parish if a rate of 3*s.* 5½*d.* in the £ produces a sum of £14,352. 1*s.* 8*d.*?

13. What principal will amount to £10,672. 9*s.* 3*d.* in 7 years and 3 months at 5½ per cent. per annum simple interest?

14. A cistern which can be filled by one tap in 28 hours is emptied by four others. The first of these alone would empty it in 10 hours, the second in 12 hours, the third in 15 hours, and the fourth in 21 hours. Supposing the cistern to be full, in what time would it be emptied if all the five taps were set running together?

15. A man buys a parcel of coffee and re-sells it, losing 3 per cent. on the transaction. If he had obtained £14 more, he would have gained 4 per cent. What was the original sum paid for the coffee?

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16. A cubic foot of Canadian elm weighs ·725 as much as a cubic foot of water; and a cubic foot of water weighs 1000 ozs.: what will be the weight of a beam of Canadian elm 12 feet 6 ins. long, 1 ft. 6 ins. deep, and 1 ft. 3 ins. thick?

17. It is desired to put a cubical case, whose content is 4019·679 cubic feet, through a square hatchway whose area is 37791·36 square inches. Shew whether this can be done.

Find the square root of $223\frac{51869}{58564}$.

18. In a rectangular piece of level ground 160 feet long and 80 feet broad it is required to make a rectangular bath 145 feet long and 65 feet broad. The earth excavated is to be placed evenly on the surrounding ground. What must be the depth of the bath in inches in order that the surrounding ground may be 8 feet above its former level?

19. A person gave £75 for 20 casks of oil, each containing 30 gallons. He sold 5 casks at three shillings per gallon, one cask was stove in and the whole of its contents lost, and 15 gallons were also lost by ordinary leakage. He then sold the remainder at a price per gallon which made his gain amount to 20 per cent. on the whole transaction. What was his selling price per gallon at the second sale?

20. *A* and *B* start at the same time from London to Blisworth, *A* walking 4 miles an hour, *B* riding 9 miles an hour. *B* reaches Blisworth in 4 hours, and immediately rides back to London. After 3 hours' rest he starts again for Blisworth at the same rate. How far from London will he overtake *A*, who has in the meantime rested for 6 hours?

21. £5000 was invested in 3 per cent. stock at par, subject to an income-tax of 6*d.* in the £. Another £5000 was invested in $3\frac{1}{4}$ per cent. stock at 104, free from income-tax. What investment was the more advantageous, and by how much?

22. Find log ·039, log $(156)^3$, and log $6\left(\frac{2\sqrt{13}}{3\sqrt{7}}\right)^2$, and find the number whose log is 2·9096844.

23. Given $7^x = 823542·4$, find the value of x.

Logarithms required.

log 1·3 = ·1139434	log 8·2354 = ·9156847
log 2 = ·3010300	log 8·2355 = ·9156900
log 3 = ·4771213	log 81·22 = 1·9096630
log 7 = ·8450980	log 81·23 = 1·9097165

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III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance will be attached to accuracy in results.*]

1. Prove the rule of signs in Algebraical Subtraction.

From

$$\{m(2m-3p)-2n(4n-3p)\}x+\{m(p-m)-p(2n+p)\}y,$$

take

$$3\left\{p\left(2n-\frac{3p}{2}\right)-\frac{p}{2}(2m-3p)\right\}x-\{p(p-m)+2n(2n+p)\}y,$$

and find the value of the difference when

$$x=n=\tfrac{1}{2},\ y=m=-2.$$

2. Divide

$$4a(a+b+c)+10bc-3(b^2+c^2) \text{ by } 2a+3b-c,$$

and find the continued product of

$$2a^{\frac{3}{2}}-3ax^{\frac{1}{2}},\ 3a^{-\frac{1}{2}}+2x^{-\frac{1}{2}},\ 4a^{\frac{1}{2}}x^{\frac{1}{2}}+9a^{-\frac{1}{2}}x^{\frac{3}{2}}.$$

3. Find the Greatest Common Measure of

$$7a^3-6a^2b-18ab^2+4b^3,\ 14a^3-19a^2b-32ab^2+28b^3,$$

and the Least Common Multiple of

$$8x^3+27,\ 16x^4+36x^2+81,\ 6x^2-5x-6.$$

4. Simplify

(*a*) $\dfrac{4}{x(x-2)}+\dfrac{1}{x^2-5x+6}-\dfrac{3}{x(x-3)}$.

(*b*) $\left\{\dfrac{x+2a}{a-2x}-\dfrac{a+2x}{x-2a}\right\}\times\left\{\dfrac{3}{2a-x}-\dfrac{1}{a-x}\right\}$.

5. Write down the fifth power of $a+\sqrt{-x}$, and shew that it will be both real and rational if $x=(5\pm2\sqrt{5})a^2$.

6. Extract the square root of

$$81(x^4+1)+36x(x^2-1)-158x^2.$$

The first and second of the three digits by which a perfect square is expressed are 1 and $2n$ respectively; find the third digit, and shew that, whatever be the scale of notation, a perfect square would be obtained either by reversing the order of these three digits, or by inserting the same number of cyphers between the first and second and between the second and third of the digits.

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7. Solve the equations:

(*a*) $\dfrac{6x-7}{9x+6}-\dfrac{5(x-1)}{12x+8}=\dfrac{1}{12}$.

(*b*) $\dfrac{4x^3(2x-3)+3}{2(x^2-1)-x}=4x(x-1)+15$.

(*c*) $\begin{cases}\dfrac{x}{a}-\dfrac{y}{b}=2.\\ \sqrt{2a(b-y)}=\sqrt{b}(\sqrt{a}+\sqrt{x}).\end{cases}$

8. Prove that a quadratic equation cannot have more than two roots.

Form an equation whose roots shall be the cubes of the roots of the equation

$$2x(x-a)=a^2.$$

9. If $a:b::b:c$, shew that

(1) $a-b:b-c::b:c$.

(2) $a:c::a^2+b^2:b^2+c^2$.

10. A grocer gains 20 per cent. by selling at 2*s.* a lb. a mixture formed by mixing with 7 lbs. of a common tea 2 lbs. of a better kind. But if he had mixed 7 lbs. of the latter with 2 lbs. of the former kind, he would have lost 20 per cent. by selling the mixture at that price. What did each kind of tea cost him per lb.?

11. Sum the series:

(1) $16\frac{1}{2}+14+11\frac{1}{2}+\ldots\ldots$to 14 terms.

(2) $1+2r+3r^2+4r^3+\ldots$to n terms.

If the arithmetic mean between two numbers $=1$, shew that the harmonic mean is the square of the geometric mean.

12. Distinguish between "permutations" and "combinations."

Write down the number of permutations, and deduce the number of combinations, of n things which are all different, taken r together.

A bag contains n sovereigns and n shillings. Prove that the number of different ways in which they can be drawn out in succession, one at a time, is

$$\frac{1\cdot3\cdot5\ldots\ldots(2n-1)}{\lfloor n}\times2^n.$$

13. Expand by means of the Binomial Theorem $\left(2a-\dfrac{x}{2}\right)^7$, and write down in its simplest form the rth term in the expansion of

$$\left(x^n-\frac{n}{x^n}\right)^{-\frac{1}{n}}.$$

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IV. PLANE TRIGONOMETRY.

(Including the solution of triangles.)

[N.B.—*Great importance will be attached to accuracy in results.*]

1. Prove that angle subtended at the centre of a circle by an arc equal in length to its radius is an invariable angle.

One angle of a triangle is $45°$, and the circular measure of another is $1\frac{1}{2}$. Find the third, both in degrees, and in circular measure.

2. Define the secant of an angle, and shew how your definition applies to angles between $180°$ and $270°$.

If $\sec A = -2$, what two values between $0°$ and $360°$ may A have?

3. Obtain a formula embracing all the angles which have a given tangent.

Determine all the values of θ which satisfy the equation:

$$\sqrt{3}\tan^2\theta + 1 = (1+\sqrt{3})\tan\theta.$$

4. Find an expression for $\tan 3A$ in terms of $\tan A$. Shew also that

$$\tan 3A \tan 2A \tan A = \tan 3A - \tan 2A - \tan A.$$

5. Prove that

$$\sin 18° = \frac{\sqrt{5}-1}{4}; \text{ and that } \sin^2 30° = \sin 18° \sin 54°.$$

Shew that in any circle the chord of an arc of $108°$ is equal to the sum of the chords of arcs of $36°$ and $60°$.

6. Demonstrate the identities:

(1) $\dfrac{(\operatorname{cosec} A + \sec A)^2}{\operatorname{cosec}^2 A + \sec^2 A} = 1 + \sin 2A,$

(2) $\sin 3A = 4 \sin A \sin (60° + A) \sin (60° - A),$

(3) $4(\cot^{-1} 3 + \operatorname{cosec}^{-1}\sqrt{5}) = \pi.$

7. What are the advantages gained by the use of logarithms calculated to the base 10?

If $\log_{10} 2 = \cdot 30103$, find the logarithms of 5, $\frac{1}{125}$, and $4\sqrt{\cdot 005}$, to the base 10.

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8. Prove that in any triangle

(1) $2bc \cos A = b^2 + c^2 - a^2$.

(2) $\dfrac{1 + \cos(A - B) \cos C}{1 + \cos(A - C) \cos B} = \dfrac{a^2 + b^2}{a^2 + c^2}$.

9. If r_1 be the radius of a circle touching the side a of a triangle and the other two sides produced, shew that

$$r_1 \cos \frac{A}{2} = a \cos \frac{B}{2} \cos \frac{C}{2}.$$

If a be the side of a regular polygon of n sides, and R, r, the radii respectively of its circumscribed and inscribed circles, prove that

$$R + r = \frac{a}{2} \cot \frac{\pi}{2n}.$$

10. Two sides of a triangle, which are respectively 250 and 200 yards long, contain an angle of $54° \, 36' \, 24''$.

Find the two other angles, having given

$L \cot 27° \, 18' = 10{\cdot}2872338$, diff. for $1' = {\cdot}3100$;

$L \tan 12° \, 8' \, 50'' = 9{\cdot}3329292$; $\log 3 = {\cdot}4771213$.

11. The eye of a soldier in a straight trench of uniform depth is 2 feet above a level plain on which he sees two men standing in the same straight line as the trench; the parts of their bodies above the level of his eye subtending at it the angles $\tan^{-1} {\cdot}00416$ and $\tan^{-1} {\cdot}004$. On walking 200 ft. towards them in the trench he notices that the height of one exactly hides that of the other; and, on approaching 596 feet 8 in. closer still he finds that the portion of the height of the nearer above the level of his eye subtends at it 45°. Find the heights of the men.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[*Great importance will be attached to accuracy.*]

Full marks may be obtained by doing about four-fifths *of this paper.*

1. If two straight lines are parallel, the common section of any two planes passing through them shall be parallel to either of them.

2. If a point be equidistant from the angles of a right-angled triangle, and not in the plane of the triangle, the line joining it with the middle point of the hypotenuse is perpendicular to the plane of the triangle.

3. If two straight lines meeting one another be parallel to two other straight lines, the first two and the other two shall contain equal angles (1) if they are in the same plane; (2) if they are not in the same plane.

4. Every solid angle is contained by plane angles which together are less than four right angles.

5. Having given the transverse and conjugate axes of an hyperbola, shew what is the easiest method of drawing its directrix. Prove the property you make use of.

6. Determine the point of intersection of the lines

$$2x+y=1 \text{ and } y+3x=2,$$

and find the equations of the bisectors of the angles between them.

7. Given a portion of the curve of a parabola, find its vertex and focus, and shew how to complete the curve.

8. A cylinder is cut by a plane which is inclined to its axis at an angle of 60°. Shew that the section is an ellipse, and that its area is double the area of the circular section of the same cylinder.

9. What loci are represented by the following equations:

(1) $2x^2+xy-3y^2+x+4y-1=0.$

(2) $x^2+y^2-2ax+a^2=0.$

(3) $r=a\operatorname{cosec}^2\frac{\theta}{2}.$

10. Prove, either geometrically or analytically, that in the hyperbola the difference between the squares of the semi-conjugates is equal to the difference of the squares of the semi-axes.

11. Transform the equation of the ellipse referred to its axes to a pair of conjugate diameters.

12. Shew that the section of a sphere by a plane is a circle.

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13. Transform the equation

$$ay^2+bxy+cx^2+f=0$$

so as to get rid of the term involving xy, and discuss its varieties, b^2 being unequal to $4ac$.

14. Normals to a parabola intersect at right angles. Determine the locus of their intersection.

VI. PURE MATHEMATICS (2).

[*Great importance will be attached to accuracy in results.*]

1. If $$a+b+c+d=0,$$

prove that

$$abc+bcd+cda+dab=\sqrt{(bc-ad)(ca-bd)(ab-cd)}.$$

2. The consumption of coal by a locomotive varies as the square of the velocity. When the speed is 16 miles an hour the consumption of coal is two tons. If the price of coal be 10*s.* per ton and the other expenses of the engine be 11*s.* 3*d.* an hour, find the least cost of a journey of 100 miles.

3. Eliminate x, y, z between the four equations:

$$x^2-yz=a^2,\quad y^2-zx=b^2,\quad z^2-xy=c^2,\ \text{and}\ x+y+z=0.$$

4. Assuming the Binomial Theorem for a positive integral index, prove it for a fractional one.

Shew that $(1-x^2)^n$

$$=(1+x)^{2n}-2nx(1+x)^{2n-1}+\frac{2n\,.\,(2n-2)}{1\,.\,2}x^2(1+x)^{2n-2}-\&c.$$

Also find the coefficients of x^{12} and x^r in the product of

$$\frac{1+x^3}{(1-x^2)(1-x)}\ \text{and}\ 1-x+x^2.$$

5. In the recurring series

$$\frac{5}{4}-\frac{1}{2}x+2x^2+lx^3+5x^4+7x^5+\&c.,$$

the scale of relation is a quadratic expression; determine the unknown coefficient of the fourth term and the scale of relation, and give the general term of the series.

6. State and prove the rule for the formation of the successive convergents to the continued fraction

$$a+\frac{1}{b+}\,\frac{1}{c+}\ \&c.$$

Find the value of

$$1+\frac{1}{3+}\,\frac{1}{2+}\,\frac{1}{3+}\,\frac{1}{2+}\ \&c.$$

in the form of a quadratic surd.

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7. Prove Demoivre's theorem for positive integers.

Find by means of it the value of

$$(-1+\sqrt{-3})^5+(-1-\sqrt{-3})^5.$$

8. Prove that

$$\frac{\sin x}{x}=\cos\frac{x}{2}\cos\frac{x}{2^2}\cos\frac{x}{2^3}\cos\frac{x}{2^4}\ldots\ldots\text{to infinity.}$$

9. Shew that

$$\theta=\tan\theta-\frac{1}{3}\tan^3\theta+\frac{1}{5}\tan^5\theta-\&c.$$

10. If by eliminating x between the equations

$$x^2+ax+b=0 \quad\text{and}\quad xy+l(x+y)+m=0,$$

a quadratic in y is formed whose roots are the same as those of the original quadratic in x, then either $a=2l$ and $b=m$, or $b+m=al$.

11. In the equation

$$x^n+p_1x^{n-1}+p_2x^{n-2}+\&c.=0,$$

p_r is the first negative coefficient and p_m the magnitude of the greatest negative coefficient. Shew that $1+\sqrt[r]{p_m}$ is a superior limit of the positive roots of the equation.

Find the integers between which the possible roots of the following equation lie:

$$x^6-x^5-6x^4+7x^3+3x^2-10x+10=0.$$

12. Solve the equations

$$x^4+x^3-2x^2+3x-1=0$$

and

$$x^6-4x^4+2x^3-4x^2+4x-1=0,$$

which have two roots common.

VII. PURE MATHEMATICS (3).

[*Great importance will be attached to accuracy in results.*]

1. Differentiate with regard to x the expressions:

$$(1)\quad \log\frac{x^2+x+1}{x^2-x+1}; \qquad (2)\quad (\tan x)^{\tan x}.$$

2. Find the nth differential coefficients, with regard to x, of the expressions:

$$(x+a)^{-1} \quad\text{and}\quad e^{x\cos\theta}\sin(x\sin\theta).$$

3. If $y=e^{x^2}\cos x$, prove that

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=0.$$

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4. Determine the greatest and least values of the expression

$$2x^3 - 21x^2 + 60x + 30.$$

From a fixed point A on the circumference of a circle of radius c, the perpendicular AY is let fall on the tangent at P; prove that the maximum area of the triangle APY is

$$\frac{3}{8}c^2\sqrt{3}.$$

5. Prove that the equation of the tangent at the point (x, y) of the curve $u = f(x, y) = 0$ is

$$(X - x)\frac{du}{dx} + (Y - y)\frac{du}{dy} = 0.$$

Find the length of the perpendicular from the origin on the tangent at the point (x, y) of the curve

$$x^4 + y^4 = c^4.$$

6. Define a point of inflexion, and state the analytical condition for the existence of such a point in a curve.

Find the two points of inflexion of the curve

$$\frac{y}{c} = \frac{x^2}{9a^2} + \left(\frac{x-a}{a}\right)^{\frac{1}{3}},$$

and draw figures shewing the characters of the inflexions.

7. Define the circle of curvature at any point of a curve, and prove that the length of its radius is equal to

$$\frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}}{\frac{d^2y}{dx^2}}.$$

Find the length of the radius of curvature at the end of the latus rectum of a parabola.

8. Define the polar subtangent, and prove that its length is $r^2\frac{d\theta}{dr}$.

Find the asymptote of the curve $r(3\theta - \pi) = a\sin\theta$.

9. Trace the curve $\quad xy^2 = (x - a)^3$.

10. Integrate with regard to x the expressions:

(1) $x\cos 2x$; (2) $\frac{1}{(x+2)(x+3)}$;

(3) $\frac{3x^2 + 4ax + a^2}{x^3 + 2ax^2 + a^2x}$; (4) $\frac{1}{4\sin^2 x + \cos^2 x}$;

(5) x^2e^{2x}.

11. Find the area of the curve

$$a^2y^2 = x^2(a^2 - x^2).$$

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VIII. STATICS.

[N.B.—*Great importance will be attached to accuracy in working.*]

1. In statics, forces are designated as *pressure*, *tension*, *resistance*, *reaction:* explain these terms. Shew how the direction of a tension may be changed without changing its magnitude.

2. It is assumed as a statical principle that when a force acts upon a body the effect of the force will be unchanged at whatever point of its direction it may be applied: illustrate this, and give some reasons for the assumption.

A string suspended from a ceiling supports three equal weights of four pounds, one at its lowest point and each of the others at equal distances from its extremities: find the tensions of the parts into which the string is divided by the weights.

3. If any number of forces acting upon a particle be represented in magnitude and way of action by the sides of a polygon taken in order, they will keep the particle at rest.

If the forces acting on a point be represented by the sides of a regular pentagon taken in order, shew at what angles the forces acting on the point are inclined to each other, and if the forces be so taken as acting on a point, prove independently of the polygon of forces that their resultant is zero.

4. When three forces acting in one plane maintain a rigid body in equilibrium, shew that their lines of action either all meet in a point or are all parallel.

In the latter case, prove the relation that must subsist amongst the forces.

The horizontal roadway of a bridge is 30 feet long and weighs 6 tons, and it rests on similar supports on its ends; what pressure is borne by each of the supports when a carriage weighing 2 tons is one-third of the way across the bridge?

5. Shew that a system of forces not in equilibrium acting in one plane on a rigid body will be equivalent either to a single resultant or to a couple.

ABC is a triangle, AE, BF, CD lines drawn from the angles to the points of bisection of the opposite sides: shew that the forces represented by AE, BF, and CD are in equilibrium.

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6. If three heavy particles are in one plane in given positions, shew how to find the distance of the common centre of gravity of the particles from either one of them.

If three heavy particles be placed in the angles A, B, C of a triangle, the weights of each being proportional to the opposite sides of the triangle a, b, c, prove that the distance of the centre of gravity of the particles from A is equal to

$$\frac{2bc}{a+b+c}\cos\frac{A}{2}.$$

7. A common steelyard supposed uniform is 40 inches long, the weight of the beam is equal to the movable weight, and the greatest weight that can be weighed by it is four times the movable weight: find the place of the fulcrum.

8. State and prove the relation between the power and the weight when they are in equilibrium on a wheel and axle.

9. A uniform ladder 70 feet long is equally inclined to a vertical wall and the horizontal ground, both rough; the weight of a man with his burden ascending the ladder is 2 cwt., and the ladder weighs 4 cwt.: how far up the ladder can the man ascend before it slips, the tangent of the angle of resistance for the wall being $\frac{1}{3}$ and for the ground $\frac{1}{2}$?

10. When is an engine said to work with one horse-power?

How many cubic feet of water will an engine of 100 horse-power raise in one hour from a depth of 150 feet, if the modulus of the engine be ·5; the weight of a cubic foot of water being 62·5 pounds?

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IX. DYNAMICS.

[*Great importance will be attached to accuracy in results.*]

[*N.B.—When needed, the acceleration due to gravity may be taken equal to 32·2 feet per second.*]

1. Define the term acceleration, and explain how it is used in measuring the rate at which a variable velocity is changing.

Prove the formula $2s = ft^2$, connecting the space moved over, the acceleration, and the time.

2. How will the measure of the acceleration due to gravity be changed, if the unit of time is altered from one second to one minute?

Find also what must be the unit of space, if gravity be represented by the number 14, when the unit of time is 5 seconds.

3. Enunciate and prove the theorem known as the Parallelogram of Velocities.

Two steamers, X and Y, are respectively at points A and B, 5 miles apart. X steams away with an uniform velocity of 10 miles an hour in a direction making an angle of 60° with AB. Find in what direction Y must start at the same moment, if it steam with an uniform velocity of $10\sqrt{3}$ miles per hour, in order that it may come into collision with X, and at what angle it will strike it.

4. Prove that the time of falling from rest down a chord of a vertical circle drawn from the highest point is constant.

If particles start from rest from a given point to run down a number of smooth inclined planes, shew that at the end of t seconds they will all be at the same distance from a point $\frac{gt^2}{4}$ feet below that from which they started.

5. Prove that a particle projected (in vacuo) in any direction not vertical, and acted on by gravity, will describe a parabola; and find the latus rectum of the parabola described.

6. A particle is projected from the foot of an inclined plane whose inclination is (β), and in a direction making an angle of 60° with the horizon. If its range on the inclined plane is equal to the distance through which another particle would fall from rest during the time which elapses before the first particle hits the plane, find β.

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7. Two balls of elasticity e impinge directly; find their subsequent velocities.

If $e=\frac{5}{6}$, and their masses are as 2 to 1, and their respective velocities before impact as 1 to 2, and in opposite directions, shew that each ball will move back after impact with $\frac{5}{6}$ths of its original velocity.

8. Two weights are connected by a string which passes over a smooth fixed pulley; determine the motion.

If the weights, being each equal to 8 oz., are in equilibrium, and $\frac{1}{10}$ oz. is then added to one of them, determine how long it will be in descending 10 feet, and what velocity it will acquire in so doing.

9. If a particle run down the arc of a smooth vertical circle, starting from rest at the highest point, prove that it will quit it on reaching a point whose perpendicular distance below the starting point is one-third of the radius.

10. If a seconds pendulum be carried to the top of a mountain half a mile high, how many seconds will it lose in a day, if gravity vary as the inverse square of the distance from the earth's centre, which is supposed to be 4,000 miles from the foot of the mountain?

11. Prove that the principle of Virtual Velocities is true for a single movable pulley when the strings are not parallel.

Define the term *horse-power*, and find that of an engine which will travel at 25 miles per hour up an incline of 1 in 100, the weight of the engine and load being 50 tons, and the resistance 10 lbs. per ton.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1882.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.). (Obligatory.)

[*Great importance will be attached to accuracy in working.*]

1. Define a right angle. Draw a straight line at right angles to a given straight line from a given point in the same.

If the given point be at the extremity of the given line, shew how to draw the perpendicular without producing the given line.

2. Prove that all the interior angles of any rectilineal figure, together with four right angles, are equal to twice as many right angles as the figure has sides.

There are two regular polygons, the number of sides of one is double the number of sides of the other, and an angle of one polygon is to an angle of the other as 9 to 8; find the number of sides of each polygon.

3. Prove that the complements of a parallelogram which are about the diameter of any parallelogram are equal to each other.

If any point P be taken on the diagonal AC of a parallelogram $ABCD$, shew that the sum of the triangles APB, PCD is half the parallelogram.

4. If a given straight line be divided into any two parts, prove that the sum of the squares of the parts, together with twice the rectangle contained by the parts, is invariable, in whatever way the line may be so divided.

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5. In an obtuse-angled triangle, if a perpendicular be drawn from either of the acute angles to the opposite side produced, prove that the square of the side subtending the obtuse angle is greater than the squares of the sides containing the obtuse angle by twice the rectangle contained by the side upon which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

If the triangle ACB be isosceles, C the obtuse angle, AD the perpendicular on BC produced, prove the square on AB is equal to twice the rectangle contained by BC and BD.

6. Prove that equal straight lines in a circle are equally distant from the centre.

Shew that all equal straight lines in a circle may be touched by another circle.

7. When are segments of circles said to be similar?

ABC is an acute-angled triangle, perpendiculars AP and BQ are drawn from A and B to the opposite sides meeting in O; prove that the angles POC and PQC are equal.

8. Describe a square about a given circle. Compare the areas of the squares described about and inscribed in the same circle.

9. An isosceles triangle, having each angle at the base double of the angle at the vertex, is inscribed in a given circle; complete the construction for inscribing a regular pentagon in the circle, and prove that it is equilateral. Shew generally that a pentagon may be equilateral without being equiangular, and equiangular without being equilateral.

10. When are rectilineal figures said to be similar, and when similarly situated?

On a given straight line describe a trapezium similar and similarly situated to a given trapezium. What must be the ratio of the given line to the homologous side of the given figure, so that the figure described upon it shall be nine times the given figure in area?

11. Prove that in any right-angled triangle any rectilineal figure described on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle. Is this true if all the figures so described be semicircles?

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II. ARITHMETIC. (Obligatory.)

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy in numerical results.*]

1. Find the value of

$$10\tfrac{1}{2}\left\{\frac{2\frac{1}{2}}{7}+\frac{3\frac{1}{2}}{9}\right\}-\frac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}+2\frac{1}{7}}\times 4\tfrac{39}{55}.$$

2. Subtract $16\frac{1}{2}\times\dfrac{1\frac{3}{5}}{3+\dfrac{1}{3\frac{1}{3}}}$ from $\dfrac{5\frac{2}{5}\text{ of }7\frac{2}{5}}{8\frac{7}{24}-3\frac{5}{12}}$.

3. Multiply together $\dfrac{1\frac{7}{9}}{9\frac{9}{11}}$, $\dfrac{\frac{27}{64}}{\frac{11}{12}}$, $\dfrac{2\frac{5}{8}}{\frac{21}{160}}$, and $\dfrac{\frac{15}{24}}{4\frac{4}{7}}$.

4. Find the quotient of $\frac{5}{7}$ divided by $1\frac{1}{14}$; divide the result by $\frac{1}{30}$.

5. Add together ·6375 of a gallon, ·01283 of a peck, and ·00412 of a bushel; subtract the result from ·93 of a gallon, and give the answer in pints and the decimal of a pint.

6. Multiply 5·3129 by 46000.

7. Divide 1 by ·00375.

8. Divide 3·8̇1̇ by 2·2̇2̇7̇, and express the answer as a decimal correct to six places.

9. Reduce $\frac{3}{110}$ of half a mile to the decimal of 6 poles.

10. Express 2 lbs. 5 oz. 15 dwts. 12 grs. as the decimal of $7\frac{1}{2}$ lbs. Troy.

11. Divide 317 days. 13 hrs. 31 mins. 57 secs. by 53.

12. Find the dividend on £4146. 12*s.* 6*d.* at 11*s.* 8*d.* in the £.

13. What principal will amount to £2775. 19*s.* in 6 years and 4 months at $7\frac{1}{2}$ per cent. per annum simple interest?

14. If 400 metres be equal to a quarter of a mile, find the number of square metres in a quarter of an acre.

15. A person sold a horse at a loss of 20 per cent.; if he had received £10 more for it, he would have gained 10 per cent. Find the cost of the horse.

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16. A man rode a bicycle from A to B, 54 miles, at an average rate of 8 miles an hour; another man started from A on horseback half an hour after the bicyclist and arrived at B 15 minutes before him. Find the ratio of their speeds.

17. Divide the cube root of $240\frac{17712}{19683}$ by the square root of 176400.

18. Find (by duodecimals) the cubical content of a block of stone 4 ft. 7 ins. long, 4 ft. 2 ins. 5 pts. wide, and 2 ft. 7 ins. 6 pts. high. Express the answer in cubic feet, cubic inches, and the fraction of a cubic inch.

19. A steamer A is in distress and stationary, and fires a gun which is heard on another steamer B coming direct towards her. A fires another gun ten minutes after the first, and this is heard on B 9 mins. 48 secs. after the first. At this moment B is 4 miles from A. How soon will the two steamers be alongside of each other? Give the answer to the nearest second, supposing sound to travel at the rate of 1130 feet per second.

20. A man sold 150 one-hundred-pound shares of railway stock which were paying 5%, at 105. With the proceeds he purchased 4 per cents. at 90 and re-sold them at 96. He then re-invested in the railway stock, which was still at 105 and paying 5%. What was the change in his income?

21. Two sums of money amounting to £1946. 5*s*. were invested, the smaller at 4 per cent. and the larger at $4\frac{1}{2}$ per cent. per annum. At the end of 18 months the simple interest on the two sums amounted together to £125. 4*s*. $10\frac{1}{2}$*d*. What were the two sums?

22. Find log ·024, log $(234)^4$, and log $\sqrt[4]{(15)^{\frac{3}{5}} \times (51)^{\frac{2}{3}}}$, and find the number whose log is 3·1187804.

23. If $5^x = 3125$, find the value of x.

log 13	$= 1{\cdot}1139434$	log 13·145	$= 1{\cdot}1187606$
log 1·7	$= {\cdot}2304489$	log 131·46	$= 2{\cdot}1187936$
log ·3	$= \bar{1}{\cdot}4771213$	log 312	$= 2{\cdot}4941546$
log ·02	$= \bar{2}{\cdot}3010300$	log 313	$= 2{\cdot}4955454$

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III. ALGEBRA. (Obligatory.)

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance will be attached to accuracy in results.*]

1. Simplify the expression

$$a-3\{b-2[a-3b]-2a\},$$

and find the value of

$$\sqrt[5]{\tfrac{3}{5}(3b+c)(b^2-2a-c)(2b-a)^2}$$

when $a=5$; $b=4$; $c=3$.

2. Multiply—

$$x^2+(a-1)x+(a+1) \text{ by } (a-1)x-a^2-a-1,$$

and divide

$$a^3+b^3+c^3-3abc \text{ by } a+b+c.$$

Prove that

$$(a-b)^3+(b-c)^3+(c-a)^3=3(a-b)(b-c)(c-a).$$

3. Find the square root of

$$\frac{a^2}{b^2c^2}+\frac{b^2}{c^2a^2}+\frac{c^2}{a^2b^2}+2\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right),$$

and the cube root of 27081081027.

4. Define a^m and prove that $(a^m)^n=a^{mn}$.

Hence shew that $a^{\frac{1}{m}}=\sqrt[m]{a}$.

Simplify the expression

$$\{(a^{\frac{4}{3}}b^{\frac{3}{4}})^{-\frac{1}{2}}\times(a^{-\frac{3}{4}}b^{-\frac{4}{3}})^{\frac{1}{2}}\}^{-24}.$$

5. Find the G.C.M. of

$$3x^3-13x^2+23x-21 \text{ and } 6x^3+x^2-44x+21$$

and the L.C.M. of

$$x^2+x-2,\ x^2-4x+3, \text{ and } x^2-x-6.$$

6. Reduce to their simplest forms

(1) $$\frac{10x-11}{3(x^2-1)}-\frac{10x-1}{3(x^2+x+1)}+\frac{x^2-2x+5}{(x^3-1)(x+1)}.$$

(2) $$(yz+zx+xy)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)-xyz\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right).$$

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7. Solve the equations:

(1) $\frac{x}{2}+\frac{5x^2-15x-8}{10(x-3)}-\frac{5x-9}{5}=1.$

(2) $3x+\frac{7y}{2}=11y-\frac{2x}{5}+2=22.$

(3) $5\sqrt{\frac{3}{x}}+7\sqrt{\frac{x}{3}}=22\frac{2}{3}.$

8. A grocer has two weights, one as much over a lb. as the other is under a lb., and he finds that on selling 511 lbs. 14 oz. of tea at 2*s.* 6*d.* a pound he gains £2 more by using the lighter weight than he would have done by using the heavier; what were the respective weights?

9. Find the sum of an arithmetic series whose first term is a and common difference d to n terms.

10. The first and third terms of an arithmetical progression are 22 and 14 respectively; how many terms must be taken that the sum may be 64? Explain the double answer.

If $x-a$, $y-a$, and $z-a$ be a geometrical progression, prove that twice $y-a$ is the harmonic mean between $y-x$ and $y-z$.

11. If $a:b::c:d$, prove that

$$ma+nb:ma-nb::mc+nd:mc-nd,$$

and that

$$a^2+c^2:b^2+d^2::ac:bd.$$

12. *Write down* the number of permutations of n things r together, and deduce the number of combinations.

Eight men sit down to two tables at whist; in how many different ways may the games be arranged?

13. Write down the term involving x^r in the expansion of $(1+x)^n$ and in the expansion of $(1-x)^{-n}$.

If p and q be two numbers whose difference $q-p$ is very small compared with either of them, prove that

$\sqrt{\frac{p}{q}}$ is nearly equal to $1-\frac{2}{n}\frac{q-p}{q+p}$.

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IV. PLANE TRIGONOMETRY. (Obligatory.)

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy in results.*]

1. Shew how to express in degrees, minutes, and seconds an angle whose circular measure is known.

Find, correct to three places of decimals, the radius of a circle in which an arc 15 inches long subtends at the centre an angle containing $71^0 36' 3{\cdot}6''$ ($\pi = 3{\cdot}1416$.)

2. Define the *sine* of an angle, and prove that

$$\sin A = \sin(180^0 - A) = \sin\{-(180^0 + A)\}.$$

Write down formulæ including all angles which satisfy

$$(1) \quad 2\sin A = 1,$$

$$(2) \quad 2\sin^2 A = 1.$$

3. Prove that $\cos(A+B) = \cos A \cos B - \sin A \sin B$, and deduce expressions for $\cos 2A$, $\cos 3A$ in terms of $\cos A$.

4. Given $\cos A = {\cdot}28$, determine the value of $\tan \frac{A}{2}$, and explain fully the reason of the ambiguity which presents itself in your result.

5. Prove that

$$(1) \quad \tan\theta + \cot\theta = \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}.$$

$$(2) \quad \sec\theta - \tan\theta = \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right).$$

$$(3) \quad \cos 20^0 + \cos 100^0 + \cos 140^0 = 0.$$

$$(4) \quad \cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}.$$

6. State and prove the rules by means of which you can determine by inspection the integral part of the logarithm of any given number.

Given $\log 4{\cdot}96 = {\cdot}6954817$, $\log 4{\cdot}9601 = {\cdot}6954904$, find the logarithms of 496010, ·000496, and 49600·25.

7. Shew that in *any* plane triangle

$$a = b\cos C + c\cos B.$$

If $c = \sqrt{2}$, $A = 117^0$, $B = 45^0$, find all the other parts of the triangle.

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8. Find the greatest angle of the triangle whose sides are 50, 60, 70 respectively, having given

$$\log 6 = \cdot 7781513,\ L \cos 39^0\ 14' = 9\cdot 8890644,\ \text{diff. } 1' = 1032.$$

9. Express the area of a triangle in terms of one side and the two angles adjacent to it.

Two angles of a triangular field are $22\frac{1}{2}^0$ and 45^0 respectively, and the length of the side opposite to the latter is a furlong. Shew that the field contains exactly two acres and a half.

10. Find an expression for the diameter of the circle which touches one side of a triangle and the other sides produced.

If d_1, d_2, d_3 be the diameters of the three escribed circles of a triangle, shew that

$$d_1d_2 + d_2d_3 + d_3d_1 = (a+b+c)^2.$$

11. A man standing at a certain station on a straight sea-wall observes that the straight lines drawn from that station to two boats lying at anchor are each inclined at 45^0 to the direction of the wall, and when he walks 400 yards along the wall to another station he finds that the former angles of inclination are changed to 15^0 and 75^0 respectively. Find the distance between the boats, and the perpendicular distance of each from the sea-wall.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS. (1.)

[*Great importance will be attached to accuracy in results.*]

1. Prove that planes which have a common normal are parallel.

2. Prove that in any trihedral angle the sum of any two of its angles is greater than the third.

3. Prove that if two straight lines in space be parallel each to a third they are parallel to one another.

4. Prove that a section of a right cone by a plane which cuts all the generating lines is an ellipse, and that the two spheres inscribed in the cone so as to touch the plane touch it in the foci of the ellipse.

5. Prove that if two tangents to a parabola are at right angles the chord of contact passes through the focus.

6. Shew how to describe an ellipse which shall have a given point for one of its foci and shall touch each of three given straight lines.

7. Shew that $ax^2+bxy+cy^2=0$ in general represents two straight lines, and find the condition that they shall be inclined at an angle α to one another.

Find the conditions that the equation $ax^2+by^2+cx+cy=0$ should represent two straight lines.

8. Find the condition that $y=m_1x+a_1$, $y=m_2x+a_2$, $y=m_3x+a_3$ shall all meet in a point.

9. Define an ellipse, and from your definition deduce its equation in the form $ax^2+by^2+c=0$.

10. Define conjugate diameters of an ellipse. Given that x_1y_1 are the co-ordinates of the extremity of a semidiameter of the ellipse $ax^2+by^2+c=0$, find the co-ordinates of the extremity of the conjugate semidiameter.

11. Prove that if an ellipse and hyperbola have the same foci they intersect one another at right angles.

12. Prove that the area is constant of the triangle contained between the axes of x and y and the tangent to the hyperbola $xy=c^2$.

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VI. PURE MATHEMATICS. (2.)

[*Great importance will be attached to accuracy in results.*]

1. Find all the values of x and y which satisfy the equations

$$x^2+y^2=61, \quad x^3-y^3=91.$$

2. Eliminate x and y between

$$\frac{x-a}{a+b}=\frac{y+b}{a-b}=\frac{x^2-y^2}{b(x+b)-a(y-a)};$$

and

$$(ax+by)(bx+ay)=c^2(a-b)(y-x).$$

3. Prove that the sum of the first and last of four quantities in Geometrical Progression will be greater than the sum of the two intermediate terms, unless the sum of the first two terms is negative.

4. Obtain an integral solution of the equation

$$11x-14y=c,$$

and in the particular case in which $c=5$, find the least positive integral values which x and y can have.

5. State and prove Fermat's Theorem respecting prime numbers.

If n be not a multiple of either 3 or 5, prove that n^4-1 must be a multiple of both.

6. Obtain the coefficient of x^{n-1} in the expansion of $\frac{1}{1-3x+2x^2}$ in a series of powers of x.

Shew also that the coefficient of x^{n-1} in the similar expansion of $\frac{1}{1-2x+2x^2}$ is equal to $2^{\frac{n}{2}} \cdot \sin\frac{n\pi}{4}$.

7. Find the limit of the fraction $\frac{\sin\theta}{\theta}$ when θ is indefinitely diminished.

Also if $\sin\phi=2\sin(\theta-\phi)$, find the limiting value of $\frac{\sin\theta}{\sin\phi}$ when θ and ϕ approach to π.

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8. Assuming the truth of Demoivre's Theorem, express the cosine of an angle in a series of powers of its circular measure.

Shew that the series is convergent.

9. The circumference of a semicircle of radius a is divided into n equal arcs. Shew that the sum of the distances of the several points of section from either extremity of the diameter of the semicircle is equal to

$$a\left(\cot\frac{\pi}{4n}-1\right).$$

10. Prove that the number of real roots (if any) of the rational integral equation $f(x)=0$, which lie between a and b, will be even or odd, according as $f(a)$, $f(b)$ have the same or contrary signs.

In what cases can you see by inspection that there must be some real roots lying between a and b?

11. Form an equation whose roots shall be the products of every two of the roots of the equation

$$x^3-ax^2+bx+c=0.$$

12. Solve completely the equation

$$2x^5+x^4+x+2=12x^3+12x^2.$$

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VII. PURE MATHEMATICS. (3.)

[*Great importance will be attached to accuracy in results.*]

1. Find from first principles the differential coefficients of $\sin x$ and ϵ^x respectively.

Do the same also from the expansions (assumed known) of $\sin x$ and ϵ^x in ascending powers of x.

2. Find $\frac{dy}{dx}$ in the following cases:

$$(1)\quad y=\sqrt{\frac{1-x}{1+x+x^2}}; \qquad (2)\quad y=\frac{x\cos^{-1}x}{\sqrt{1-x^2}}.$$

$$(3)\quad (\cos x)^y=(\sin y)^x.$$

3. Find the nth differential coefficient of $\frac{1}{x+a}$.

Apply the result or any other method to find the nth differential coefficient of

$$\frac{1}{x^2+(a+b)x+ab}.$$

Find the coefficient of x^n in the expansion of $\frac{1}{ab+(a+b)x+x^2}$ in ascending powers of x.

4. If $x=\sin\sqrt{y}$ prove that

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2.$$

5. Shew from the meanings of the forms 0^0, ∞^0, 1^∞, that they are indeterminate, and that the first two are restricted in value and the third not so.

Evaluate $$lt_{x=1}\frac{1+\cos\pi x}{\tan^2\pi x}.$$

6. *State fully* the conditions that $f(x)$ shall be a maximum or minimum for any given value of x.

If a parallelogram be inscribed in an ellipse the greatest possible value of its perimeter is equal to twice the diagonal of the rectangle described on the axes.

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7. Find the equation of the tangent at the point x, y, of the curve $y=f(x)$ and deduce that of the normal.

If p_1 and p_2 be the perpendiculars from the origin on the tangent and normal respectively, and if

$$\tan\theta=\frac{dy}{dx},$$

prove that

$$p_1=y\cos\theta-x\sin\theta \text{ and } p_2=x\cos\theta+y\sin\theta.$$

Hence prove that $$p_2=-\frac{dp_1}{d\theta}.$$

8. If r and p have their ordinary meanings in polar curves, prove that the radius of curvature at any point is $r\dfrac{dr}{dp}$.

In the curve $r=a\sec 2\theta$ prove that the radius of curvature is

$$-\frac{1}{3}\frac{r^4}{p^3}.$$

9. Find the oblique asymptote of the curve

$$\left(\frac{y}{x+a}\right)^3=\frac{x-a}{x+2a},$$

and prove that it cuts the curve at the angle whose tangent is 8.

10. Trace the curve whose equation is given in the last question.

11. Find the integrals of the following differentials:

(1) $\sqrt{a^2-x^2}dx$; (2) $\dfrac{dx}{1+x-2x^2}$;

(3) $\sec x dx$; (4) $\dfrac{dx}{(e^x+e^{-x})^2}$.

12. Find the length of the arc between the origin and any given point on the curve $ay^4=x^5$.

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VIII. STATICS.

[N.B.—*Great importance will be attached to accuracy in working.*]

1. State what are the axioms assumed in the Theory of Statics.

2. Shew that if three forces acting on a point keep it in equilibrium, each force is proportional to the sine of the angle between the directions of the other two.

3. A weight Q hanging freely over a pulley supports P, which rests on an inclined plane. Determine the position of equilibrium and the pressure on the inclined plane.

4. Two forces act upon a rigid body in the same plane but not at the same point. Determine the magnitude, the point of application, and the direction of the resultant.

5. Explain what is meant by the *arm*, the *moment*, and the *axis* of a *couple;* and prove that the effect of a couple upon the equilibrium of a body is not altered if its *arm* be turned through any angle about one extremity in the plane of the couple.

6. Draw two diagrams representing the system of pullies in which $W=2^n P$, and that in which $W=(2^n-1)P$, and state what are their respective advantages.

7. Weights in the proportion of 1, 2, 3 are placed at the three angles of a triangle; find by a geometrical construction the centre of gravity.

8. What laws have been established by experiment as regards the friction of plane surfaces? How is the friction estimated? Shew that the coefficient of friction between two given substances is equal to the tangent of the inclination of the plane of one of them when the body formed of the other is about to slide down.

9. The lengths of the arms of a false balance are (a) and (b), and the weight W appears to balance P at the shorter arm (b), and Q at the longer arm (a). Shew that if the balance be of uniform density and thickness,

$$\frac{a}{b}=\frac{P-W}{W-Q}.$$

10. A uniform beam AB of given length and weight has its extremity A resting in a horizontal groove AC, and its extremity B in a vertical groove BC, and is kept in equilibrium by a string DC fixed at a given point D on the beam. Find the tension of the string, and the limits, as to the length and point of attachment of the string, under which equilibrium is possible.

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IX. DYNAMICS.

[*Great importance will be attached to accuracy of working.*]

(N.B.—*When needed the measure of the force of gravity may be taken as 32 feet.*)

1. If a particle moves uniformly in a circle, distinguish between the angular and linear velocities. What is the relative velocity of two particles moving uniformly in a straight line (1) in the same, (2) in opposite directions? What is the relative angular velocity of the hour and minute hands of a clock, and their relative linear velocity, if the minute hand be 9 inches and the hour hand 3 inches in length?

Two trains whose lengths respectively were 130 and 110 feet, moving in opposite directions on parallel rails, were observed to be 4 seconds in completely passing each other, the velocity of the longest train being double that of the other; find at what rate per hour each train is moving.

2. Enunciate the first law of motion. State briefly the evidence on which we accept the truth of the law. How is the velocity of a body affected when acted on by a uniformly accelerating force? If a body projected upwards with a velocity (u), ascend through a space (s), obtain the equation $v^2 = u^2 - 64s$.

A tower is 288 feet high; at the same instant one body is dropped from the top of the tower and another projected vertically upwards from the bottom, and they meet half way; find the initial velocity of the projected body, and its velocity when it meets the descending body.

3. Prove that the times of descent down all chords in a vertical circle, whether drawn from the highest or the lowest points of the circle, are constant.

Two vertical circles whose radii are 10 and 6 feet touch each other at the highest point; a straight line is drawn from the point of contact to meet the outer circle; find the time of describing from rest the portion of this line intercepted between the two circles.

4. What is understood by the parallelogram of velocities? Find the resultant velocity of two uniform component velocities.

A particle moves in a straight line along a horizontal smooth plane with a velocity of 3 feet per second; after 2 seconds a velocity of 8 feet per second is imparted to it in a direction at right angles to its original motion; find the distance of the particle from its starting point after it has been in motion for 4 seconds.

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5. Find the time of flight and greatest height of a projectile with reference to the horizontal plane passing through the point of projection.

If (t) be the time in which the projectile reaches a point P, and (t') the time from P until it strikes the horizontal plane through the point of projection, prove that the height of P above that plane is $\frac{1}{2}gtt'$. Hence verify the expression for the greatest height.

6. State the third law of motion. If (W) be the weight of a body in pounds, what assumptions are made and what units are referred to in obtaining the expression $\left(\frac{W}{g}\right)$ for the mass of the body?

A heavy body is placed on a smooth horizontal table, a pressure of 6 lbs. acts continuously upon it; at the end of three seconds the body is moving with a velocity of 48 feet in a second; find the weight of the body.

7. A weight (W) is drawn up a smooth inclined plane by means of a string, to the other end of which a weight (P) is attached that hangs freely over the top of the plane; find the accelerating force and the tension of the string.

If both (P) and (W) be 8 lbs., the inclination of the plane 30°, and the string be just on the point of breaking, find the greatest weight which the string would support if it were suspended from a fixed point vertically.

8. Define an impulsive force. How is such a force estimated? When one elastic ball impinges directly on another, describe briefly the action supposed to take place during their impact.

An elastic ball (m) moving with a given velocity impinges in direct impact on (m') at rest; find the velocity of (m') after impact, and determine the ratio of the relative velocity of the balls after impact to the original velocity of (m).

9. Point out briefly how a simple pendulum may be used to determine the force of gravity at the place where it swings.

A pendulum, whose length is L, makes (m) oscillations in a day: its length is changed so that it makes ($m+n$) oscillations in a day; shew that $\frac{2n}{m}L$ is the measure of this change nearly.

10. In the theory of work what is meant by a foot-pound? Shew that the kinetic energy of a body in motion is equal to half its "vis viva."

A train is moving on a horizontal rail at the rate of 15 miles an hour; if the steam be suddenly turned off, how far will it run before it stops, the resistances being taken at 8 pounds per ton?

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1883.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Great importance will be attached to accuracy.*]

1. If two straight lines cut one another, the vertical, or opposite, angles shall be equal.

ABC is a triangle, BD, CE lines drawn making equal angles with BC, and meeting the opposite sides in D and E and each other in F: prove that if the angle AFE is equal to the angle AFD the triangle is isosceles.

2. Triangles on equal bases, and between the same parallels, are equal to one another.

ACB is a triangle, CD, BE parallel lines meeting AB and AC produced respectively in D and E: prove that if the triangles BCE, ACB are equal D is the middle point of AB.

3. In any right-angled triangle, the square which is described on the side subtending the right angle is equal to the squares described on the sides which contain the right angle.

If the squares on the first and third sides of a quadrilateral are together equal to those on the second and fourth, the diagonals intersect at right angles.

4. If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

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5. If a straight line drawn through the centre of a circle, cut a straight line in it, which does not pass through the centre, at right angles, it shall bisect it.

Two equal circles have a common chord AB. If a chord AC of one of them, equal to AB, produced backwards pass through the centre of the other, AB is equal to the radius of either circle.

6. The angle at the centre of a circle is double of the angle at the circumference on the same base, that is, on the same part of the circumference.

ABC is an isosceles triangle; on BC as base construct a triangle whose vertical angle shall be equal to either of the equal angles B or C, and which shall be similar to the given triangle.

7. In equal circles, equal angles stand on equal circumferences, whether they be at the centres or circumferences.

AB is a fixed chord in a circle $APQB$, PQ another chord of given length; shew that if AP, BQ meet in R, R will be on the circumference of the same circle for all positions of PQ.

8. Describe a circle about a given triangle.

If $ABCD$ is a parallelogram, and BE makes with AB the angle ABE equal to the angle BAD, and meets DC produced in E, the circles described about the triangles BCD, BED will be equal.

9. If the radius AB of a circle is divided in C so that the rectangle AB, BC is equal to the square on AC, and the chord BD is equal to AC, the circle of which AD is a chord and which touches BD will pass through C, and the triangle ACD will be isosceles.

10. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or those sides produced, proportionally.

Prove the following construction for trisecting a line AB in G and H:

On AB as diagonal construct a parallelogram $ACBD$; bisect AC, BD, in E and F. Join DE, FC, cutting AB in G and H.

11. Triangles which have one angle of the one equal to one angle of the other, and their sides about the equal angles reciprocally proportional, are equal to one another.

If ABC is a triangle right-angled at B, and BD the perpendicular on AC is produced to E so that DE is a third proportional to BD and DC, the triangle ADE will be equal to the triangle BDC.

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II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Find the value of

$$\frac{11}{221}\left\{\left(\frac{4\frac{1}{11}}{20}+\frac{70}{8\frac{9}{4}}\right)-1\frac{11}{16}\left(\frac{3\frac{2}{3}+6\frac{1}{10}}{4\frac{1}{9}-3\frac{2}{3}}\div 4\frac{79}{200}\right)\right\}.$$

2. Subtract

$$1\frac{1}{1\frac{1}{5}}\text{ of }\frac{2\frac{2}{7}}{4\frac{1}{9}-\frac{2}{2\frac{1}{3}}}\text{ from }\left\{3+\frac{2\frac{1}{3}-1\frac{5}{8}}{1\frac{1}{40}\left(2\frac{1}{4}+\frac{3\frac{1}{2}}{6}\right)}\right\}.$$

3. Multiply together

$$\frac{6\frac{9}{11}}{1\frac{1}{14}},\ \frac{7\frac{10}{19}}{\frac{29}{57}},\ \frac{3\frac{5}{18}}{\frac{17}{25}},\text{ and }\frac{1\frac{5}{46}}{9\frac{2}{5}}.$$

4. What fraction of 2 is the quotient of $11\frac{1}{111}$ divided by $16\frac{19}{37}$?

5. Add together ·5125 of a yard, ·62734 of a pole, and ·018325 of a furlong; subtract the result from ·0049 of a mile, and express the answer as the decimal of a yard.

6. Multiply 17·081 by 3·00091.

7. Divide 121110 by ·011.

8. Divide $9{\cdot}1\dot{6}$ by $\frac{1}{19{\cdot}\dot{6}\dot{3}}$.

9. Reduce $\frac{7}{121}$ of half a rood to the decimal of $\frac{10}{11}$ of an acre.

10. Express 9 hours 19 minutes and 3 seconds as the decimal of 9 days.

11. Divide 110 tons 11 cwt. 2 qrs. 16 lbs. 8 oz. by 79.

12. Find the dividend on £9,648. 8*s.* at 16*s.* $0\frac{1}{2}$*d.* in the £.

13. At what rate per cent. per annum will £1,885. 15*s.* amount to £2,569. 6*s.* $8\frac{1}{4}$*d.* in 7 years and 3 months, simple interest?

14. The French unit of volume is the stère, which is a cube whose side is a mètre.

Supposing a linear yard to be $\frac{22}{25}$ of a mètre, find to two places of decimals the difference, in cubic inches, between the volume of a cubic yard and $\frac{1}{8}$ that of a stère.

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15. A person bought an estate and subsequently sold it for £625 less than he gave for it, thereby losing $1\frac{1}{4}$ per cent. What should he have received in order to gain $12\frac{1}{2}$ per cent.?

16. The first of a series of cogged wheels, working into each other in a straight line, has a certain number of teeth; the number of teeth in the second is to that of the first as 6 : 7; of the third to the second as 5 : 6; and of the fourth to the second as 2 : 3. If the wheels are set in motion, how many revolutions must each wheel make before they are simultaneously in their original positions?

17. A hare sees a hound 176 yards away from her and scuds off in the opposite direction at a speed of 12 miles an hour; thirty seconds later the hound perceives her and gives chase at a speed of 18 miles an hour. How soon will he overtake the hare, and at what distance from the spot whence the hare took flight?

18. Express, as a decimal, the square root of $6{\cdot}24\dot{9}$; also find the length of the edge of a cube of metal which cost £5,407. 8*s*. $11\frac{1}{2}$*d*.; one cubic inch being valued at 8*s*. 4*d*.

19. Find, by duodecimals, the cubical content of a block 2 ft. 9 ins. long, 1 ft. 8 ins. wide, and 1 ft. 4 ins. deep. If the weight of this block is 11 cwt., find what would be the length of a bar of the same material, having a sectional area of $\frac{5}{6}$ of a square foot and weighing 18 cwt.

20. If a pipe of 9-inch bore discharges a certain quantity of water in 6 hours, how long would 4 pipes of 6 inches bore take to discharge three times the quantity? (The rates of discharge are as the squares of the diameters of the pipes.)

21. Stock to the amount of £2,700 was sold at 90 and reinvested in the 5 per cents. at 125; what will the annual income be? (To be worked entirely by logarithms.)

22. Find $\log 210$, $1\frac{1}{6} \times \log {\cdot}182$, and $\log \sqrt{13\sqrt{5} \div \sqrt{7}}$.
By means of logarithms find the number which is equal to

$$\left(\frac{4^3 \times 25^2}{2^6 \times 10^3}\right)^{\frac{1}{2}}.$$

23. $\text{Log } 2^{x+3} = 1{\cdot}2221818$, find the value of x.

$\log 2 = {\cdot}3010300$	$\log 1{\cdot}3 = {\cdot}1139434$
$\log 3 = {\cdot}4771213$	$\log 9{\cdot}72 = {\cdot}9876665$
$\log {\cdot}7 = \bar{1}{\cdot}8450980$	$\log 25 = 1{\cdot}3979400$

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III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Simplify the expression

$$a-[3a+c-\{4a-(3b-c)\}+3b],$$

and find the value of

$$\frac{a}{b}-\sqrt{\frac{1+a}{1-b}}+\sqrt[5]{\frac{5a^2}{2b^4}} \text{ when } a=\tfrac{1}{4} \text{ and } b=\tfrac{1}{8}.$$

2. Multiply

$$\frac{x^3}{2}-5\frac{x^2}{3}+\frac{x}{12}+3 \text{ by } \frac{x^2}{2}-x+3.$$

Prove that

$$a\left(\frac{b^3-c^3}{b-c}\right)+b\left(\frac{c^3-a^3}{c-a}\right)+c\left(\frac{a^3-b^3}{a-b}\right)=(a+b+c)(bc+ca+ab).$$

3. Resolve into factors

$$x^2-7x+12;\ 3x^2-6x-9;\ 2x^2-6x-8.$$

Also find their Least Common Multiple.

4. Extract the fourth root of

$$x^8-\frac{3a^2x^7}{b}+\frac{27a^4x^6}{8b^2}-\frac{27a^6x^5}{16b^3}+\frac{81a^8x^4}{256b^4}.$$

5. Obtain a meaning for the expressions

$$a^{-m},\quad a^0.$$

Divide

$$x^{\frac{5}{2}}-x^2-4x^{\frac{3}{2}}+6x-2x^{\frac{1}{2}} \text{ by } x^{\frac{3}{2}}-4x^{\frac{1}{2}}+2.$$

Simplify

$$\sqrt{12}+\frac{1}{10}\sqrt{75}+6\sqrt{\frac{1}{12}}.$$

6. Reduce to their simplest forms

(1) $\dfrac{1}{x-1}-\dfrac{3}{x+1}+\dfrac{2(x-2)}{x^2+1}.$

(2) $\left(2-\dfrac{3n}{m}+\dfrac{9n^2-2m^2}{m^2+2mn}\right)\div\left(\dfrac{1}{m}-\dfrac{1}{m-2n-\dfrac{4n^2}{m+n}}\right).$

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7. Solve the equations

(1) $\dfrac{1}{3x-1}+\dfrac{2(x+1)}{x-1}-\dfrac{3x^2+1}{3x^2-4x+1}=1.$

(2) $7x-13y=6x-10y-8=0.$

(3) $x+4y=7;\ x^2+y^2=10.$

8. A debt which might have been paid exactly with $5x$ half-sovereigns and x half-crowns was paid out of a £10 note, and the change was found to be equal to $15x$ half-crowns and x half-sovereigns. Find x and the amount of the debt.

9. Divide 183 into three parts in Geometrical Progression, and such that the sum of the first and third is $2\frac{1}{10}$ times the second.

10. Transform 1022634 from the septenary to the denary scale, and 124·96 from the denary scale to that with radix 5.

11. If $a+b:b+c::c+d:d+a$,

prove that $$a=c$$

or $$a+b+c+d=0.$$

12. Assuming the number of permutations of n things r together to be known, find a formula for the corresponding number of combinations; and shew that the number of the latter is the same as when the things are taken $n-r$ together.

If there are in a bag 10 white and 6 red balls, in how many different ways may 6 balls be drawn out, so that in each drawing there may be at least 2 red balls?

13. Write down the coefficient of x^9 in $(5a^3-4x^3)^7$; also the middle term of $(1+x)^{2n}$, and shew that this latter is the same as the $\overline{n+1}|^{\text{th}}$ term of $(1-4x)^{-\frac{1}{2}}$.

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IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Assuming that a circle may be treated as a regular polygon with an infinite number of sides, shew that the ratio of the circumference of a circle to its diameter is constant. What is the circular measure of the least angle whose sine is $\frac{1}{2}$, and what is the measure in degrees, &c., of the angle whose circular measure is $\frac{3\cdot1416}{20}$?

2. Shew that the sine and the cosecant of any angle are respectively the same as the sine and cosecant of the supplement of the angle. Is this true of the cosine and secant of the angle?

Find the least positive value of A that satisfies the equation

$$2\sqrt{3}\cos^2 A = \sin A,$$

and write down the general value of A if $(\sec A)^2 = \frac{4}{3}$.

3. Prove by a geometrical construction

$$\cos 2A = (\cos A)^2 - (\sin A)^2,$$

without assuming or proving the ordinary formula for $\cos(A+B)$.

Prove $$\cos 2A = \frac{1-(\tan A)^2}{1+(\tan A)^2}.$$

4. Prove $$\tan A = \frac{2\tan\frac{A}{2}}{1-\left(\tan\frac{A}{2}\right)^2}.$$

From this formula find $\tan\frac{A}{2}$ if $\tan A = \sqrt{3}$, and explain the double answer.

Prove $$\tan 75^\circ + \cot 75^\circ = 4.$$

5. Prove

(1) $2\cos\frac{A}{2} = -\sqrt{1+\sin A} - \sqrt{1-\sin A}$, if A is between 270° and 360°.

(2) $\sin^{-1}\frac{2}{2\cdot5} + \sin^{-1}\frac{1\cdot5}{2\cdot5} = 90^\circ$.

(3) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cdot\cot\frac{B}{2}\cdot\cot\frac{C}{2}$, if $A+B+C=180^\circ$.

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6. What is meant by a system of logarithms?

Shew that in all systems log $1=0$, and that the logarithms of all proper fractions are negative.

Given $\log_{10}2=\cdot3010300$ find $\log_{10}125$, and express with its proper characteristic $\log_{10}\cdot0005$; find the tabulated logarithm of sin 30°.

Given Log sin $60°=9\cdot9375306$, find $\log_{10}3$.

7. If A, B, C be the angles of a triangle, and a, b, c the sides opposite to them respectively, prove

(1) $\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$.

(2) $b-c=a\cos\theta$; where $\tan\theta=\frac{2\sqrt{bc}}{b-c}\sin\frac{A}{2}$.

8. Find the radius of the circle circumscribing a triangle, in terms of its sides a, b, c.

If $c^2=a^2+b^2$, shew that the expression obtained for the radius becomes $\frac{c}{2}$.

9. For what data will the solution of a triangle become ambiguous? Explain this.

Given $B=30°$, $c=150$, $b=50\sqrt{3}$, shew that of the two triangles that satisfy the data one will be isosceles and the other right-angled. Find the third side in the greatest of these triangles.

Would the solution be ambiguous if the data had been

$$B=30°,\ c=150,\ b=75?$$

10. A flagstaff 100 feet high stands in the centre of an equilateral triangle which is horizontal; from the top of the flagstaff each side of the triangle subtends an angle of 60°; find a side of the triangle.

AB is a horizontal line whose length is 400 yards; from a point in the line between A and B a balloon ascends vertically, and after a certain time its altitude is taken simultaneously from A and B; at A it is observed to be 64° 15′; at B 48° 20′; find the height of the balloon when the observations are taken.

Given L sin 64° 15′ $=9\cdot9545793$.
L sin 48° 20′ $=9\cdot8733352$.
L sin 67° 25′ $=9\cdot9653532$.
$\log_{10}2\cdot29149=\cdot4646213$.
$\log_{10}2\qquad=\cdot30103$.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[*Great importance will be attached to accuracy.*]

1. If four straight lines meet in a point and one be at right angles to the other three, shew that these three lines lie in a plane.

2. If two straight lines are parallel, the line of intersection of any two planes passing through them is parallel to either.

3. Shew that every solid angle is contained by plane angles whose sum is less than four right angles.

4. Shew that the section of a cylinder by a plane inclined at an angle α to the axis is an ellipse, and express the eccentricity in terms of α.

5. Prove that the semi-latus rectum of a parabola is a harmonic mean between the segments of a focal chord.

6. Shew that the straight lines joining the extremities of two focal chords of a conic intersect in the directrix.

Deduce a relation respecting tangents from this theorem.

7. Define supplemental chords of an ellipse, and shew that they are parallel to conjugate diameters.

Hence find the position of the equi-conjugate diameters.

8. Shew that the linear equation $lx+my+n=0$ represents a straight line, and give the geometric meaning of the several constants.

9. Find an expression for the tangent of the angle between the lines

$$lx+my+n=0\,;\ l_1x+m_1y+n_1=0\,;$$

and deduce the condition that they may be (1) parallel, (2) perpendicular to one another.

10. Define the terms pole and polar.

Find the polar of the point (hk) with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; and find the locus of the pole when the perpendicular on the polar from the origin is constant.

11. Find the conditions that the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

should represent an ellipse, hyperbola, and parabola respectively.

12. A straight line of given length slides between two straight lines at right angles to one another. Find the locus of a point dividing it in a fixed ratio.

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13. Find the equation to the tangent to the rectangular hyperbola $xy=c^2$.

Shew that its pole with respect to the circle $x^2+y^2=a^2$ lies on a concentric and similarly placed rectangular hyperbola.

VI. PURE MATHEMATICS (2).

[*Great importance will be attached to accuracy in results.*]

1. Solve the equations

$$2x+y=2z,\ 9z-7x=6y,\ x^3+y^3+z^3=216.$$

2. If x be a real quantity, determine the limits of value between which the expression $\dfrac{2x^2+6x+3}{2x+1}$ must lie.

3. Find the coefficient of x^r in the expansion of $\dfrac{a-bx+cx^2}{e^x}$.

4. Prove that every convergent is nearer to the continued fraction than any of the preceding convergents. Hence determine limits to the error made in taking any convergent for the continued fraction.

Find the 5th convergent to the positive root of $2x^2-5x-4=0$.

5. If C_r denote the number of combinations of n things taken r together, prove that

$$\frac{1}{n}C_1+\frac{1}{n-1}C_2+\frac{1}{n-2}C_3+\&c.+\frac{1}{1}C_n=\frac{2}{n+1}(2^n-1).$$

6. If the n quantities $a, b, c, \ldots k$ be positive and unequal, prove that

$$\left(\frac{a+b+c+\ldots+k}{n}\right)^n \text{ is greater than } (abc\ldots k).$$

7. If θ be the circular measure of a positive angle less than a right angle, prove that $\sin\theta$ is greater than $\theta-\dfrac{\theta^3}{4}$; and that $\cos\theta$ is less than

$$1-\frac{\theta^2}{2}+\frac{\theta^4}{16}.$$

Given $\dfrac{\sin\theta}{\theta}=\dfrac{1013}{1014}$; prove that the value of θ is $4^0\ 24'$ nearly.

8. Expand $(\sin\theta)^{10}$ in terms of cosines of multiples of θ; and prove that

$$\log_e\left(2^{\frac{1}{2}}\cos\frac{\theta}{2}\right)=\frac{\cos\theta}{2}-\frac{\cos^2\theta}{4}+\frac{\cos^3\theta}{6}-\&c.$$

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9. Expand θ in terms of $\tan\theta$; and prove that

$$\frac{\pi}{4}=4\tan^{-1}\tfrac{1}{5}-\tan^{-1}\tfrac{1}{70}+\tan^{-1}\tfrac{1}{99}.$$

10. Shew how to transform an equation into another, the roots of which shall be less than those of the proposed equation by a constant difference.

Transform $x^3-9x^2+4x-8=0$ into an equation wanting the second term.

11. Solve the equation $x^3+6x^2-13x-42=0$, which has roots in Arithmetical Progression.

12. Enunciate Des Cartes' rule of signs; and shew that the equation

$$x^4+3x^2-10x-5=0,$$

has two real roots of contrary signs; that it cannot have more real roots; and that these roots lie between -1 and 2.

VII. PURE MATHEMATICS (3).

[*Great importance will be attached to accuracy in results.*]

Full marks may be obtained by doing about three-quarters of this paper.

1. Find, without applying the rules for differentiation, but assuming the ordinary expansions of Trigonometry, the differential coefficients of

$$\sqrt[3]{1+x^2+x^4},\text{ and }\log(\sin x).$$

2. Find $\frac{dy}{dx}$ in the following cases:—

(1) $y=\cos\left(a\sin^{-1}\frac{1}{x}\right)$.

(2) $\operatorname{Tan} y=\sin x\,.\,e^{(\cos x)^2}$.

(3) $e^y=\frac{(a+bx^n)^{\frac{1}{2}}-a^{\frac{1}{2}}}{(bx^n)^{\frac{1}{2}}}$.

3. Enunciate Leibnitz's theorem for the successive differentiation of the product of two functions of x.

If $y=x\sin x$, find the value of $\frac{d^4y}{dx^4}$ when $x=\frac{\pi}{2}$.

Shew also that whatever be the value of x

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+y(x^2+2)=0.$$

4. Shew that the values of x which make $f(x)$ a maximum or minimum must generally be looked for among the roots of $f'(x)=0$ and $f'(x)=\infty$.

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Point out the other necessary conditions, if any, and give tests to distinguish between maxima and minima values.

O is a fixed point without a circle, A one of the extremities of the diameter through O, OQQ' a chord through O; find its position when the area of the triangle QAQ' is a maximum.

Does it ever become a minimum?

5. Apply Maclaurin's theorem to obtain the three first terms in the expansion of $\log(1+\tan x)$.

6. Shew how to determine the existence and position of asymptotes in curves given by polar equations.

Example:— $$r=\frac{a\sin 2\theta}{\sin^3\theta+\cos^3\theta}.$$

7. If $y=f(x)$ is the equation to a curve, obtain an expression for p, the perpendicular on the tangent from the origin. What form does this assume when the equation to the curve is homogeneous in x and y?

In the case of the curve

$$\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1,$$

find when p is the greatest possible.

8. Prove that the envelope of a system of curves touches all the curves of the system which intersect.

Find the envelope of the curves

$$\frac{a^2\cos\theta}{x}-\frac{b^2\sin\theta}{y}=\frac{c^2}{a}.$$

9. Shew how to find the position and magnitude of the circle which has the closest possible contact with a given curve at a given point. Does it necessarily cross the curve?

Obtain the equation to the circle of curvature of a parabola at one extremity of the latus rectum, and find where it again cuts the curve.

10. Trace the curve

$$2x(x^2+y^2)=a(2x^2+y^2).$$

Find the position of the points of inflection by means of polar co-ordinates.

11. Integrate the following differentials:

$$(1)\ \frac{dx}{\sin x};\quad (2)\ \frac{dx}{\sqrt{1-x-x^2}};\quad (3)\ \sqrt{\frac{x}{a+x}}\cdot dx;$$

$$(4)\ e^{ax}\sin rx\,dx.$$

12. Find the area included between two equal parabolas having their vertices at the same point and their axes at right angles.

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VIII. STATICS.

[*Great importance will be attached to accuracy.*]

1. When are statical forces said to be in equilibrium? When three forces act upon a point in one plane, state all the geometrical conditions of their equilibrium. Could three forces of 9 lbs., 4 lbs., 3 lbs., acting in one plane, have their directions so adjusted as to be in equilibrium at a point? If three equal forces acting on a point are represented by the three equal sides taken in order of a regular hexagon, shew geometrically what their resultant will be, and find its magnitude if each of the equal forces be 10 pounds.

2. Shew how the truth of the parallelogram of forces may be verified by experiment.

If $ABCD$ is any quadrilateral figure, and O be the point of intersection of the two straight lines bisecting the opposite sides of the quadrilateral, then the forces represented in magnitude and direction by OA, OB, OC, OD will be in equilibrium at O.

3. If three forces acting in one plane are in equilibrium they either pass through a point or are parallel. In the latter case prove the conditions of equilibrium if the two extreme forces have the same direction.

A uniform lever is 18 inches long, and each inch in length weighs one ounce; find the place of the fulcrum when a weight of 27 ounces at one end of the lever balances a weight of 9 ounces at the other end. If the least of the weights be doubled, how must the position of the fulcrum be shifted to preserve the equilibrium?

4. What is the moment of a couple? Shew that two unlike couples in the same plane will balance each other if their moments are equal.

If any two polygons are of equal areas and in one plane, shew that the forces represented by the sides of the one, taken in order, will be in equilibrium with the forces represented by the sides of the other, taken in order, in the opposite direction.

5. Given the centre of gravity of a plane area and the centre of gravity of any portion of it, find the centre of gravity of the remainder.

From a circular area of radius (r) a smaller circular area one-quarter the size is so stamped out that its centre bisects a radius of the larger circle; find the centre of gravity of the remainder.

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6. In the use of the simple machines called "mechanical powers," what is meant by a mechanical advantage? In any two of these machines state numerically the mechanical advantage.

Describe the construction of a pair of toothed wheels used as mechanical power, and find the equilibrium ratio between the power and the weight; express also that ratio when the teeth are very small compared with the radii of the wheels.

7. Shew how the two following laws of statical friction may be established by experiment:

(1) Limiting friction is proportional to the pressure:

(2) Limiting friction is independent of the surfaces in contact.

A weight of 30 lbs. is just supported on a rough inclined plane (coefficient of friction $\frac{3}{4}$) whose height is $\frac{3}{5}$ths of its length. Shew that it will require a force of 36 lbs. acting parallel to the plane just to be on the point of moving the weight up the plane.

8. A uniform ladder rests between a vertical wall and the horizontal ground, both rough; if the coefficient of friction for the ladder and wall be $\frac{1}{3}$, and for the ladder and ground $\frac{3}{7}$; find the angle which the ladder makes with the ground when it just begins to slide.

9. State the principle of virtual velocities as applicable to the mechanical powers, and prove that it holds good on a smooth inclined plane when the power acts at any angle to the plane.

Explain the result if the expression for the virtual velocities of the power and weight in this case are equal.

10. How is work measured? How is horse-power estimated numerically?

An engine is required to raise in 3 minutes a weight of 13 hundredweight from a pit whose depth is 840 feet; find the horse-power of the engine.

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IX. DYNAMICS.

[*Great importance will be attached to accuracy of working.*]

[N.B.—*When needed the measure of the accelerating force of gravity may be taken as* 32.]

1. How is velocity measured?

Find to three decimal places the velocity of the extremity of the minute hand of a watch an inch long, if $\pi = \frac{22}{7}$, the units of space and time being inches and minutes respectively.

2. If the velocity of the point A in fixed space is known, and also the velocity of the point B relative to A, how is B's velocity in fixed space determined?

If a watch be laid face upwards on a table and moved without rotation parallel to the line from 9 to 3 o'clock with the velocity with which the extremity P of the minute hand moves when the watch is at rest, prove that the velocity of P in fixed space is perpendicular and proportional to the line joining P at the instant with the position of P at any half-hour.

3. Find the space described by a moving particle in any time when there is uniform acceleration in the line of motion, and apply the result to determine the motion of a projectile.

A wet open umbrella is held with the handle upright and made to rotate round that handle at the rate of 14 revolutions in 33 seconds. If the rim of the umbrella be a circle of one yard diameter and its height above the ground be 4 feet, prove that the drops shaken off from the rim meet the ground in a circle of 5 feet diameter, π being $\frac{22}{7}$; the effect of the air being neglected.

4. Three bodies are projected simultaneously from the same point and in the same vertical plane, one vertically upwards, another at the angle of elevation 30°, and the third horizontally. If their velocities be in the ratio of $1 : 1 : \sqrt{3}$, prove that they will always be in a straight line.

How does this line move in space?

5. If APB be a vertical circle whose highest and lowest points are A and B, prove that the time down either AP or PB from rest is equal to the time down AB.

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If the point Q be taken in AB such that $AQ=AP$, and if AP produced meet the tangent at B in R and a body slide down APR from rest, prove that the times of the body being within and without the circle are in the ratio of AQ to BQ.

6. Define the terms Energy (Kinetic and Potential), Work, and Power, and state what is meant by the "Conservation of Energy."

If the unit of Energy be that required to raise 1 lb. through 1 foot (without gain of velocity), find the number of units of Kinetic Energy in a mass of 1 oz. moving 10 feet per second.

7. Find the velocities of two equal perfectly elastic spheres after oblique impact.

Prove that the directions of the relative velocities of the spheres before and after impact are equally inclined to the line of centres at the instant of impact.

8. Find the acceleration of a particle describing a circle with uniform velocity.

In question 3 prove that the force required to keep a drop of water attached to the umbrella's rim makes an angle whose tangent is $\frac{1}{8}$ with the vertical. If the drop weighs ·01 of an oz., find to four decimal places the magnitude of this force.

9. State the Third Law of Motion.

A train weighing 50 tons is moving on a level at 30 miles an hour when the steam is shut off, and the brake being applied to the brake-van the train is stopped in a quarter of a mile. Find the weight of the brake-van, taking the coefficient of friction between its wheels and the rails to be one-sixth, and supposing the unlocked wheels of the train to roll without any sliding.

10. Explain the principle of the pendulum.

Two pendulums oscillating at two different places lose t and τ seconds a day respectively, and if the places at which they oscillate be interchanged they lose t' and τ' seconds; prove that $t+\tau=t'+\tau'$ nearly.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1883.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.). (Obligatory.)

[*Great importance will be attached to accuracy.*]

1. Draw a straight line perpendicular to a given straight line of unlimited length, from a given point without it.

2. If a side of a triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles are equal to two right angles.

If ABC be any triangle, and through D the middle point of AB, DE is drawn parallel to BC, and BE be drawn to bisect the angle ABC and meet DE in E, AEB will be a right angle.

3. The complements of the parallelograms which are about the diameter of any parallelogram are equal to one another.

If $ABCD$ be a parallelogram, and from K, a point on the diagonal AC, EKF be drawn parallel to AD to meet AB in E and DC in F, and HKG be drawn parallel to AB to meet AD in H and BC in G; the triangles AGF, AEH are together equal to the triangle ABC.

4. If a straight line be divided into any two parts, the squares on the whole line, and on one of the parts, are equal to twice the rectangle con-

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tained by the whole and that part, together with the square on the other part.

If AB be divided in C so that the square on AC is double the square on CB, the sum of AB and CB will be equal to the diameter of the square on AB.

5. If two circles touch each other internally, the straight line which joins their centres, being produced, shall pass through the point of contact.

6. The angles in the same segment of a circle are equal to one another.

A and B are the extremities of an arc of a circle, and Q any point on the arc is joined to A and B. If P be a point on AQ or AQ produced such that $QP = QB$, prove that P will lie on an arc of a fixed circle passing through A and B.

7. If from any point without a circle two straight lines be drawn, one of which cuts the circle but does not pass through the centre, and the other touches it: the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it.

8. About a given circle describe a triangle equiangular to a given triangle.

9. If the angle ACB of the triangle ACB is bisected by CE, cutting AB in E, and another point D be taken in AB produced, such that the angle ECD is equal to CED, CD will touch the circle described about ACB.

10. If two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportionals, the triangles shall be equiangular to one another, and shall have those angles equal which are opposite to the homologous sides.

If APB is a semicircle of which AB is the diameter, and C the centre, N a point on CB, and AB is produced to T so that

$$AT : AC = AN : CN,$$

and PT is the tangent drawn from T, CNP will be a right angle.

11. If four straight lines be proportionals, the similar rectilineal figures similarly described upon them shall be proportionals.

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II. ARITHMETIC. (Obligatory.)

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Find the value of

$$\frac{65}{565}\left\{\left(\frac{3\frac{11}{13}}{25}+\frac{40}{4\frac{1}{6}}\right)-11\tfrac{1}{7}\left(\frac{4\frac{3}{5}+6\frac{8}{9}}{7\frac{5}{12}-3\frac{6}{7}}\right)\div 33\tfrac{4}{35}\right\}.$$

2. Subtract

$$4\,\frac{4}{4\frac{4}{9}}\text{ of }\frac{22\frac{1}{7}}{13\frac{1}{15}-12\frac{1}{12}}\text{ from }7\div\frac{1\frac{1}{7}-1\frac{1}{8}}{\frac{27}{28}\left(3\frac{1}{2}+\frac{2\frac{1}{2}}{7}\right)}.$$

3. Multiply together

$$\frac{16\frac{9}{15}}{1\frac{11}{36}},\ \frac{1\frac{8}{21}}{2\frac{18}{22}},\ \frac{2\frac{18}{21}}{1\frac{3}{35}},\ \frac{4\frac{1}{11}}{1\frac{9}{40}},\text{ and }\frac{7}{32}.$$

4. Divide $43\frac{1}{43}$ by $28\frac{88}{139}$ and express the result as the fraction of $\frac{1}{13}$.

5. Add together ·00125 of an acre, ·0625 of a rood, and ·00375 of a perch.

Subtract the result from 91·6634375 yards, and give the answer in square feet, and the decimal of a square foot.

6. Multiply 150·079 by 14·00014.

7. Divide ·036387 by 1·17.

8. Divide 4·761 by $\frac{1\cdot73\dot{1}}{\cdot01}$, and express the answer as a decimal.

9. Reduce $\frac{17}{18}$ of 1 hour 54 minutes to the decimal of $8\frac{1}{2}$ days.

10. Express 3 pecks 2 qts. $\frac{4}{5}$ pt. as the decimal of 11 bushels.

11. Divide 18 miles 23 poles 6 inches by 5872.

12. Find the dividend on £7,566. 16*s.* at 13*s.* $10\frac{1}{4}$*d.* in the £.

13. In what time will £1,675 amount to £1,995. 6*s.* $10\frac{1}{2}$*d.* at $4\frac{1}{4}$ per cent. per annum simple interest?

14. Explain the rule for finding the Least Common Multiple.

Find the smallest sum of which 14*s.* 7*d.*, £1. 11*s.* 6*d.*, and £3. 15*s.* are exact parts.

15. What principal will amount to £498. 18*s.* $5\frac{1}{2}$*d.* in 3 years at $3\frac{1}{2}$ per cent. per annum compound interest, neglecting fractions of one halfpenny?

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16. In what proportion must a merchant mix one kind of tea at 3*s.* per lb. with another at 1*s.* 6*d.* in order that by selling the mixture at 2*s.* 8*d.* per lb. he may make a profit of 25 per cent.?

17. If a rectangular pathway which measures 787·4 mètres in length and 1·526 mètres in width is made at a cost of $1\frac{1}{2}$ francs per square mètre, find the length in English measure of a similar pathway 6 feet wide, which costs 1*s.* $1\frac{1}{2}$*d.* per square yard, the total cost being the same.

(A mètre may be taken as equal to 39·37 inches and £1 as equal to 25 francs. Fractions of an inch may be neglected.)

18. *A* and *B* start from the same point to run in opposite directions round a circular race-course, 9755 feet in circumference, *A* not starting until *B* has run 105 feet. They pass each other when *A* has run 4850 feet. Which will first come round again to the starting point (their speeds being uniform throughout) and what distance will they then be apart?

19. There are four vessels of equal capacity; the first is filled with spirit to the extent of $\frac{1}{3}$th, the second to $\frac{1}{4}$th, the third to $\frac{1}{5}$th, and the last to $\frac{1}{6}$rd. The first is then filled up with water, and from this mixture the second is filled up, again from this second mixture the third is filled up, and in like manner the fourth from the third. What proportion of spirit to water is there in the fourth vessel?

20. What sum at compound interest will amount to £650 at the end of the first year and £676 at the end of the second year?

21. Find by logarithms the value of

$$\frac{403{\cdot}09 \times {\cdot}002317 \times 17}{18{\cdot}543}.$$

22. Find the logarithms of 165 and $3{\cdot}\dot{7}\dot{7}$; and reduce the expression

$$x = \frac{\sqrt[4]{7^2 \cdot 8 \cdot 9^5}}{\sqrt{11^2 \cdot 3^3}}$$

to logarithmic form without calculating the result.

23. Log $3^{x-2} = 2{\cdot}404691352$; find the value of x.

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log 2 = ·3010300	log 1·1 = ·0413927
log ·3 = $\bar{1}$·4771213	log 17 = 1·2304489

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log 4·0309 = ·6054020
log 2·317 = ·3649260
log 1·8543 = ·2681800
log 8·5624 = ·9325955. Diff. = 51.

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III. ALGEBRA. (Obligatory.)

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Simplify the expression

$$\tfrac{1}{4}\{a-5(b-a)\}-\tfrac{3}{2}\left\{\tfrac{1}{3}\left(b-\frac{a}{3}\right)-\tfrac{2}{9}\left[a-\tfrac{3}{4}\left(b-\frac{4a}{5}\right)\right]\right\},$$

and find the value of

$$\{a-(b-c)\}^2+\{b-(c-a)\}^2+\{c-(a-b)\}^2$$

when $a=1$, $b=3$, $c=5$.

2. Multiply

$$a+b+\frac{b^2}{a}+\frac{a^2}{b} \text{ by } a-b+\frac{b^2}{a}-\frac{a^2}{b}.$$

Divide

$$9a^2b^3-12a^4b+3b^5+2a^3b^2+4a^5-11ab^4 \text{ by } 3b^3+4a^3-2ab^2,$$

and resolve into elementary factors the expression

$$(1+a)^2(1+b^2)-(1+b)^2(1+a^2).$$

3. Find the highest common divisor of

$$6x^4-2x^3+7x^2-x+2 \text{ and } 6x^4-12x^3+21x^2-6x+9.$$

4. If $x+y=xy=1$, prove that

$$x^3=y^3=-1,$$

and if $$\frac{x}{y+z}=a,\quad \frac{y}{z+x}=b,\quad \frac{z}{x+y}=c,$$

prove that $$ab+bc+ca+2abc=1.$$

5. Extract the square root of

$$x^4+2x^3(y+z)+x^2(y^2+z^2+4yz)+2xyz(y+z)+y^2z^2.$$

6. Multiply

$$a^{\frac{2}{3}}+b^{\frac{4}{3}}+c^2-cb^{\frac{2}{3}}-ca^{\frac{1}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}} \text{ by } a^{\frac{1}{3}}+b^{\frac{2}{3}}+c,$$

and reduce the following expressions to their simplest forms:

(1) $\{ab^2(ab^3)^{\frac{1}{2}}(a^2b^3)^{\frac{1}{6}}\}^{\frac{1}{2}}$.

(2) $1+\sqrt{8}+\sqrt{2}-\sqrt{27}-\sqrt{12}+\sqrt{75}-\sqrt{19+6\sqrt{2}}$.

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7. Simplify the expressions:

(1) $\left\{\frac{x}{1+x}+\frac{1-x}{x}\right\} \div \left\{\frac{x}{1+x}-\frac{1-x}{x}\right\}$.

(2) $\frac{7}{2(x+1)}-\frac{1}{6(x-1)}-\frac{10x-1}{3(x^2+x+1)}$.

8. Solve the equations:

(1) $\frac{2x}{3}-\frac{1-\frac{x}{2}}{4x}=\frac{8x+1}{12}$.

(2) $\begin{cases}\frac{x-1}{3}-\frac{y+5}{12}=\frac{x+2}{60}, \\ (x-1\frac{1}{2})(y-1\frac{1}{2})=xy-5.\end{cases}$

(3) $(a^2-b^2)(x^2+1)=2(a^2+b^2)x$.

9. Three men, A, B, and C, run a race, the betting being even on A, 2 to 1 against B, and 3 to 1 against C. Find the respective sums which a man must lay against each runner in order that he may win £5 in every event, dead heats being excluded.

10. Prove that in the quadratic equation

$$x^2+px+q=0$$

the sum of the roots is $-p$ and their product is q.

B and C each attempt the same quadratic equation. B after reducing has only a mistake in the numerical term, and finds for roots $+8$ and $+2$; C after reducing has only a mistake in the coefficient of x, and finds for roots -9 and -1. Find the roots of the correct equation.

11. Define Arithmetical, Geometrical, and Harmonical Progression, and find n arithmetic means between a and d.

If a, b, c, d be in Arith. Prog., a, e, f, d in Geom. Prog., a, g, h, d in Harm. Prog. respectively; then

$$ad=ef=bh=cg.$$

12. Find the number of combinations of n things r together.

A person has 20 acquaintances, 12 of whom are relatives. In how many ways may he invite 15 guests from among them so that exactly 8 of these are relatives?

13. Enunciate the binomial theorem and find the coefficient of x^r in $(1-x)^{-n}$ and $(1+x)^3(1-x)^{-2}$ respectively.

Find also the greatest term in the expansion of $(1-x)^{-3}$ when $x=\frac{3}{4}$.

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IV. PLANE TRIGONOMETRY. (Obligatory.)

(Including the solution of triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Define the "unit of circular measure," and find how many degrees, minutes, and seconds it contains, neglecting fractions of a second, and assuming the ratio of the circumference to the diameter of a circle to be 22 : 7.

Also find the circular measure of $49° 41' 15''$.

2. Prove that $\cos A = -\cos(180° - A)$.

If the sum of four angles is 180°, shew that the sum of the products of their sines taken two together is equal to the sum of the products of their cosines taken two together.

3. Find an expression for all angles which have a given tangent.

Find all the values of θ which satisfy

$$\operatorname{cosec}\theta = \cot\theta + \sqrt{3}.$$

4. Prove that

(1) $\cot^2(45° + A) = -1 + \ldots 1 + \sec^2(45° - A)$.

(2) $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$.

(3) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}\frac{3}{5}$.

5. Find the acute angles of a right-angled triangle whose hypotenuse is four times as long as the perpendicular let fall upon it from the opposite angle.

6. Find the values of $\sin 18°$ and $\cos 36°$.

Prove that

$$\cos 12° + \cos 60° + \cos 84° = \cos 24° + \cos 48°.$$

7. Shew how the characteristic of any logarithm to the base 10 may be found by inspection.

Given $\log 5 = \cdot 69897$, find $\log 200$, $\log \cdot 025$, $\log \sqrt[3]{62\cdot 5}$, and also $L \sin 30°$ and $L \cos 45°$.

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8. Shew that in any plane triangle

(1) $2bc \cos A = b^2 + c^2 - a^2$;

(2) $(a+b+c)\left(\tan\frac{A}{2} + \tan\frac{B}{2}\right) = 2c \cot\frac{C}{2}$.

9. Given $a = 2b$, $C = 120°$, $\log 3 = \cdot 4771213$,

$L \tan 10° \, 53' = 9\cdot 283907$, diff. $1' = 6808$,

find A, B, and the ratio $c : a$.

10. At each end of a base of length $2a$ it is found that the angular altitude of a certain peak is θ, and at the middle point of the base the altitude is ϕ. Prove that the vertical height of the peak above the plane is

$$\frac{a \sin\theta \sin\phi}{\sqrt{\sin(\phi+\theta)\sin(\phi-\theta)}}.$$

11. Express the radius of the inscribed circle of a triangle in terms of one side and the two angles adjacent to it.

O is the centre of the inscribed circle of the triangle ABC. If R_1, R_2, R_3 be the radii of the circles circumscribing the triangles OBC, OCA, OAB respectively, and r, R be the radii of the inscribed and circumscribed circles of the triangle ABC, shew that $R_1R_2R_3 = 2rR^2$.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS. (1.)

[*Great importance will be attached to accuracy in results.*]

1. If two straight lines be parallel, and one of them at right angles to a plane, the other also shall be at right angles to the same plane.

2. Two straight lines AC, BC intersect at right angles in C, and from A a straight line AD is drawn at right angles to the plane in which they are: shew that CD is at right angles to BC.

3. If a solid angle be contained by three plane angles, any two of them are greater than the third.

4. Shew that in an elliptic section of a cone the semi-axis minor is a mean proportional between the perpendiculars from the vertices on the axis of the cone.

The section of a cone through its axis is an equilateral triangle: compare the area of the elliptical section, whose vertices are the middle point of a side and the extremity of the base, with those of the circular sections through the vertices.

5. Prove that in any conic if PG the normal at P meet the axis in G; $SG : SP :: SA : AX$.

6. Prove that in the hyperbola the conjugate semi-axis is a mean proportional between the lengths of the perpendiculars let fall from the foci on any tangent.

7. The portion of the tangent of an hyperbola which is terminated by the asymptotes is bisected at the point of contact, and is equal to the parallel diameter.

8. Shew that the equation $4x^2 - 24xy + 11y^2 = 0$ represents two straight lines, and find the tangent of the angle between them.

9. Investigate the length of the perpendicular from a point (hk) on the straight line $\frac{x}{a} + \frac{y}{b} = 1$.

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10. Find the condition that the straight line $y=mx+c$ may touch the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and hence find the locus of the points of intersection of tangents at the extremities of conjugate diameters.

11. Find the magnitude and position of the principal axes of the ellipse

$$3(x^2+y^2)+2xy=8.$$

12. Find the equation to the hyperbola referred to the asymptotes as axes.

A circle is drawn touching the asymptotes of a rectangular hyperbola and the curve at its vertex; find its equation referred to the asymptotes as axes.

13. Pairs of tangents are drawn from points on the circle

$$x^2+y^2=(a+b)^2$$

to the ellipse $$\frac{x^2}{a}+\frac{y^2}{b}=a+b:$$

prove that the chords of contact will all touch a second ellipse, and find its equation.

VI. PURE MATHEMATICS. (2.)

[*Great importance will be attached to accuracy in results.*]

1. Prove that, if the three fractions

$$\frac{2x-3y}{3z+y},\ \frac{z-y}{z-x},\ \frac{x+3z}{2y-3x}$$

be all equal, each is also equal to $\frac{x}{y}$ and hence shew that either $x=y$, or $z=x+y$.

2. A sum of money borrowed at the beginning of a year was returned by equal instalments paid at the end of that and of each succeeding year, and interest at the rate of 4 per cent. was paid at the end of each year upon the amount of debt remaining uncancelled at the beginning of that year. The total sum received by the creditor was exactly double of the sum originally borrowed. What was the number of instalments?

3. Expand $\frac{1+nx^n}{(1-x)^2}$ in a series of ascending powers of x, and write down the coefficients of x^{n-r} and x^{n+r}.

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4. State and prove the law of formation of the successive convergents of a continued fraction.

Prove that

$$\sqrt{9a^2+3}=3a+\frac{1}{2a+}\,\frac{1}{6a+}\,\frac{1}{2a+}\,\frac{1}{6a+}\cdots\cdots$$

and write down the first five convergents.

5. If a, b be two prime numbers, shew that the number of positive integers less than and prime to the integer $a^m b^n$ is

$$a^{m-1}b^{n-1}(a-1)(b-1).$$

6. Prove that

$$\tan 3a=\tan\left(\frac{\pi}{3}-a\right).\tan a.\tan\left(\frac{\pi}{3}+a\right).$$

7. The circumference of a semicircle is divided into two arcs such that the chord of one is double that of the other. Shew that the sum of the areas of the two segments cut off by these chords : area of the semicircle :: 27 : 55. ($\pi=\frac{22}{7}$.)

8. Prove that the expression $\{\cos\theta+\sqrt{-1}\sin\theta\}^{\frac{m}{n}}$, where m and n are positive integers, has n different values and no more.

Express in the form $\alpha+\beta\sqrt{-1}$ each of the roots of the equation $x^4+1=0$.

9. Find the sum of n terms of the series

$$\cos\frac{\pi}{2n+1}+\cos\frac{3\pi}{2n+1}+\cos\frac{5\pi}{2n+1}+\cdots\cdots$$

and expand $\log(\sec\theta)$ in a series of powers of $\tan\theta$.

10. State the relations which exist between the coefficients and the roots of the equation

$$x^n+p_1x^{n-1}+p_2x^{n-2}+\ldots+p_{n-1}x+p_n=0.$$

Shew that the numerically greatest negative coefficient increased by unity is a superior limit of the positive roots of the equation.

11. Investigate the condition that an algebraical equation may have equal roots.

The roots of the equation

$$3x^2(x^2+8)+16(x^3-1)=0$$

are not all unequal: find them.

12. Solve the equation

$$6x^3+23x^2-72x+36=0,$$

having given that its roots are in Harmonical progression.

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VII. PURE MATHEMATICS. (3.)

[*Great importance will be attached to accuracy in results.*]

[*Full marks may be obtained by doing about* four-fifths *of this paper.*]

1. Explain the meaning, and establish the truth, of the equation

$$\frac{dy}{dx} \cdot \frac{dx}{dy} = 1.$$

Find from first principles the differential coefficient of $\sin x$, and deduce the differential coefficient of $\sin^{-1} x$.

2. Find $\frac{dy}{dx}$ in the following cases:

(1) $y = \sin(m \cos y)$.

(2) $y = \log\left(\frac{x}{a^x}\right)$.

(3) $y = \sin^{-1} \frac{a + b\cos x}{b + a\cos x}$.

3. If $y = \cos m\theta$, and $x = \cos\theta$, prove that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2 y = 0.$$

4. State Leibnitz's theorem for finding $\frac{d^n}{dx^n}(u, v)$, where u and v are given functions of x.

Prove that

$$\frac{d^n}{dx^n}\left\{(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2 y\right\}$$

is

$$(1 - x^2)\frac{d^{n+2}y}{dx^{n+2}} - (2n + 1)x\frac{dy^{n+1}}{dx^{n+1}} + (m^2 - n^2)\frac{d^n y}{dx^n}.$$

5. State and prove Maclaurin's theorem.

Expand $\log_e(1 + \cos x)$ as far as the term in x^4.

6. Find the value of

(1) $\dfrac{1 - \tan x}{1 - \sqrt{2} \cdot \sin x}$; when $x = \frac{\pi}{4}$.

(2) $\{1 + \phi(x)\}^{\psi(x)}$; when $x = a$,

given that $\phi(a) = 0$; $\psi(a) = \infty$; $lt_{x=a}\, \phi(x)\, \psi(x) = m$.

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7. State without proof the method of determining maxima and minima values of given functions of x.

If A be the least of the three angles of a triangle whose sides are a, b, and c, prove that the length of the shortest line which bisects the area is

$$\sqrt{2bc}\sin\frac{A}{2}.$$

8. If χ be the angle between the radius vector and tangent of a curve and p the perpendicular upon the latter from the pole, prove that

$$\cos\chi=\frac{dr}{ds};\ \sin\chi=r\frac{d\theta}{ds};\ p=r^2\frac{d\theta}{ds}.$$

9. Shew how to find the envelope of a system of curves whose equation contains one arbitrary parameter.

Prove that the polar equation between p and r of the envelope of the lines

$$x\cos 2\theta+y\sin 2\theta=2a\cos\theta,$$

is

$$p^2=\tfrac{4}{3}(a^2-r^2).$$

10. Find the co-ordinates of the centre of curvature of the curve $y=f(x)$ at the point x, y.

11. Trace the curve

$$y^2=\frac{x(a^2-x^2)}{a}.$$

12. Integrate the expressions:

$$(1)\quad \frac{dx}{\sqrt{a^2-x^2}};$$

$$(2)\quad \frac{dx}{1+\cos x}.$$

13. Find the area of the loop of the curve $r=a\theta\log_e\theta$.

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VIII. STATICS.

[*Great importance will be attached to accuracy.*]

1. Enunciate the "Polygon of Forces" and deduce it from the "Parallelogram of forces."

Forces of 3, 4, 5 and 6 lbs. act in directions respectively north, east, south, and west; find the magnitude and direction of their resultant.

2. Find the magnitude and direction of the resultant of two parallel forces acting in opposite directions on a rigid body.

Explain the meaning of your results in the case where the two forces are equal.

3. What is meant by the Resolved part of a given force in a given direction?

Prove that, if any number of forces meet in a point, the sum of their resolved parts in any direction is the same as the resolved part of their resultant in that direction.

4. What is meant by the Moment of a force about a straight line?

Prove that the sum of the moments of two forces acting in a plane about a point in that plane is equal to the moment of their resultant about that point.

5. Prove that, if the sum of the moments of a number of forces acting in one plane about each of three points in that plane not in the same straight line is zero, the forces are in equilibrium.

The magnitude of a force is known, and also its moments about two given points A and B. Find by a geometrical construction its line of action.

6. Find the centre of gravity of a number of heavy particles in a plane.

At the centres of the three circles escribed to a triangle particles are placed of masses inversely as the radii of those circles. Find the centre of gravity of the system.

7. Find the position of equilibrium of a balance when the weights in each scale pan, the length of the arms, the weights of the beam and of the scale pans and the position of the centre of gravity are given.

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8. Explain what is meant by the principle of virtual velocities.

A pulley is suspended by a vertical loop of string to a wheel and axle, and supports a weight, one end of the string being wound round the axle, the other in a contrary direction round the wheel. Find by virtual velocities the power which, acting at right angles to one end of an arm a so as to turn the axle, will support the weight.

9. A string without weight is suspended from two pegs not in the same horizontal line and passes through a small smooth heavy ring free to slide along the string. Find the position of equilibrium of the ring.

If the ring, instead of being free to slide, be tied to a given point in the string, find equations to give the ratio of the tensions of the two portions of the string.

10. A continuous string, without weight, length l, hung over two smooth pegs in the same horizontal line, distant a apart, hangs in two loops, on each of which is placed a small smooth heavy ring, one of weight W, the other of weight W'. Find an equation to determine the tension of the string.

IX. DYNAMICS.

[*Great importance will be attached to accuracy.*]

[N.B.—*When needed the measure of the accelerating force of gravity may be taken as* 32.]

1. If a body start with a velocity V under the influence of a uniform acceleration f in the direction of motion, find its velocity after a time t and the distance over which it will have gone in that time.

2. A ball is dropped from a height h under the influence of gravity, and another ball is discharged from the point, where the first would strike the ground, with velocity sufficient to carry it to the height h. At what moment ought it to be discharged so as to meet the other half way?

3. What is meant by the work done by a given force?

Find how many foot-pounds of work are required to change the velocity of a body of W lbs. weight from v to v' feet per second.

4. Describe any means by which the acceleration due to gravity may be ascertained.

What is meant when we say that gravity is an acceleration of 32? If an hour were the unit of time, and 4000 miles were the unit of length, what number would represent the acceleration due to gravity?

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5. Prove that a body moving freely under the influence of gravity moves in a parabola.

Given the velocity and direction of its projection, find the velocity and direction of its motion at any subsequent time t.

6. A body is projected from a given point with velocity v. Find the direction of its projection so that it may pass through another given point, distant h horizontally and k vertically, from the point of projection.

7. Two heavy particles of equal weight are connected by a string, one lies on a rough table, and the other hangs over the edge; find the distance over which they will have moved after a given time t from rest.

8. Two imperfectly elastic balls impinge on one another in a direct line; given their velocities before impact, find their velocities after impact.

Prove that the motion of the centre of gravity of the bodies is the same after impact as before it.

9. Prove that where a body descends from rest down a smooth curve in a vertical plane its velocity depends only on the vertical height through which it has descended.

An elliptical wire of eccentricity $\sqrt{\frac{2}{3}}$ is placed in a vertical plane with its major axis inclined to the horizontal at an angle 60°. If a small ring is allowed to slide along the wire from rest at the higher extremity of the major axis, shew that its velocities at the two ends of the minor axis are as $\sqrt{2} : 1$.

10. A body moves uniformly in a circle: find the acceleration towards the centre.

With what number of turns per minute must a weight of 10 lbs. revolve horizontally at the end of a string 15 inches long, in order to cause the same tension on the string as if a 1 lb. weight were hanging at rest held by the string vertically?

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JULY, 1884.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.). (Obligatory.)

[*Great importance will be attached to accuracy in working.*]

1. IF two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases equal; the angle which is contained by the two sides of the one shall be equal to the angle contained by the two sides equal to them of the other.

Two equal straight lines AB and CD are joined towards opposite parts by the equal straight lines AD and CB intersecting in O. Prove that both the triangles OAC and OBD are isosceles.

2. If a straight line fall upon two parallel straight lines, it makes the alternate angles equal to one another; and the exterior angle equal to the interior and opposite upon the same side; and likewise the two interior angles upon the same side together equal to two right angles.

No straight line can be placed within a parallelogram greater than the greater diameter.

3. Triangles upon equal bases, and between the same parallels, are equal to one another.

The line AD is drawn from the vertex of the isosceles triangle ABC perpendicular to the base, and is bisected in E. If BE produced meet AC in F, prove that the triangle BCF is double of the triangle BAF.

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4. If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square of half the line bisected, is equal to the square of the straight line which is made up of the half and the part produced.

5. In obtuse-angled triangles, if a perpendicular be drawn from any of the acute angles to the opposite side produced, the square of the side subtending the obtuse angle is greater than the squares of the sides containing the obtuse angle by twice the rectangle contained by the side upon which, when produced, the perpendicular falls, and the straight line intercepted without the triangle between the perpendicular and the obtuse angle.

6. Equal straight lines in a circle are equally distant from the centre, and those which are equally distant from the centre are equal to one another.

The chords AB and CD of the circle $ACBD$ intersect in O. Prove that if the difference of the segments OA and OB is equal to the difference of the segments OC and OD, the chords AB and CD are also equal.

7. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

The line AD bisects the angle BAC of the triangle ABC, and meets the base BC in D, and a circle is described passing through A and touching BC in D. Prove that this circle will touch the circle circumscribing ABC in the point A.

8. Describe a circle about a given square.

9. Similar triangles are to one another in the duplicate ratio of their homologous sides.

10. In a right-angled triangle if a line be drawn from the right angle perpendicular to the base it will divide the triangle into two triangles similar to the whole and to one another.

If two straight lines touch a circle at opposite extremities of a diameter, and a third line be drawn cutting the two former in A and B, and touching the circle in C, prove that the rectangle $AC \cdot BC$ is equal to the square upon the radius of the circle.

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II. ARITHMETIC. (Obligatory.)

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Simplify

$$\left\{4\frac{1\frac{3}{4}}{\frac{7}{9}} - \left(\frac{26}{4\frac{1}{3}} - \frac{\frac{4}{5}}{1\frac{3}{5}}\right)\right\} \div \left(\frac{3\frac{4}{5}}{2\frac{1}{7}} - \frac{1\frac{4}{9}}{2\frac{3}{7}+\frac{3}{8}}\right).$$

2. Add $\frac{4\frac{3}{4}}{6\frac{3}{8}}$ of $\frac{1\frac{1}{2}-\frac{5}{8}}{1\frac{1}{2}+\frac{1}{6}}$ of £2. 10*s*. to $\frac{6\frac{9}{11}}{11\frac{4}{11}}$ of $\frac{2\frac{1}{4}+1\frac{1}{8}}{2\frac{1}{4}-1\frac{1}{8}}$ of 7*s*. 6*d*.

3. Find the value of $\frac{3\frac{1}{2}\times 3\frac{1}{2}\times 3\frac{1}{2}-3}{3\frac{1}{2}\times 3\frac{1}{2}-3} \div 8\frac{23}{37}$.

4. What number added to $\frac{37}{52}+\frac{15}{56}$ will make $3\frac{9}{14}$?

5. Add together 346·07643, 3·974928, and ·00647328, and subtract the result from 359·038622.

6. Multiply 34·00376 by ·070605.

7. Divide 1·25837 by 34·01.

8. Divide ·37592 by ·05, and express the answer as a decimal.

9. Add together ·375 of 13*s*. 4*d*. and ·07 of £2. 10*s*., and subtract the result from ·45 of £1.

10. Express 1 day 4 hrs. 31 mins. 52½ secs. as the decimal of 3 days 4 hrs. 5 mins.

11. A wheel makes 1,028 revolutions in passing over 2 miles 4 fur. 9 pls. 5 yds. 6 ins. What is the circumference of the wheel?

12. Find the dividend on £5,471 at 5*s*. 9¾*d*. in the £.

13. At what rate per cent. per annum, simple interest, will £3,775 amount to £4,105. 6*s*. 3*d*. in 2½ years?

14. How often is each of the quantities 19⅓, 19⅝, and 21¼ contained in their Least Common Multiple?

15. I row against a stream flowing 1½ miles an hour to a certain point, and then turn back, stopping two miles short of the place whence I originally started. If the whole time occupied in rowing be 2 hrs. 10 mins., and my uniform speed in still water be 4½ miles an hour, find how far upstream I went.

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16. With what two quantities of spirits, that cost me 14*s.* 6*d.* and 17*s.* 6*d.* a gallon respectively, did I make a blend of 12 gallons, if by selling the mixture at 16*s.* 3*d.* a gallon I should have incurred a loss of $2\frac{1}{2}$ per cent.?

17. A merchant sells an article at an advance of 12 per cent. on the cost price; how much per cent. of the selling price is profit?

18. What sum would in three years amount to £2,811. 18*s.* at compound interest, if the rate were 3 per cent. per annum for the first year, 4 per cent. for the second year, and 5 per cent. for the third?

19. A certain piece of work was done by A, B, and C, all working together, with the exception that B left off half an hour before the completion of the work. A alone could have done the whole in $7\frac{1}{2}$ hours, B in $8\frac{1}{2}$ hours, C in 10 hours. In what time was the work done?

20. Find the logarithms of 105 and $18\cdot\dot{7}$.

21. Find by logarithms the value of

$$\frac{\cdot 01283 \times 1\cdot 63 \times 39}{\sqrt{24}}.$$

22. If $\dfrac{\log x}{\log 26} = \cdot 8567$, what is the value of x?

23. If $7^x = \frac{1}{10}$ of 36^2, what is the value of x?

Given logarithms:—

$\log 2 = 0\cdot 3010300$	$\log 1283 = 3\cdot 1082267$
$\log 7 = \bar{0}\cdot 8450980$	$\log 1664\cdot 8 = 3\cdot 2213621$
$\log \cdot 13 = \bar{1}\cdot 1139434$	Diff. 261.
$\log 15 = 1\cdot 1760913$	$\log 16300 = 4\cdot 2121876$
	Diff. 266.

III. ALGEBRA. (Obligatory.)

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Find the value of

$$\frac{3y}{x-y} - \frac{x}{x+y} - \frac{x^2+2xy}{2xy-y^2},$$

when $x=2$, $y=-\frac{1}{2}$; and express as the product of two simple factors

$$(3a+5b)^2+b^2-\{16a^2+(5a-3b)^2\}.$$

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2. From $x(x+a-2b)(x-2a+b)$

subtract $(x+a)(x+b)(x-2a-2b)$;

and divide $x^4-(y^2-3z^2)x^2+xyz^2+2z^4$ by $x^2-xy+2z^2$.

3. Shew that, if the squares of any two consecutive integers and the square of their product be added together, the sum will itself be a perfect square.

4. Prove that the product of two quantities is equal to the product of their Greatest Common Measure and Least Common Multiple.

Find the Greatest Common Measure and Least Common Multiple of

$$5a^4-3a^2-8a-3 \text{ and } a^5-a^3-4a^2-3a-2.$$

5. Divide the difference between $\frac{(1+x)^3}{1+x^3}$ and $\frac{(1-x)^3}{1-x^3}$ by the sum of

$$\frac{1+x}{1-x} \text{ and } \frac{1-x}{1+x}.$$

Also find the value of

$$\frac{x+a}{x+b}\times\frac{x+b+c}{2x+a+c} \text{ when } x=\frac{c^2-ab}{a+b-2c}.$$

6. Simplify

(1) $\sqrt{75-12\sqrt{21}}$,

(2) $\sqrt[3]{21+7\sqrt{2}}\times\sqrt[3]{21-7\sqrt{2}}$,

and find the coefficients of x and x^3 in the product of

$$x^{\frac{5}{2}}+p^{\frac{1}{2}}x^3+2px^{\frac{3}{2}}-3p^{\frac{3}{2}}x+2p^2x^{\frac{1}{2}}+p^{\frac{5}{2}} \text{ and } (x^{\frac{1}{2}}-p^{\frac{1}{2}})^3.$$

7. Solve the equations:—

(1) $\frac{5}{4}(3x-3\frac{10}{13})-\frac{7x-8\frac{6}{13}}{3}=6+\frac{13x-12}{39}$.

(2) $\frac{2x-1}{(x-1)^2}-\frac{2x+1}{(x+1)^2}=4$.

(3) $x^2+xy=y^2-9x^2y+64=0$.

(4) $\sqrt{2x+5a}-\sqrt{2x}=\frac{a}{\sqrt{5a-2x}}$.

8. A gentleman arrives at the railway station nearest to his house an hour and a half before the time at which he had ordered his carriage to meet him. He sets out at once to walk at the rate of 4 miles an hour, and meeting his carriage when it had travelled 2 miles, reaches home exactly an hour earlier than he had originally expected. How far is his house from the station, and at what rate was his carriage driven?

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9. Eliminate x, y, and z between the equations

$$a^2x^3+b^2y^3+c^2z^3=p^5, \quad ax^2=by^2=cz^2,$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{p}.$$

10. If $a:b::c:d$, shew that

$$a-b:a+b::c-d:c+d,$$

and if $z^2=x^2+y^2$, prove that

$$y-z+x:z-x+y::x-y+z:x+y+z.$$

11. Two terms are taken equidistant from the beginning and end of a series. Prove that their sum will be constant if the series is arithmetical, and their product will be constant if it is geometrical.

Sum the series

(1) $1\frac{4}{5}+1\frac{1}{5}+\frac{4}{5}+\ldots\ldots$ to 10 terms.

(2) $1\frac{4}{5}+1\frac{1}{5}+\frac{3}{5}+\ldots\ldots$ to 12 terms.

12. Twenty-six people are to travel by an omnibus which can carry 12 inside and 14 outside. If 8 of them will not ride outside and 6 will not go inside, in how many different ways can the party travel?

13. Write down the continued product of n factors—

$$x+a_1,\ x+a_2,\ x+a_3\ldots\ldots x+a_n,$$

and deduce the expansion of $(x+a)^n$ in powers of x.

Expand $(1-x)^{\frac{1}{2}}$ to 5 terms, and shew that the sum of n terms of the series

$$\frac{1}{2}+\frac{1}{\underline{|2}}\left(\frac{1}{2}\right)^2+\frac{1\cdot 3}{\underline{|3}}\left(\frac{1}{2}\right)^3+\frac{1\cdot 3\cdot 5}{\underline{|4}}\left(\frac{1}{2}\right)^4+\ldots\ldots$$

is equal to

$$1-\frac{1\cdot 3\cdot 5\cdot 7\ldots(2n-1)}{\underline{|n}\cdot 2^n}.$$

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IV. PLANE TRIGONOMETRY. (Obligatory.)

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Assuming 3·1416 as the value of π, express in circular measure $3^\circ\ 12'$, and find to seconds the angle whose circular measure is ·8.

Find approximately the distance of a tower whose height is 51 feet, and which subtends at the eye an angle of $5\frac{5}{11}'$.

2. Prove that $\cos(135^\circ + A) + \sin(135^\circ - A) = 0$.

If $\cos A = \tan B$, and $\sin B = \cot A$, find A and B.

3. Find the smallest positive value of θ which satisfies the equation

$$\cos\theta = \sin\left\{(4n+3)\frac{\pi}{2} + \alpha\right\}.$$

Trace the changes in the sign and value of $\cos(\pi \sin\theta)$ as θ increases from 0° to 180°.

4. Prove that

$$\cos 2A = \frac{1 - 6\tan^2\frac{A}{2} + \tan^4\frac{A}{2}}{\left(1 + \tan^2\frac{A}{2}\right)^2},$$

and that the value of

$$\sin(n+1)B \sin(n-1)B + \cos(n+1)B \cos(n-1)B$$

is independent of n.

If $\dfrac{\tan(\theta - 15^\circ)}{\tan(\theta + 15^\circ)} = \dfrac{1}{3}$, find θ.

5. From the extremities of a diameter of a circle, AB, chords AD, BE are drawn intersecting in C at an angle of 120°; if the area of ABD is double that of AEB, find the angles at A and B.

6. If $\sin A = a$, prove that

$$2\cos\frac{A}{2} = \pm\sqrt{1+a} \pm \sqrt{1-a},$$

and find in what quadrant A is when the two negative signs have to be taken.

If $\cos A = \tan A$, prove that $\sin A = 2\sin 18^\circ$.

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7. Find $\log_{10} \cdot 0001$ and $\log_{\sqrt[8]{11}} \{297\sqrt{11}\}^{\frac{8}{3}}$.

Find also $\log \frac{1}{147}$ and $\sqrt[3]{1 \cdot 323}$,

having given

$$\log 3 = \cdot 4771213,$$
$$\log 7 = \cdot 845098,$$
$$\log 3 \cdot 6593 = \cdot 563399.$$

8. Prove that in any plane triangle

(1) $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\left(\text{assuming } \cos A = \frac{b^2+c^2-a^2}{2bc}\right)$.

(2) Area of triangle $= \frac{a^2}{2} \cdot \frac{\sin B \cdot \sin C}{\sin (B+C)}$.

9. The angles A, B, C of a triangle ABC are 40°, 60°, and 80° respectively, and CD is drawn from C to the base bisecting the angle ACB; find CD: having given

$$AB = 100 \text{ inches} \qquad L \sin 40° = 9 \cdot 8080675,$$
$$\log 2 = \cdot 30103 \qquad L \sin 50° = 9 \cdot 884254,$$
$$\log 5 \cdot 73979 = \cdot 7588951 \qquad L \sin 60° = 9 \cdot 9375306.$$

10. Two lines of straight railway, ABC, DEC, meet at C, telegraph posts being situate at A, B, D, E; the angles DAE, DBE are each equal to α; and the angles EAB, EBC are β and γ respectively; shew that

$$BC = AB \frac{\sin \beta \sin (\alpha+\beta)}{\sin (\gamma-\beta) \sin (\alpha+\beta+\gamma)}.$$

11. Find the radius of a circle inscribed in a right-angled triangle in terms of the sides.

Two equal right-angled triangles ABC, $A'B'C$ have a common right angle C and their hypotenuses meet in D, CA, CA' being equal; compare the radii of the circles inscribed in ABC and $A'BD$.

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V. PURE MATHEMATICS. (I.)

[*Great importance will be attached to accuracy in results.*]

1. Draw a straight line perpendicular to a given plane, from a given point without it.

2. Every solid angle is contained by plane angles, which are together less than four right angles.

3. Determine the length of the perpendicular from the vertex upon the base of a triangular pyramid, all the sides of which are equilateral triangles of a given area.

4. Prove that the tangents drawn from any point to a conic subtend equal angles at the focus.

An ellipse is inscribed in a triangle, so that the line joining the points of contact of two sides is a focal chord. Shew that the third side subtends a right angle at this focus.

5. Shew that any section of a cylinder by a plane not parallel or perpendicular to the axis is an ellipse.

6. If, in a parabola, QVQ' be a double ordinate of a diameter PV, and QD the perpendicular from Q upon PV, prove that QD is a mean proportional between PV and the latus rectum.

7. Define the asymptotes of an hyperbola; and prove that the asymptotes intersect the directrices in the same points as the auxiliary circle, and the lines joining the corresponding foci with the points of intersection are tangents to the circle.

8. Prove that the area of the triangle formed by the straight lines whose equations are

$$2y+x-5=0,\ y+2x-7=0,\ x-y+1=0, \text{ is } \tfrac{3}{4}.$$

9. Determine the distance of the point of intersection of the lines

$$2x-3y+5=0,\ 3x+4y=0,$$

from the line $\frac{x}{2}-\frac{y}{5}=0$.

10. Find the equation to a parabola which touches the axes at distances h, k from the origin respectively, and determine the equation to the tangent at any point on the curve.

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11. Investigate the equation to the tangent to an ellipse in terms of the tangent of the angle which the tangent line makes with the major axis. Hence shew that the locus of the intersection of the tangent with the perpendicular on it from the focus is a circle described on the major axis as diameter.

12. The equation to an ellipse being

$$x^2+y^2+xy+x+y=1;$$

find the co-ordinates of its centre, and the semi-axes.

13. Having given one side of a triangle, and the difference between the tangents of the adjacent sides, prove that the locus of the vertex is an hyperbola, and find its transverse and conjugate axes.

VI. PURE MATHEMATICS. (2.)

[*Great importance will be attached to accuracy in results.*]

1. Eliminate x, y, z from the equations

$$x^{-1}+y^{-1}+z^{-1}=a^{-1},$$
$$x+y+z=b,$$
$$x^2+y^2+z^2=c^2,$$
$$x^3+y^3+z^3=d^3,$$

and shew that, if x, y, z are all finite and numerically unequal, b cannot be equal to d.

2. Prove that the coefficient of x^{n+r} in the expansion of

$$\frac{(1+3x)^n}{(1+2x)^2} \text{ is } (-2)^r(r-2n+1).$$

3. Investigate the present value of an annuity to commence at the end of m years, and continue n years, the rate of interest being r.

4. Sum the series of which the nth term is

$$\{an+b\}\{a(n+1)+b\}\{a(n+2)+b\}\ldots\{a(n+m-1)+b\}.$$

Find the nth convergent to the continued fraction

$$\frac{x}{x+1}-\frac{x}{x+1}-\frac{x}{x+1}-\ldots$$

5. Prove that the arithmetical mean of any number of positive quantities is greater than the geometrical mean.

6. Find all the values of θ which satisfy the equation

$$\sin^2 n\theta-\sin(n-1)\theta=\sin^2\theta.$$

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7. Two similar and similarly situated triangles are such that the same circle can be described in the one and about the other; compare their areas, and prove that, if $A_1 = 4A_2$, the triangles must be equilateral.

8. Assuming Demoivre's theorem, express $\sin \alpha$ and $\cos \alpha$ in ascending powers of α.

Sum the series $\cos \alpha + \frac{1}{2} \cos 2\alpha + \frac{1}{3} \cos 3\alpha + \&c.$

9. Find x from the equation

$$3 \tan^{-1}(x+1) = 2 \tan^{-1}(x-1) + \tan^{-1} \frac{x}{2-x}.$$

10. Prove that impossible roots enter equations with real coefficients in pairs.

Hence prove that if $e^{\theta\sqrt{-1}}$ is a root of the equation $x^3 - px^2 + qx - r = 0$,

$$\sin 3\theta - p \sin 2\theta + q \sin \theta = 0.$$

11. Find the equation whose roots are the squares of the differences of the roots of the equation $x^3 + qx + r = 0$, and hence find the condition that two of its roots may be equal.

12. Prove that $x^4 + px^3 + qx^2 + rx + s$ may always be expressed as the difference of two squares.

Solve the equation $\quad x^4 + 2x^3 + 3x^2 + x + \frac{5}{4} = 0.$

VII. PURE MATHEMATICS. (3.)

[*Great importance will be attached to accuracy in results.*]

Full marks may be obtained by doing about four-fifths *of this paper.*

1. Define a *differential coefficient*, and find from your definition the differential coefficients of $\log x$ and $\sin^{-1} x$.

2. If u, v be two functions of the same variable x, prove that

$$\frac{d(uv)}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}.$$

Write down (without proof) the expression for $\frac{d^n(uv)}{dx^n}$ in terms of u, v, and their differential coefficients, and apply it to find the fifth differential coefficient of $e^x \cdot \log x$.

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3. Differentiate with respect to x

$$\frac{(x+a)^{n+1}}{(x-a)^{n-1}}, \quad \frac{a^{-\frac{x}{2}}}{\sqrt{1-x\log a}}, \quad \cot^{-1}\sqrt{\frac{a-x}{x}}.$$

4. If $y=\sin(\epsilon^{-x})$, prove that

$$\epsilon^{2x}\cdot\frac{d^3y}{dx^3}-(\epsilon^{2x}-1)\frac{dy}{dx}=3y.$$

5. Find the first four terms in the expansion of $\cos^{-1}x$ in powers of x.

6. Find the value of each of the following functions when $x=1$:—

(1) $\dfrac{2x^3-3x^2+1}{3x^5-5x^3+2}$.

(2) $\{1-x^2\}^{\frac{1}{\log(1-x)}}$.

7. Investigate the conditions which must be satisfied in order that a given function of x may have a maximum or minimum value.

An open tank is to be constructed with a square base and vertical sides, so as to contain a given quantity of water. Shew that the expense of lining it with lead will be least if the depth is made half of the width.

8. Write down the equations of the tangents and normals to the curve $y(x^2+a^2)=ax^2$ at the points where $y=\dfrac{a}{4}$.

Shew that these are points of inflexion.

9. Define the *polar subtangent*, find an expression for its length, and explain its use in drawing asymptotes.

Find the equation of the asymptote of the curve

$$r=a\tan\theta\sin\theta.$$

10. Find the radius and co-ordinates of the centre of curvature at the point whose abscissa is a on the curve—

$$x^2y=a^2(x-y).$$

11. Explain what is meant by the *envelope* of a system of curves.

Prove that the envelope of the circles obtained by varying the arbitrary parameter a in the equation

$$c^2(y-a)^2+(cx-a^2)^2=(a^2+c^2)^2$$

consists of a straight line and a circle.

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12. Integrate the functions

$$\sec x,\ x \cos x,\ \frac{1}{(a-3x)^n}.$$

13. Trace the curve $y^2(x+a)^3=a^5$, and find the area bounded by the arc of this curve and the positive directions of the axes of co-ordinates.

VIII. STATICS.

[*Great importance will be attached to accuracy.*]

1. Prove the parallelogram of forces for the magnitude of the resultant, assuming it true for direction.

2. Enunciate the proposition of the triangle of forces, and deduce therefrom that of the polygon of forces.

The sides AB, BC, CD, and DA of the quadrilateral $ABCD$ are bisected at E, F, G, and H respectively. Prove that the resultant of the two forces represented in magnitude and direction by EG and HF is represented in magnitude and direction by AC.

3. Define the term moment of a force about a point.

Prove that the difference of the moments of any force P about two points in the same plane with each other and the force is equal to the moment about either of the points of a force parallel and equal to P applied at the other point.

4. Find the resultant of two parallel forces in the same direction.

Prove that the moment of this resultant about any point in the plane of the forces is equal to the algebraical sum of the moment of the forces about that point.

5. Three equal strings of no sensible weight, are knotted together to form the equilateral triangle ABC, and a weight, W, is suspended from A. If the triangle and weight be supported, with BC horizontal, by means of two strings at B and C, each at the angle of $135°$ to BC, prove that the tension in BC is $\frac{W}{6}(3-\sqrt{3})$.

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6. The ends of a driving rein are passed through two smooth rings which are fastened one to each side of the bit. They are then doubled back and tied to fixed points in the head-piece, one on each side of the horse's head. Find the pressure produced by the bit on the tongue by a given pull of the driver.

7. Find the centre of gravity of a uniform triangle, and shew how to find that of a given quadrilateral.

The uniform quadrilateral $ABCD$ has the sides AB, AD and the diagonal AC all equal, and the angles BAC and CAD 30° and 60° respectively. If a weight equal to one-sixth of the triangle ABC be attached to the point B, and the whole rest suspended from A, prove that the diagonal AC will be vertical.

8. Find the mechanical advantage in a system of weightless moveable pulleys in which each string is attached to a bar supporting the weight. Find also at what point of the bar the weight should be suspended.

9. Find the relation between the power and the weight when a body is supported on a smooth inclined plane by a force at any angle to that plane, and prove that the principle of virtual velocities holds good in this case.

A sphere of given weight rests between two smooth planes, one vertical and the other inclined at a given angle to the vertical; find the pressures on the planes.

10. State the principal laws of friction.

If in the case mentioned in the second part of Question 9 the inclined plane be one of the faces of a prism which rests with another face upon a horizontal plane, and if the coefficient of friction between the horizontal face and plane be μ, find the least weight of the prism consistent with equilibrium.

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IX. DYNAMICS.

[*Great importance will be attached to accuracy.*]

[N.B.—*When needed the measure of the accelerating force of gravity may be taken as* 32.]

1. Describe briefly Attwood's machine; and explain how it may be used to prove that the force of gravity is constant in the same place.

2. A bucket containing a hundredweight of coal is being drawn up from a coal-pit, so that the pressure of the coal on the bottom of the bucket is equal to 126 lbs. Find the acceleration of the bucket.

3. State the meaning of the terms "potential energy" and "kinetic energy." A rope 500 feet long, and weighing 2 lbs. a foot, is wound on a roller. What is the difference of its potential energy in this position, and in its position when 200 feet of the rope have rolled out, neglecting friction and the weight of the roller, and supposing that no part of the rope touches the ground?

4. If a particle start from rest under the influence of a constant force f, shew that the space described in time t is $\frac{1}{2}ft^2$.

Two balls of equal mass are connected by an inextensible string, the one rests on a smooth plane inclined to the horizon at an angle α, and the other hangs over the higher edge. Find the acceleration of each of the balls. If $\alpha = 30°$, find the space described from rest during the time which the balls have taken to acquire a velocity of 48 ft. per second.

5. A ball impinges in succession on two adjacent sides of a smooth, perfectly elastic, billiard table; shew that the direction of motion, after rebounding from the second side, will be parallel, but opposite, to the original direction.

6. Shew that the height of the directrix of the parabolic path of a projectile depends on the initial velocity only, and shew that the velocity at any point of the orbit is equal to that of a particle falling vertically from the directrix to this point.

7. A bullet is shot horizontally from a gun at the top of a high mountain with velocity u. Find the range on a horizontal plane below if the inclination of the tangent at the point of impact be 45°; and the velocity at the point of impact. The resistance of the air is to be neglected.

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8. Explain what is meant by centrifugal force, and give the mathematical expression for it. If a stone 4 oz. in weight be hurled from a sling which has been swung in a horizontal circle of 3 ft. radius at a height of 6 feet from the ground at the steady rate of 21 revolutions in two seconds, find the centrifugal force and kinetic energy of the stone at the moment of leaving the sling, and the distance of the point at which it strikes the ground.

9. If a weight suspended by an inextensible string swing as a conical pendulum, prove that the time of revolution varies as the square root of the height of the cone.

Find the inclination to the vertical when the string is 20 inches long and makes 200 rotations per minute.

10. The acceleration produced by gravity being 32 when a second and foot are the units of time and length; find its measure when $\frac{1}{10000}$ of an hour and a centimetre are taken for units (one centimetre = ·0328 feet).

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1884.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.). (Obligatory.)

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. Prove that the interior angles of every triangle are equal to two right angles: that the sum of the angles of any rectilineal figure together with four right angles is equal to twice as many right angles as the figure has sides: and that the sum of the distances of any point from the angular points of the figure is greater than half the perimeter.

2. Parallelograms upon the same base and between the same parallels are equal to one another.

On the sides AB, BC of a triangle parallelograms $ABDE$, $BFGC$ are described so as to fall without the triangle, and ED, GF, produced if necessary, intersect in H. Show that the parallelogram described on AC, with adjacent sides equal and parallel to HB, will be equal in area to these two parallelograms.

3. If a straight line be divided into any two parts the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by those parts.

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Hence prove that the straight line drawn from the vertex of a right-angled triangle perpendicular to the base is a mean proportional between the segments of the base.

4. In an obtuse-angled triangle, if a perpendicular be drawn from one of the acute angles on the opposite side produced, the square of the side subtending the other acute angle is less than the squares of the sides containing it by twice the rectangle contained by the side, on which when produced the perpendicular falls, and the straight line intercepted between the perpendicular and the acute angle.

If C be the obtuse angle of a triangle ABC, D and E the feet of the perpendiculars from A and B on the opposite sides: prove that the square on AB is equal to the sum of the rectangles contained by BC and BD, and AC and AE.

5. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

Find the condition that a circle may be inscribed in a quadrilateral.

6. If two straight lines cut one another in a circle, but not at right angles, one of which passes through the centre, and the other does not, the rectangles contained by their segments are equal to one another.

PCQ is a chord of a circle passing through a fixed point C. CR is drawn at right angles to PQ to meet the circle described on PQ as diameter in R. On what curve does R lie?

7. Inscribe a square in a given circle.

Compare the magnitudes of the inscribed and circumscribed squares.

8. Make the construction for describing an isosceles triangle, having each of the angles at the base double of the third angle.

Show that the base of the triangle is equal to the side of a regular pentagon inscribed in the smaller circle of the figure.

9. What is meant by a line being cut in extreme and mean ratio?

In right-angled triangles, the rectilineal figure described upon the side opposite to the right angle is equal to the similar and similarly described figures upon the sides containing the right angle.

10. If the exterior angle CBD of a triangle be bisected by a straight line, which cuts the base produced in E, prove that the square on BE is equal to the difference of the rectangles contained by AE, EC, and AB, BC.

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II. ARITHMETIC. (Obligatory).

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Simplify—

$$\frac{3\frac{1}{5}}{7+\frac{\frac{7}{5}}{1+\frac{1}{2}}}+\frac{\frac{11}{20}-\frac{5}{12}}{\frac{14}{15}-\frac{9}{10}}-\left(1-\frac{188}{145}\right).$$

2. Find the value of

$$\frac{10\times\sqrt{2+\frac{3}{4}}\times\sqrt{1-\frac{3}{7}}}{13\times\sqrt{1-\frac{3}{7}}\div\sqrt{2+\frac{3}{5}}}\times\frac{\sqrt{3+\frac{1}{4}}}{\frac{9\frac{3}{4}}{\sqrt{3+\frac{1}{4}}}}\div 1\tfrac{1}{2}.$$

3. A person who owned $\frac{1}{3}$ of an estate sold $\frac{2\frac{7}{11}}{8\frac{7}{10}}$ of $\frac{11}{24}$ of his share for £96: what would be the value of $\frac{2\frac{1}{2}}{3\frac{1}{3}}$ of $\frac{5}{12}$ of the whole estate at the same rate?

4. What number is that from which if $\frac{7}{15}-\frac{3}{12}$ be deducted, and to the remainder $\frac{\frac{4}{5}}{1\frac{1}{4}}$ be added, the sum will be $3\frac{38}{45}$?

5. Add together 4·00607, ·617634, and 376·473, and subtract the result from 400.

6. Multiply ·00734 by 7·06004.

7. How many lengths of ·0375 of a foot are contained in 31·7296875 feet? Express the remainder as the decimal of a foot.

8. Reduce ·6761904 to a vulgar fraction in its lowest terms.

9. Add ·16875 of £1 to ·375 of 3*s.* 4*d.*, and from the result subtract $\frac{3}{16}$ of 5*s.*

10. Express 6 cwt. 1·125 qrs. as the decimal of a ton.

11. Distribute £630 among *A*, *B*, *C* and *D*, so that *B* and *C* may together have half what *A* and *D* have together, that *B*'s money may be $\frac{7}{8}$ of *C*'s, and *D*'s money $\frac{3}{5}$ of *A*'s.

12. Find the value of 2 tons 17 cwt. 3 qrs. 14 lbs. at £5. 9*s.* 6*d.* per cwt.

13. At 4 per cent. per annum the simple interest on a certain sum of money amounted to £1,087. 14*s.* 6*d.* in $7\frac{1}{2}$ years. What was the sum?

14. Find the square root of the Least Common Multiple of $7\frac{1}{6}$, $19\frac{9}{25}$, and $20\frac{1}{4}$.

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15. A builder (A) sells to an agent (B) a house for £4,860, by which he loses 19 per cent.; B disposes of it to C at a price which would have given A 17 per cent. profit. Find B's total gain.

16. A milkseller pays 1*s.* 1*d.* per gallon for his milk; he adds water and sells the mixture at 2*d.* per pint, thereby making altogether 40 per cent. profit. Calculate the proportion of water to milk his customers receive.

17. A certain turnip-field can be hoed by 10 women in 4 days, or by 6 boys in 10 days, or by 2 men in 12 days. One man, three boys, and three women are employed; what is the total expense, a man's daily pay being 2*s.* 8*d.*, a woman's 1*s.* 8*d.*, and that of a boy 1*s.*?

18. A merchant sells silk of two qualities which cost him 5*s.* 4*d.* and 4*s.* 4*d.* per yard respectively. The selling price of the latter is two-thirds that of the former, but the quantity sold is double. He gains 25 per cent. on the whole. Calculate the selling price per yard of each.

19. A person holding £5,250 stock in the 3 per Cents. and £4,750 in $6\frac{1}{2}$ per cent. Bank stock sells out, the price of the former being $91\frac{7}{8}$ and of the latter $185\frac{3}{8}$. With part of the proceeds he buys ninety 8 per cent. £100 railway bonds at $142\frac{3}{8}$ (the usual brokerage $\frac{1}{8}$ per cent. being paid in each transaction), and invests the balance in a mortgage at 4 per cent. Find the difference in his income.

20. Find, without the use of logarithm tables, the value of

$$\frac{1}{6}\sqrt{\left\{\frac{3\log 1728}{1+\frac{1}{2}\log \cdot 36+\frac{1}{3}\log 8}\right\}}.$$

21. Find the logarithms of ·00114 and $1\cdot\dot{7}\dot{2}$.

22. Find by logarithms the value of

$$\frac{1089\times \cdot 01881\times \cdot 405}{\sqrt[3]{(729)^4}}.$$

23. A solid cube of lead weighs 126·44 lbs. 998 ounces of water occupy one cubic foot, and a cubic foot of lead is 11·352 times as heavy as a cubic foot of water. Find by logarithms the length of a side of the cube of lead correctly to six places of decimals of a foot.

Given logarithms:—

log 4 = 0·6020600	log 1·1 = 0·0413927
log 60 = 1·7781513	log 19 = 1·2787536
log 49·9 = 1·6981005	log 129 = 2·1105897
log ·012644 = $\bar{2}$·1018845	log 56·311 = 1·7505932
Diff. 343.	Diff. 78.

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III. ALGEBRA. (Obligatory).

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Explain the use of brackets, and prove that

$$a+(b-c)=a+b-c,$$
$$a-(b-c)=a-b+c.$$

Simplify

$$\frac{a}{4}-\left[\frac{2c-3a}{4}-\left\{c-\frac{a-2c}{2}-\left(2c-\frac{3a+c}{2}\right)\right\}\right].$$

2. Multiply $a^4-4a^2b^2+16b^4$ by $a^2-2ab+4b^2$,
and divide $8a^3-b^3+c^3+6abc$ by $2a-b+c$.

3. Find the highest common factor of

$$x^4-8x^2-9 \text{ and } x^3-x^2-4x-6,$$

and the least common multiple of

$$12\,(a^7b-ab^7) \text{ and } (a^2+b^2)^4-(a^4-6a^2b^2+b^4)^2.$$

4. Simplify the expressions

(1) $$\left\{\frac{5a}{a-6b}-\frac{2b}{3a-2b}\right\}\div\left\{\frac{2a}{a+2b}-\frac{2b-a}{2b-3a}\right\}.$$

(2) $$\frac{1}{a^2-3b^2+2ab}+\frac{1}{b^2-3a^2+2ab}-\frac{2}{3a^2+10ab+3b^2}.$$

5. If $ab=c^2$ prove that $(a+c)(b-c)=(b+c)(c-a)$
and $(a^2+b^2+2c^2)(a+b+2c)^2=\{(a+c)^2+(b+c)^2\}^2$.

6. Extract the square root of

$$1-3x+\frac{11x^2}{12}+2x^3+\frac{4x^4}{9},$$

and find the fourth power of $a^{\frac{1}{4}}-a^{-\frac{1}{4}}$.

7. Transform the fraction $\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}-3\sqrt{6}}$ so that it may have a rational denominator, and express, in the simplest possible form, the value of

$$\sqrt{x+\tfrac{1}{2}}+\sqrt{x-\tfrac{1}{2}}, \text{ when } 4x=\sqrt{a}+\frac{1}{\sqrt{a}}.$$

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8. Solve the equations :—

(1) $\dfrac{2x+13}{10} - \dfrac{2(x-2)}{2x-15} = \dfrac{x-21}{5}$.

(2) $\dfrac{x+2}{x-2} + \dfrac{2x-3}{2(x-1)} = \dfrac{23}{6}$.

(3) $\begin{cases} 2xy + y^2 = 16. \\ 2x^2 - xy = 12. \end{cases}$

9. Form the equation whose roots are

$$\frac{1}{3+\sqrt{5}} \text{ and } \frac{1}{3-\sqrt{5}},$$

and determine the limits between which n must lie in order that the equation

$$2ax(ax+nc) + (n^2-2)c^2 = 0$$

may have real roots.

10. The figures which express the pounds and the pence in a certain sum of money will change places if £2. 19*s*. 9*d*. be added to it, and those which express the shillings and the pence would be interchanged by subtracting 2*s*. 9*d*. What alteration would be produced in the sum of money by interchanging the figures which express the pounds and shillings?

11. Find the sum of a given arithmetical series.

300 trees are planted regularly in rows in the shape of an isosceles triangle, and the numbers in successive rows decrease by one. How many trees are there in the row which forms the base of the triangle?

12. Find the number of permutations of the letters forming the word *treasurers* taken all together.

13. Write down the middle terms in the expansions of

$$(a-x)^{10} \text{ and } \left(x - \frac{1}{x}\right)^{4n}.$$

Show also that only one term in the expansion of

$$(1-px)^{\frac{p+1}{p}}$$

has a negative sign, if p is any positive number not less than unity.

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IV. PLANE TRIGONOMETRY. (Obligatory.)

(Including the solution of triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Express in circular measure and in degrees to three places of decimals an angle which is subtended at the centre of a circle 3 square inches in area by an arc of 1 inch. [$\pi = \frac{22}{7}$.]

2. Define the trigonometrical ratios of A involved in the equation—

$$\cot A + \tan A = \sec A \operatorname{cosec} A,$$

and establish its truth by a geometrical construction.

3. Show how the formula for $\tan(A+B)$ in terms of $\tan A$, $\tan B$, may be deduced from the formula for $\sin(A+B)$.

Evaluate $\tan\left(\frac{\pi}{4}+A\right) \times \tan\left(\frac{3\pi}{4}+A\right)$.

4. Prove that $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$.

Hence find an expression for $\sin 3^\circ$.

5. Prove that—

(1) $\text{Tan}^2 60^\circ - 2\tan^2 45^\circ = \cot^2 30^\circ - 2\sin^2 30^\circ - \frac{8}{4}\operatorname{cosec}^2 45^\circ$.

(2) $\text{Sec}\, 2A - \frac{1}{2}\tan 2A \sin 2A = \frac{\cot^2 A + \tan^2 A}{\cot^2 A - \tan^2 A}$.

(3) $\text{Tan}^{-1}\frac{1}{2} + \cot^{-1}\frac{1}{8} + \sin^{-1}\frac{7\sqrt{2}}{10} = 0$.

6. Find all the values of A which satisfy the equation—

$$\cos^3 A - \cos A \sin A - \sin^3 A = 1.$$

7. If x be the least positive angle of which the sine is equal to $\sin A$, find a general formula for A in terms of x.

Find also all possible values of $\sin\frac{A}{6}$ in terms of the trigonometrical ratios of $\frac{x}{6}$.

8. Prove that in any triangle ABC, with the usual notation,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

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If AD be drawn perpendicular to the plane of ABC, and DB, DC be joined, show that

$$\sin DCB \,.\, \sin DBA = \sin DCA \,.\, \sin DBC.$$

9. If the sides of a triangle be 7·152 in., 8·263 in., 9·375 in., find its area.

log 1·2395 = ·0932465.
log 5·243 = ·7195799.
log 4·132 = ·6161603.
log 3·02 = ·4800069.
log 2·8477 = ·4544942. Diff. = 152.

10. Find the radii of the three circles escribed to a triangle in terms of the sides.

Show that the sum of the products of these radii two and two is equal to the square on the semi-perimeter of the triangle.

11. A straight flagstaff, leaning due east, is found to subtend an angle α at a point, in the plain upon which it stands, a yards west of the base. At a point b yards east of the base the flagstaff subtends an angle β. Find at what angle it leans.

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[*Great importance will be attached to accuracy in results. Full marks may be obtained by doing about* three-quarters *of this paper.*]

1. If two straight lines be at right angles to the same plane they shall be parallel to one another.

Only one line can be drawn from a given point perpendicular to a given plane, and it is the shortest line from the point to the plane.

2. If a solid angle be contained by three plane angles any two of them are together greater than the third.

3. Find the radius of the sphere inscribed in a given tetrahedron.

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4. Define a right cone and prove that every section of it by a plane meeting it on opposite sides of the vertex is a hyperbola. Prove that the distance between the foci of the hyperbola is equal to the sum of the distances of its vertices from the vertex of the cone.

5. In the parabola prove the propositions

(1) $PN^2 = 4AS \cdot AN$,

(2) $AT = AN$,

the letters having their ordinary signification.

6. PV is any chord of a parabola. Through O, O', two points on PV equidistant from its extremities, chords ROQ, $R'O'Q'$ are drawn parallel to one another. Show that the rectangle contained by RO, OQ is equal to the rectangle contained by $R'O'$, $O'Q'$.

7. In the ellipse prove that the locus of Y, the foot of the perpendicular from S upon the tangent at P, is the auxiliary circle.

8. Find the polar equation of the straight line joining the points (1, 2) and (3, 4), the axis of y being the prime radius.

9. Find the equation of a circle referred to rectangular axes.

Prove that the circles

$$x^2 + y^2 - 2ax + c^2 = 0 \text{ and } x^2 + y^2 - 2by - c^2 = 0$$

intersect at right angles.

10. Find the equation of the tangent at any point Q of an ellipse referred to conjugate diameters PCp and DCd as axes.

Let this tangent meet the diagonals, or the diagonals produced, of the parallelogram circumscribing the ellipse at the extremities of Pp and Dd in R and S, and let RS be bisected in T.

If X, Y are the co-ordinates of Q referred to Pp and Dd as axes, and if X', Y' are the co-ordinates of T, show that

$$\frac{X}{Y} = -\frac{X'}{Y'}.$$

11. Find the equation of the hyperbola referred to the asymptotes as axes.

12. Prove that the equation of the axes of the central conic

$$ax^2 + 2bxy + ay^2 = d$$

is $x^2 - y^2 = 0$,

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and that the lengths of the semi-axes are

$$\sqrt{\frac{d}{a+b}} \text{ and } \sqrt{\frac{d}{a-b}} \text{ respectively.}$$

13. Find the polar equation to the tangent to an ellipse, the focus being the pole and the major axis the initial line.

VI. PURE MATHEMATICS (2).

[*Great importance will be attached to accuracy in results.*]

1. In the expansion of $(x-y-z)^9$ find the coefficient of $x^3y^2z^4$, and the number of terms.

2. Prove that the arithmetic mean of n positive quantities

$$a, b, c, d \ldots \text{ \&c.}$$

is greater than their geometric mean.

3. Find the annuity, to continue for four years, which can be purchased for £750, supposing that a perpetual annuity is worth 25 years' purchase. The first payment is made at the expiration of a year from the purchase of the annuity.

4. Show that—

$$\sqrt{a^2+4a} = a + \frac{2}{1+}\,\frac{1}{a+}\,\frac{1}{1+}\,\frac{1}{a+}\,\frac{1}{1+}\ldots.$$

5. Find the coefficient of x^r in the expansion of $\frac{1-9x+32x^2}{(1-5x)^2(1+7x)}$ in ascending powers of x.

6. Solve the equation—

$$1-\cos 2\theta = 2(\cos\alpha\cos\theta - \cos 2\alpha).$$

7. Prove that

$$\left(1-\tan^2\frac{\phi}{2}\right)\left(1-\tan^2\frac{\phi}{2^2}\right)\left(1-\tan^2\frac{\phi}{2^3}\right)\ldots\ldots \text{ to infinity} = \frac{\phi}{\tan\phi}.$$

8. Show that the perimeter of a triangle : perimeter of the inscribed circle :: area of the triangle : area of the circle

$$:: \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2} : \pi.$$

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9. Investigate the expression for $\cos n\theta$ in a series of descending powers of $\cos\theta$, when n is a positive integer.

10. Show how to transform an equation into another, the roots of which shall be less than those of the proposed equation by a constant difference.

Transform $x^3 - 6x^2 + 5 = 0$ into an equation without its second term, and hence find the roots of the original equation.

11. Determine the roots of the equation

$$2x^4 - 5x^3 + 6x^2 - 5x + 2 = 0.$$

12. Describe fully the process known as Des Cartes' method of solving biquadratic equations.

Example. $x^4 - 3x^2 - 42x - 40 = 0$.

VII. PURE MATHEMATICS (3).

[*Great importance will be attached to accuracy in results. Full marks may be obtained by doing about* four-fifths *of this paper.*]

1. Explain the terms *dependent variable, independent variable, limit of a function.*

Find the limit of $\dfrac{a^x - 1}{x}$ when $x = 0$.

2. Find the differential coefficients of x^n and $\sin^{-1} x$.

3. Differentiate with respect to x

$$\frac{(2x^2-1)\sqrt{1+x^2}}{x^3}, \quad \tan^{-1}\sqrt{\frac{a-x}{a+x}}, \quad x^{\cos x}.$$

4. Find the nth differential coefficients of

$$\frac{1}{1-x}, \quad \frac{3}{2-x-x^2}, \quad x^2 \sin x.$$

5. If $f(x)$ be a function which vanishes when $x = a$ and $x = b$, and $f(x)$ and $f'(x)$ be both continuous between those values of x, prove that $f'(x)$ will vanish for some value of x between a and b.

Deduce a proof of Taylor's Theorem.

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6. Find the values of

(1) $\dfrac{x^2 \sin x}{2x - \sin 2x}$ when $x = 0$.

(2) $\cot x \times \log(\cos x)$ when $x = \dfrac{\pi}{2}$.

7. The ordinate of any point P of an ellipse, whose centre is C, meets the axis major in N. Find the positions of P which make the area PCN a maximum.

8. Draw all the asymptotes of the curve

$$xy(y - x) = a(3x^2 + 2y^2).$$

9. If p be the perpendicular let fall from the pole upon the tangent at any point of the curve $r = f(\theta)$, prove that

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2.$$

In the curve $r^2 = a^2 \sin 2\theta$ show that $p \propto r^3$ and that the radius of curvature $\propto \dfrac{1}{r}$.

10. Explain what is meant by the *evolute* of a plane curve.

Show that $4y^3 = 27ax^2$ is the equation of the evolute of the parabola $x^2 = 4a(y + 2a)$.

11. Find the integrals of the functions

$$\sqrt{a^2 - x^2}, \quad \frac{\log x}{x}, \quad \frac{1}{x^4 - x}.$$

12. Show how to find by integration the length of a curve whose equation is given in rectangular co-ordinates.

Find the length of the curve

$$(y - b)^3 = 27ax^2$$

between the points whose abscissæ are 0 and a.

13. Trace the curve $r = a \tan \dfrac{\theta}{2}$, and prove that the area of its loop is $a^2\left(2 - \dfrac{\pi}{2}\right)$.

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VIII. STATICS.

[*Great importance will be attached to accuracy.*]

1. If the parallelogram of forces be true for the direction of the resultant of one force and each of two other forces taken separately, prove that it will be true for the direction of the resultant of the former force and the two others taken together.

2. State and prove the triangle of forces.

The resultant of two forces P and Q acting in the directions AB and CB respectively meets the line AC in the point D, and the resultant of the same forces acting in the directions AB and BC respectively meets the same line AC produced in the point E; prove that AC is divided harmonically at D and E.

Where are D and E respectively situated when AB and CB are proportional to P and Q?

3. Define the moment of a force about a point, and prove that the moments of two non-parallel forces, about any point in the line of their resultant, are equal and opposite.

State and prove the converse of this proposition.

4. Four forces, P, Q, R, and S, no two of which are parallel, act in a plane; the resultant of P and Q meets that of R and S in A, the resultant of P and R meets that of Q and S in B, and the resultant P and S meets that of Q and R in C; prove that A, B, and C are in the same straight line.

5. Three weightless rods, AB, AC, and BC, are fastened together at the angles to form the isosceles triangle ABC, whose vertical angle BAC is $30°$. The figure is kept at rest by a force P at A bisecting BAC, and two forces, each equal to P, at B and C. Prove that the equilibrium will not be affected if the rods be replaced by flexible strings knotted together at the ends, the forces acting outwards from the triangle. Find the tensions of the strings.

6. Show how to determine the centre of gravity of known weights rigidly connected.

Prove that, if weights 1, 2, 3, 4, 5, 6 are situated at the angles of a regular hexagon, the distance of their centre of gravity from the centre of the circumscribing circle is two-sevenths of the radius of that circle.

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7. Show how to graduate the common steelyard.

The length of the rod is 2 feet, its weight 2 lbs., the distance of its centre of gravity from the fulcrum 1 inch towards the end of the shorter arm, the distance of the point where the weight is suspended from the fulcrum 2 inches, and the moveable weight 6 ozs. Find the greatest weight which can be weighed.

8. Find the ratio of the power to the weight in the system of pulleys wherein each hangs by a separate string, neglecting the weights of the pulleys themselves.

If the weights of the pulleys be taken into account, and be all equal, find the least value of the weight of each in order that there may be any mechanical disadvantage in supporting a weight W.

9. State the laws of friction.

A weightless rod of length ($2a$) is loaded with equal weights (W) at its extremities, and supported with its middle point on the highest point A of the circumference of a rough vertical circle.

The coefficient of friction is such that a particle of the same material as the rod would be just upon the point of sliding if placed on the point B of the circle where the arc $AB=b$. If the weight at one end of the rod be very slowly increased, prove that the greatest weight which can be thus added for equilibrium to be possible is $\frac{2bW}{a-b}$.

If half this weight be added, find the position of equilibrium.

10. Explain the principle of Virtual Velocities. Apply it to find the ratio of the power to the weight in the Differential Screw.

IX. DYNAMICS.

[*Great importance will be attached to accuracy.*]

[N.B.—*When needed, the measure of the accelerating force of gravity may be taken as* 32.]

1. Give some explanation why different measures of force are adopted in Statics and Dynamics. Show how the one measure may be derived from the other.

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A body that weighs 2 cwt. is moved along a smooth horizontal table by a constant pressure which, acting for two seconds, generates a velocity of 16 feet per second in the body; find the weight which the pressure would statically support.

2. If a body be projected vertically downwards with a velocity V, find the space described and the velocity acquired at the end of (t) seconds.

A body is projected downwards with a velocity (V), and at the end of (t) seconds has a velocity (v); show that the space described is the same as that described by the body moving uniformly for (t) seconds with a velocity that is the arithmetic mean of (V) and (v).

3. State the second law of motion, and show how it is applicable to the theory of projectiles.

If a projectile be projected from the foot of a given inclined plane, find the distance from the point of projection at which the projectile strikes the inclined plane.

There is a hill whose inclination is 30°. From a point on the hill one projectile is projected up the hill and the other down it with equal velocities; the angle of projection in each case is inclined to the horizon at 45°; show that the range of one projectile is nearly $3\frac{3}{4}$ times the range of the other.

4. A weight (W) is placed on a rough horizontal table, and is moved along the table by a vertical weight (P) which hangs over the edge of the table, and is attached to (W) by an inextensible string; show that the accelerating force on (W) is $\frac{P-\mu W}{P+W}g$, where (μ) is the coefficient of friction.

If $W=5P$, find (μ) when the roughness of the plane is such that it takes twice the time from rest to acquire the same velocity in the moving body that it acquires when the plane is smooth.

5. A ball weighing 12 lbs. leaves the mouth of a cannon horizontally, with a velocity of 1000 feet per second; the gun and carriage, together weighing 12 cwt., slide upon a smooth plane whose inclination to the horizon is 30°; find the space through which the gun and carriage will be driven up the plane by the recoil, it being given that the pressure caused by the explosion on the ball and on the end of the bore of the gun at each instant is the same.

6. How is the work accumulated in a moving body estimated? If one inelastic ball overtake another inelastic ball in direct impact, show that work is lost by the impact. If the balls be equal, and the velocity of one be double the velocity of the other, find the work lost.

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7. Given (l) the length of a simple pendulum, $\pi\sqrt{\frac{l}{g}}$ the time of an oscillation, show how to find, approximately, the height of a mountain, when a seconds pendulum, by being taken from sea-level to its summit, loses (n) beats in 24 hours. If (n) = 15 what is the height of the mountain, the radius of the earth being taken as 4000 miles?

8. A weight of 200 lbs. is to be raised through a height of 40 feet by a cord passing over a fixed smooth pulley; it is found that a constant force P pulling the cord at its other end for three-fourths of the ascent communicates sufficient velocity to the weight to enable it to reach the required height; find P.

9. What is the "vis-viva" of a body in motion? How is the kinetic energy of a body expressed in terms of the vis-viva?

A train runs down an incline of 1 in 100 for the distance of a mile; how far will it be carried along the level at the foot of the incline, supposing all the resistances on the train to amount to 8 lbs. per ton: express the result in miles.

10. If a railway carriage without flanges to its wheels move on a circular curve, show how the effect of the centrifugal force may be counteracted by a rise of the outer rail, and find what the rise of the outer above the inner rail should be if the radius of the circle be 1320 feet, the velocity of the train be 30 miles an hour, and the breadth of the rail 5 feet.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

APRIL, 1885.

PRELIMINARY EXAMINATION.

EUCLID (Books I.—IV. and VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If a side of a triangle be produced, the exterior angle is equal to the two interior and opposite angles.

Through the vertices of a triangle ABC straight lines, falling within the triangle, are drawn making equal angles BAL, CBM, ACN: if these lines intersect, two and two, in points A', B', C', prove that the triangle $A'B'C'$ is equiangular to ABC.

2. To a given straight line apply a parallelogram equal to a given triangle and having an angle equal to a given rectilineal angle.

3. If there be two straight lines, one of which is divided into any number of parts, the rectangle contained by the two straight lines is equal to the rectangles contained by the undivided line and the several parts of the divided line.

If A, B, C, D are points in a straight line, taken in order, prove that the rectangle $AC \cdot BD = AB \cdot CD + BC \cdot AD$.

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4. Divide a given straight line into two parts, so that the rectangle contained by the whole line and one of the parts shall be equal to the square of the other part.

5. Equal straight lines in a circle are equally distant from the centre.

Through the middle points of two equal chords of a circle a third chord is drawn: show that the portions of this chord intercepted between the middle points and the circumference are equal.

6. The angles in the same segment of a circle are equal to one another.

AB is a diameter of a circle, and C a given point in AB: find a point in the circumference at which AC, CB will each subtend half a right angle.

7. Inscribe a circle in a given triangle.

If O is the centre of the circle inscribed in the triangle ABC, and AO produced meets the circumference of the circle circumscribed about the triangle in H, prove that HB and HC are each equal to HO.

8. Inscribe an equilateral and equiangular hexagon in a given circle.

Prove that the area of the hexagon is $\frac{3}{4}$ of the equilateral and equiangular hexagon circumscribed about the circle.

9. The sides about the equal angles of equiangular triangles are proportionals.

Straight lines AOD, BOE, intersecting at O, being drawn so that $AO=2OD$ and $BO=2OE$, AE and BD are drawn and produced to meet in C: prove that AC and BC are bisected at E and D.

10. Define *compound ratio.*

Equiangular parallelograms have to one another the ratio which is compounded of the ratio of their sides.

11. The rectangle contained by the diagonals of a quadrilateral figure inscribed in a circle is equal to the two rectangles contained by the opposite sides.

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II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Simplify

$$\tfrac{3}{2}-[\tfrac{1}{8}+\{1-(\tfrac{1}{3}-\tfrac{1}{4})\}]+\tfrac{1}{2}\left(\frac{\frac{3}{4}-\frac{1}{2}}{\frac{2}{3}-\frac{1}{4}}\right)+\tfrac{19}{20}.$$

2. Find the value of

$$\frac{1\frac{29}{35}}{2\frac{7}{60}}\text{ of }\frac{\frac{2}{5}+\frac{3}{4}-\frac{4}{15}}{\frac{3}{3\frac{3}{4}}-\frac{1}{2\frac{5}{8}}+\frac{5}{8\frac{3}{4}}}.$$

3. *A* had at first £1. 8*s*., *B* paid him $2\frac{3\frac{1}{7}}{1\frac{2}{5}}$ of £1. 11*s*. 6*d*., and found that he had remaining $\frac{1}{45}$ of what *A* then had. How much had *B* at first?

4. What fraction multiplied by $3\frac{1}{2}$ of $\frac{4}{5}$ produces $2\frac{2}{5}$ of $\frac{4}{15}$?

5. Add together 3·40765, 537·063 and ·84379. Subtract the result from 601·0473.

6. Find the area of a square slab, the side of which measures ·2868 of a metre.

7. Divide 157·505 by ·03706.

8. Subtract $\frac{5}{77}$ of $2\frac{3}{4}$ of £3. 6*s*. 6*d*. from ·0475 of £100.

9. Divide ·27 by 75·75 and express the answer as a decimal.

10. Express the difference between $3\frac{1}{4}$ guineas and £1·7625 as the decimal of 5 shillings.

11. If with sales amounting to £1000 a tradesman gains £100 in 7 months, what sales would he have to make in order to gain £60. 10*s*. in 11 months?

12. What would be the amount of a tax on £575 at $11\frac{3}{4}$*d*. in the £?

13. At what rate per cent. per annum must £5,750 be invested so that the simple interest on it may amount to £646. 17*s*. 6*d*. in $2\frac{1}{2}$ years?

14. Find the square root of the least integer that is a common multiple of $1\frac{194}{121}$, $22\frac{11}{40}$, and $1\frac{9}{16}$.

15. A person (*C*) buys an estate, and some time afterwards disposes of it to another (*D*) for £5,400. At this price it yields *D* $2\frac{3}{4}$ per cent. *C* previously obtained 3 per cent. for his investment, but his income, owing to less careful management was £36 less than *D*'s. What did *C* originally pay for it?

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16. A druggist buys a certain commodity by avoirdupois weight and sells it by troy weight. The buying price is 3*d.* per oz., and he sells at 6*d.* What is his gain per cent?

17. Strong spirit is mixed with inferior spirit, valued at 5*s.* per gallon, in the proportion of 6 to 1. The mixture is then worth 9*s.* the gallon. Find the value of a gallon of the strong spirit.

18. The annual average depth of rainfall for the three years 1879, 1880, 1881, at a certain place was 24·98 inches; the succeeding three years it was 29·62. The year 1883 was the rainiest, when there fell 4·8 inches more than in 1882, 6·36 inches more than in 1884, and 7·47 more than in 1880. The year 1881 was short of the preceding year by only ·17 inches. Find the depth of rain that fell in each of the six years.

19. A fundholder directs his broker to purchase eight £100 shares in a certain mine, quoted at $272\frac{3}{8}$ per share. To accomplish this he authorises him to sell out £850 stock in the 3 per cents. at $95\frac{5}{8}$, and £1300 stock in the $4\frac{1}{2}$ at $105\frac{1}{8}$. The broker's charge on each of the three transactions is $\frac{1}{8}$th per cent. What had the broker to receive upon the whole?

20. Show without the use of logarithm tables that

$$\frac{\log \sqrt{27} + \log 8 - \log \sqrt{1000}}{\log 1{\cdot}2} = 1\tfrac{1}{2}.$$

21. Find the logarithms of ·000208 and $2{\cdot}01\dot{6}$.

22. Find by logarithms the value of

$$\sqrt[5]{\frac{6300 \times {\cdot}00117 \times 42{\cdot}9}{\frac{1}{2}(2197)^{\frac{1}{3}}}}.$$

23. A person borrowed £11,000 for two months at 5 per cent. per annum. At the end of the time the interest was added on and the debt renewed for another two months. This was continually repeated till at the end of two years the debt and interest were paid. Find by logarithms what this debt and interest together amounted to.

Given logarithms

$\log 2 = 0{\cdot}3010300$	$\log {\cdot}011 = \bar{2}{\cdot}0413927$
$\log 7 = 0{\cdot}8450980$	$\log 90 = 1{\cdot}9542425$
$\log 13 = 1{\cdot}1139434$	$\log 217{\cdot}47 = 2{\cdot}3373994$
$\log 121{\cdot}51 = 2{\cdot}0846120$	Diff. 200.
Diff. 357.	

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III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Add together $x+y$, $3x-y-3z$, $2y-2x+z$ and multiply the result by $x-y-z$.

Find the value of $\dfrac{x^5-x^4y+x^3y^3}{y^5-y^4x+y^3x^3}$ when $x=2y$, and prove that

$$(x+y)(x^2+y^2)(x^4+y^4)=\frac{x^8-y^8}{x-y}.$$

2. Divide $15x^5-17x^4-24x^3+138x^2-130x+63$ by $5x^3+6x^2-9x+7$ and verify your result by multiplication.

3. Prove that any common factor of two expressions is a factor of the sum or difference of any multiples of them.

What use is often made of this fact in the process of finding the highest common divisor of two algebraic expressions?

Express $4x^2-6yz-(9y^2+z^2)$, $9y^2+4xz-(4x^2+z^2)$, $z^2-12xy-(4x^2+9y^2)$ in factors, and hence write down their L.C.M.

4. Simplify

(1) $\dfrac{x^6+y^6}{x^6-y^6}\times\dfrac{x-y}{x+y}\div\dfrac{x^4-x^2y^2+y^4}{x^4+x^2y^2+y^4}$.

(2) $\dfrac{1}{2(x-1)}-\dfrac{x-5}{x^2-7x+10}+\dfrac{1}{2}\cdot\dfrac{x-6}{x^2-9x+18}$.

5. Extract the square root of the expression

$$(a-b)^2\{(a-b)^2-2(a^2+b^2)\}+2(a^4+b^4).$$

6. Show how to determine when the roots of the equation

$$ax^2+bx+c=0$$

are impossible.

If α, β be the roots of the equation $x^2+px+q=0$, prove that the roots of the equation $qx^2+p(1+q)x+(1+q)^2=0$ are $\alpha+\dfrac{1}{\beta}$ and $\beta+\dfrac{1}{\alpha}$.

7. Prove that when m and n are positive integers $(a^m)^n=a^{mn}$, and explain why the symbol a^{-n} is used to denote $\dfrac{1}{a^n}$.

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Divide $a^{\frac{5}{2}}+a^2b^{\frac{1}{3}}-a^{\frac{3}{2}}b^{\frac{2}{3}}-ab+a^{\frac{1}{2}}b^{\frac{4}{3}}+b^{\frac{5}{3}}$ by $a^{\frac{1}{2}}+b^{\frac{1}{3}}$, and find the value of

$$\frac{\sqrt{12}}{(1+\sqrt{2})(\sqrt{6}-\sqrt{3})}.$$

8. Solve the equations:

(1) $\dfrac{2x+1}{3}-\dfrac{3x-2}{4}=\dfrac{x-2}{6}.$

(2) $\left.\begin{array}{l} xy+\dfrac{x}{y}=10 \\ xy^2-x=6y \end{array}\right\}.$

(3) $\dfrac{y}{x^2-3}=-\dfrac{x}{y^2-3}=\dfrac{7}{x^3-y^3}.$

9. Find two numbers which are in the ratio of $\frac{1}{2}$ to $\frac{2}{3}$, but which, if respectively increased by 6 and 5, will be in the ratio of $\frac{3}{5}$ to $\frac{1}{2}$.

10. A slow train takes 5 hours longer in journeying between two given termini than an express, and the two trains when started at the same time, one from each terminus, meet 6 hours afterwards. Find how long each takes in travelling the whole journey.

11. Explain the meaning of "fourth proportional," "mean proportional," "duplicate ratio."

If $a : b :: c : d$ prove that $\left(\frac{a}{b}\right)^2+\left(\frac{c}{d}\right)^2=2\frac{ac}{bd}$, and show that if x and y are unequal, and $x : y$ is the duplicate ratio of $x-z : y-z$, then z is a mean proportional to x and y.

12. If the first term of a Geometric series be (a) and the last term be (l), the number of terms (n) being odd, what is the middle term?

Sum the series:

(1) $\sqrt{\frac{2}{3}}+\frac{1}{3}\sqrt{2}+\frac{2}{9}\sqrt{\frac{2}{3}}+$ &c. to infinity.

(2) $(2n-\frac{1}{2})+(4n+\frac{1}{8})+(6n-\frac{1}{18})+$ &c. to n terms.

13. Prove that the number of different arrangements of n things, taken three together is $n(n-1)(n-2)$.

In how many ways can a picket of 3 men and an officer be chosen out of a company consisting of 80 men and 3 officers?

14. Expand $(1-x^2)^{-5}$ to 6 terms by the binomial theorem.

Show that the nth terms of $(1-x)^{-n}$ and $(1+x)^{2n-2}$ are equal.

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IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Define the *tangent* and *secant* of an angle, and obtain a formula connecting them. Deduce the corresponding formula connecting the cotangent and cosecant, and prove that

$$(\tan\alpha + \operatorname{cosec}\beta)^2 - (\cot\beta - \sec\alpha)^2 = 2\tan\alpha\cot\beta\,(\operatorname{cosec}\alpha + \sec\beta).$$

2. Prove by geometrical construction that

$$\sin A = \sin(180^\circ - A), \text{ and}$$
$$\cos(180^\circ + A) = \cos(180^\circ - A).$$

3. Express $\cos 2A$ in terms of $\cos A$ or $\sin A$, and show that

$$8(\sin^4 A - \sin^2 A) + 1 = \cos 4A.$$

If $4A$ contains 40°, what four angles less than 180° will have their sines determined by this formula?

4. Prove that

(1) $2\cot 2A = \cot A - \tan A.$

(2) $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{16}{65}.$

(3) $\cot(A + 15^\circ) - \tan(A - 15^\circ) = \dfrac{4\cos 2A}{2\sin 2A + 1}.$

5. Find all the values of θ which satisfy the equation

$$4\cos\theta - 3\sec\theta = 2\tan\theta.$$

6. Prove that the circular measure of an angle less than a right angle is intermediate in value between the sine and the tangent of the angle.

Find approximately the number of minutes denoting the inclination to the horizon of an incline which rises $5\frac{1}{2}$ feet in 420 yards ($\pi = \frac{22}{7}$).

7. Show that in any plane triangle

(1) $a = b\cos C + c\cos B.$

(2) $a\sin(B - C) + b\sin(C - A) + c\sin(A - B) = 0.$

8. Two adjacent sides of a parallelogram, 5 inches and 8 inches long respectively, include an angle of 60°. Find the length of the two diagonals and the area of the figure.

9. In the triangle ABC, $a = 13$, $b = 7$, $C = 60^\circ$, find A and B.

Having given $\log 3 = \cdot 4771213$. $L\tan 27^\circ . 27' = 9\cdot 7155508$.
Diff. $1' = 3087$.

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10. A man observes that, when he has walked c feet up an inclined plane, the angular depression of an object in the horizontal plane through the foot of the slope is α; and that, when he has walked a further distance of c feet, the angular depression of the same object is β. Show that the inclination of the slope to the horizon is $\cot^{-1}\{2\cot\beta - \cot\alpha\}$, and determine the distance of the object observed from the foot of the slope.

11. Find the radius of the circle inscribed in a triangle in terms of the lengths of the sides.

If d_1, d_2, d_3 be the distances of the centre of this circle from the angular points A, B, C respectively, prove that

$$\frac{d_1^2}{bc} + \frac{d_2^2}{ca} + \frac{d_3^2}{ab} = 1.$$

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

(*In answering the questions on Geometry ordinary abbreviations may be employed, but the method of proof must be geometrical.*)

[*Great importance will be attached to accuracy in results. Full marks may be obtained by doing about* three-quarters *of this paper.*]

1. If two straight lines are at right angles to the same plane, prove that they are parallel to one another.

2. Draw a straight line perpendicular to a given plane from a given point without it.

Draw a straight line perpendicular to each of two given straight lines which are not in the same plane.

3. Show that there cannot be more than five different regular polyhedra.

Find the volume of a regular octohedron in terms of one of its edges.

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4. Show how a right circular cone may be cut by a plane so that the curve of section may be a parabola.

Determine the relative positions of two such cutting planes if they produce parabolas of which the one latus rectum is double the other.

5. TP, TP' being two tangents to a parabola whose focus is F, prove that TP, TP' subtend equal angles at F; and that the triangles TFP, TFP' are similar.

6. Any point P on an ellipse being joined with A, A' the extremities of the major axis and PK, PK' being drawn respectively perpendicular to AP, $A'P$ to meet the major axis in K and K'; prove that KK' is equal to the latus rectum.

7. If SY, HZ be the perpendiculars from the foci S and H upon a tangent to a hyperbola, prove that $CY = CZ = CA$: and that

$$SY \cdot HZ = BC^2.$$

8. Find the equations to the straight lines which pass through a given point and make a given angle with a given straight line.

Show that the equation $y^3 - x^3 + 3xy(y - x) = 0$ represents three straight lines equally inclined to one another.

9. Find the equation to a circle referred to oblique axes, and deduce that for rectangular axes.

Obtain the equations to two circles passing through a point h, k and touching the two straight lines $y \pm x = 0$.

10. Find the equation to a tangent at any point of the parabola $y^2 = 4ax$, and the length of the perpendicular to it drawn from the focus.

11. If CP, CD are two conjugate semi-diameters of the ellipse

$$a^2y^2 + b^2x^2 = a^2b^2,$$

find the coordinates of D in terms of those of P.

If S be a focus and θ and ϕ the angles SP, SD make with the tangents at P and D, prove that $\cot 2\theta + \cot 2\phi$ is constant.

12. Find the equation of a hyperbola referred to its asymptotes as axes.

Find the eccentricity and the equations of the asymptotes of the hyperbola $4xy - 3x^2 - 2ay = 0$, the axes being rectangular.

13. Obtain the equation to the normal at the origin of the curve

$$ax^2 + bxy + cy^2 + dx + ey = 0.$$

Hence show that all chords of any conic section which subtend a right angle at a given point of the curve intersect on the normal at that point.

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VI. PURE MATHEMATICS (2).

[*Full marks may be obtained for about* three-quarters *of this paper. Great importance will be attached to accuracy in results.*]

1. Eliminate x, y from the equations,

$$\left.\begin{aligned} \frac{b}{x}+\frac{a}{y}&=1 \\ \frac{x^2}{a^2}+\frac{y^2}{b^2}&=2 \\ xy+ab&=0 \end{aligned}\right\},$$

and find the resulting equation between a and b.

2. Prove that

$$1+mx+\frac{m.(m-1)}{1.2}x^2+\&c.$$
$$=\left(1-mx^2+\frac{m(m-1)}{1.2}x^4-\&c.\right)\left(1+mx+\frac{m(m+1)}{1.2}x^2+\&c.\right),$$

for all values of m.

3. In any arithmetical series prove that the product of two terms equidistant from the beginning and end increases as the difference between the orders of the terms diminishes.

Hence or by any other means prove that

$$1.2.3\ldots\ldots n>n^{\frac{n}{2}} \text{ if } n>2.$$

4. If $\frac{p_n}{q_n}$, $\frac{p_{n-1}}{q_{n-1}}$, $\frac{p_{n-2}}{q_{n-2}}$ be successive convergents to the continued fraction $\frac{1}{a_1+}\,\frac{1}{a_2+}$ &c., state and prove the rule for finding $\frac{p_n}{q_n}$ when $\frac{p_{n-1}}{q_{n-1}}$, $\frac{p_{n-2}}{q_{n-2}}$ are known.

5. State and prove Fermat's theorem.

If m be any integer and n be a prime number, prove that

$$(m-1)^n-m^n+1$$

is divisible by n.

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6. Find the sum of the series

$$\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\frac{1}{7\cdot 9}+\&c.$$

to n terms, and to infinity.

7. If $\sin\phi=\mu\sin\phi'$, and $\sin\psi=\mu\sin\psi'$, where μ is greater than 1, and each of the angles is less than a right angle, prove that

$$\phi-\psi>\psi'-\phi'.$$

8. If $\tan\beta$, $\tan 2\beta$, and $\tan\alpha$ are in arithmetical progression, prove that $\tan(\alpha-\beta)=\sin 2\beta$.

9. If D, E, F be the feet of the perpendiculars from the angles A, B, C of the triangle ABC upon the opposite sides respectively, and if S, and σ be the perimeters of ABC and DEF respectively, prove that

$$\sigma : S :: r : R,$$

where r and R are the radii of the inscribed and circumscribed circles of ABC.

10. Find the expansion of x in powers of $\tan x$.

If $\tan\frac{y}{2}=\sqrt{-1}\tan\frac{x}{2}$, prove that

$$\log\tan\left(\frac{\pi}{4}+\frac{y}{2}\right)=\sqrt{-1}\,.\,x.$$

11. If the roots of the equation $x^3+px^2+r=0$ be a, b, c, form the equation whose roots are a^2-bc, b^2-ac, and c^2-ab.

12. Prove that the equation

$$x^{n-1}-\frac{n-1}{2}x^2+\frac{n-3}{2}=0$$

has two roots equal to one another if n be any integer greater than 3.

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VII. PURE MATHEMATICS (3).

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance will be attached to accuracy in results.*]

1. Find the differential coefficients of

$$\log(x+\sqrt{x^3+a^2}),\quad \frac{(x+1)^{\frac{2}{3}}(x^2+1)^{\frac{1}{2}}}{x^2},\quad \sin^x x.$$

2. Find the fourth differential coefficient of $(1-x^2)\sin x$.

3. Find, by any method, the expansion of $\sin^{-1}x$ in ascending powers of x.

If $\sin\alpha=\frac{1}{10}$, find the value of α, in circular measure, to six decimal places.

4. If α and β be small quantities, find the value of $\sin(a+\alpha)\sin(b+\beta)$, neglecting small quantities of the third order in α and β.

5. Find the value of $\dfrac{x\log(1+x)}{1-\cos x}$ when $x=0$, and of

$$\frac{(x\sin^2\theta+a\cos^2\theta)^n-a^n}{x^n-a^n},$$

when $x=a$.

6. Find the minimum value of $\dfrac{(x+a)(x+b)}{(x-a)(x-b)}$, and of $\dfrac{a^2}{\sin^2 x}+\dfrac{b^2}{\cos^2 x}$.

7. If p be the length of the perpendicular from the origin on the tangent to a curve at a point whose radius vector is r; prove that

$$\rho=\frac{r\,dr}{dp},$$

where ρ is the radius of curvature at the point.

Apply this method to find the length of the radius of curvature at any point on an ellipse.

8. Find the equations of the asymptotes to the curve

$$(a+x)(b^2+x^2)=(x+c)y^2.$$

9. Find the integrals of the expressions

$$\sin^3 x\cos^4 x\,dx,\quad \sin^{-1}x\,dx,\quad \frac{x\,dx}{x^4+1}.$$

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10. Find, by any method, the length of an arc of the cycloid; and prove that the whole area of the curve is three times that of its generating circle.

11. Find the values of the definite integrals

$$\int_0^1 \frac{dx}{1+x^2}, \quad \int_0^{\frac{\pi}{2}} x^2 \sin x \, dx, \quad \int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}\, dx.$$

12. Trace the curves $r^2 = a\cos\theta$, $r = a\cos^2\theta$ and find the whole area of the latter curve.

VIII. STATICS.

1. Assuming that the sum of the moments of two forces about any point is equal to the moment of their resultant about that point, prove that the sum of the moments of any system of forces in equilibrium, about any point in their plane, is zero.

2. Show that a force and a couple in one plane may be reduced to a single force. Given in position a force of 10 lbs., and a couple consisting of two forces of 4 lbs. each, at a distance of two inches asunder, acting with the hands of a clock, draw the equivalent single force.

3. Four forces form the sides of a quadrilateral $ABCD$; three acting in the directions AB, BC, CD and the fourth acting from A to D. Show that their resultant is parallel to AD, and find its distance from AD by a geometrical construction.

4. Find the least horizontal force applied at the centre of a wheel which will cause it to surmount a given obstacle, given the vertical load acting at the centre of the wheel, and supposing the wheel to have no velocity when in contact with the obstacle. Is the horizontal direction the best for the force and is the centre the best point of the wheel at which to apply it, the wheel being supposed to be rough?

5. Give a rough sketch of Roberval's balance and apply the theory of couples to prove that two equal weights will balance on it, wherever they are laid on the two scales.

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6. A heavy string rests on two given rough inclined planes, of the same material, passing over a smooth pulley at their common vertex. If the string is on the point of motion, show that the line joining its two ends is inclined to the horizon at the angle of friction.

7. A heavy triangular plate lies on the ground. If a vertical force applied at the vertex A is just great enough to begin to lift that vertex off the ground, prove that the same force will suffice, if applied at B or C, the two remaining vertices.

8. A uniform bar is placed in a sloping position, its lower end on the ground, its upper in the air, the bar being supported by a smooth fixed peg against which it rests. If the ground is smooth can it rest in this position?

If the coefficient of friction at the ground is μ, if l = length of bar, and h = height of the peg from the ground, and if α be the angle the bar makes with the horizon when on the point of slipping, prove that α is to be found from the equation

$$\cos\alpha \sin^2\alpha + \mu \sin\alpha \cos^2\alpha = 2\frac{h}{l}\mu,$$

(supposing the centre of the bar below the peg).

9. Three bars of the same material and the same section, OA, OB, OC have one extremity O common. Find their centre of gravity. If the centre of gravity be at O, show that the sines of the three angles they form are as the squares of the lengths of the bars.

10. A heavy particle, weight W, is in a given position on the outside of a smooth vertical circle, being supported there by a string passing round a pulley situated on the vertical diameter produced, and which supports a weight. Find this weight by a geometrical construction (or otherwise); and determine whether the equilibrium is stable or unstable. Show that the pressure on the circle is the same wherever W is placed.

11. Four rods without weight form a foursided frame $ABCD$, being freely jointed together at the angles. If two equal and opposite forces act respectively at two points of the rods AB, CD, prove that they cannot keep the frame in equilibrium unless their direction passes through the intersection of AD and BC.

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IX. DYNAMICS.

1. How is velocity measured, (1) uniform, (2) variable?

The Derby course is 1 mile 5 furlongs in length. If the race is run in 2 minutes 30 secs., compare the average velocity of the winning horse with that of a train moving at the rate of 40 miles an hour.

2. A particle starts with a velocity of 20 yards a minute, under a uniform acceleration, producing an increase of velocity of 100 miles an hour in each hour, find the number of feet described by it in 6 seconds.

3. A projectile is projected with velocity V from a point in a plane which is inclined at the angle α to the horizon in a direction making an angle β with the plane measured upwards: find the range on the plane.

If the plane be smooth, find the greatest distance measured along the plane to which the particle will ascend, and the time before it again reaches the point of projection.

4. How is an impulsive force measured?

If a given impulse acting on a stone causes it to rise to a height h, how high will a stone of half the weight of the former rise under the action of the same impulse?

5. Assuming that the resistance of all kinds to a train on the level is equal to 6 lbs. per ton of the total weight of the train, when it is moving with less speed than 10 miles an hour, with an addition of $\frac{1}{8}$ lb. per ton for every mile of speed above 10, that the total weight of a train and engine is 200 tons, and the maximum speed obtainable 40 miles an hour, find the time from rest of reaching a velocity of 10 miles an hour, the engine always exerting its full force.

6. A person wishes to throw a stone so as to produce the greatest possible blow, at a point in a smooth vertical wall, at a height h from the ground. His strength is sufficient to throw the stone vertically upwards to a height $2h$. Prove that he must throw from a point distant $2h$ from the foot of the wall, the resistance of the air and height of his hand in throwing being neglected.

7. Explain *generally* the reasoning by which it is proved that the time of oscillation of a simple pendulum of length l is $\pi\sqrt{\frac{l}{g}}$.

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If a seconds pendulum be formed of a heavy particle suspended by a string of length (l) from a point A, in a vertical wall, and if a nail jut out from the wall at a distance $\frac{l}{n}$ vertically below A, so as to catch the string once in each complete oscillation, find the time of a complete oscillation (n being large).

8. Find the acceleration of a body on a smooth inclined plane.

Prove that the time of falling down the plane from rest is the same as the time of moving over the same distance, with a uniform velocity equal to half that acquired in falling down the plane.

9. Define angular velocity, and state how it is measured. Find the linear velocity and acceleration of a point on the circumference of a circle of radius (a) revolving with angular velocity (ω) round the centre fixed.

If the centre be at the same time moving along a given straight line with uniform velocity (μ) find the velocity and acceleration of any given point on the circumference.

10. Find, after direct impact, the velocities of two spheres of equal radii but different masses, their elasticity being given.

If the impact be oblique and if θ be the inclination of the relative velocity to the line of centres before impact: find the corresponding inclination after impact.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JULY, 1885.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If two triangles have two angles of the one equal to two angles of the other, each to each, and one side equal to one side, viz., sides which are opposite to equal angles in each; then shall the other sides be equal, each to each, and also the third angle of the one to the third angle of the other.

$ABCD$ is a rectangle, and AE, BF are drawn to meet the diagonals BD, AC in E and F respectively, and in such a direction that the angles AEB, AFB are equal to one another. Show that the triangles AEB, AFB are equal in all respects.

2. The opposite sides and angles of a parallelogram are equal to one another, and the diameter bisects the parallelogram, that is, divides it into two equal parts.

AB, CD, EF are three parallel straight lines: and the points A, C, E are in a straight line, and also the points B, D, F. Prove that if AC is equal to CE, then BD is equal to DF.

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3. If a straight line be bisected and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

4. In obtuse angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

What is the value in similar terms of the square on the side subtending one of the acute angles of this triangle?

$ABCDE$ is a straight line so divided that $AB=BC=CD=DE$, and O is an external point; show that the difference of the squares on OA and OE is twice the difference of the squares on OB and OD.

5. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

If any eight-sided figure be inscribed in a circle, show that the sum of either four alternate angles is equal to six right angles.

6. If from a point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it.

Two equal circles intersect in A and B; show that if from an external point O a tangent be drawn to each circle, the tangents will be unequal unless O lies on AB produced.

7. Inscribe a circle in a given triangle.

Find that straight line which would, if produced, bisect the angle between two given straight lines, without producing the given straight lines to meet.

8. Describe an isosceles triangle having either of the angles at the base double of the third angle.

9. What is Euclid's test for *Proportion?*

Triangles and parallelograms of the same altitude are to one another as their bases.

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10. In a right-angled triangle, if a perpendicular be drawn from the right angle to the base, the triangles on each side of it are similar to the whole triangle, and to one another.

If AD be the perpendicular, and AB, AC the two sides including the right angle: prove that

$$\frac{1}{AD^2} = \frac{1}{AB^2} + \frac{1}{AC^2}.$$

11. Similar triangles are to one another in the duplicate ratio of their homologous sides.

II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Simplify $\dfrac{(423 \div \frac{3}{8}) + (1\frac{4}{5} \div 3) + (4\frac{1}{2} \div 1\frac{3}{4})}{(\frac{3}{7} \times 41) + (32 \times \frac{3}{8}) - (1 \div \frac{7}{10})}$.

2. Express the value of $\dfrac{6{\cdot}604\dot{1}\dot{6}}{33{\cdot}0208\dot{3}} \div \dfrac{2.7\dot{7}\dot{2}}{11{\cdot}\dot{0}\dot{9}}$ as a decimal.

3. A cube has an edge 2 ft. 6 in. long: find the ratio between the sum of the areas of the semicircles described on its edges, and the whole surface of the cube, having given that the area of a circle $= 3{\cdot}1416$ times the square of the radius.

4. Extract the square root of ·02010724, and the cube root of 18·609625.

5. Find the value of $\cdot\dot{5}\dot{4}$ of 8*s.* 3*d.* + $\cdot\dot{0}\dot{2}\dot{7}$ of £2. 15*s.* + ·3125 of £2. 2*s.*

6. If 24 oxen require 6 acres of turnips to supply them for 10 weeks, how many acres would supply 6 score of sheep for 15 weeks, 3 oxen eating as much as 10 sheep?

7. What weight must be added to $\frac{3}{8}$ of $\frac{5}{4}$ of half a cwt. to make it up to $\frac{3}{10}$ of $3\frac{3}{4}$ quarters avoirdupois?

8. Reduce 15*s.* $9\frac{3}{4}$*d.* to the decimal of £1, and £5. 7*s.* $6\frac{3}{4}$*d.* to the decimal of one shilling.

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9. Define a common measure and a common multiple, and show how to find the Least Common Multiple of two numbers. Find the least number which contains the numbers 12, 20, 42, and 56.

10. If 3 tons 13 cwt. 3 qrs. 12 lbs. cost £8. 12*s*. 4*d*., how much will 1 ton $15\frac{3}{4}$ cwts. cost?

11. (*a*) How many superficial feet of inch plank can be sawn out of a log of timber 20 ft. 7 in. long, 1 ft. 10 in. wide, and 1 ft. 8 in. deep? Find this by duodecimals. (*b*) Show that an acre contains 10 square chains, assuming that 1 chain = 100 links, that 1 link = 0·66 of a foot, and that 640 acres = 1 square mile.

12. Find, by practice, the cost of making a road 29 m. 7 f. 200 yds. at £34. 16*s*. 8*d*. per mile.

13. Find the difference between the simple and compound interest on £378. 15*s*. for 2 years at $3\frac{1}{8}$ per cent.

14. Find the rate of interest when the discount on £226. 2*s*. 8*d*. due at the end of $1\frac{1}{2}$ years is £12. 16*s*.

15. A person has a sum of money invested in Consols (3 per cent.). He sells out at $87\frac{1}{4}$, and invests the proceeds in railway shares, when £100 sells for $£174\frac{1}{2}$. He thus increases his income from £120 to £200. What dividend per cent. does the railway pay?

16. An open tank measuring on the outside 6 yds. 2 ft. in length, 3 yds. 2 ft. in width, and 7 ft. deep, is made with sides and floor of brick 1 ft. thick, and is filled with water. Find the weight of the tank and its contents, it being given that one cubic foot of water weighs 1000 oz., and that brick is one and a half times as heavy as water.

17. An astronomical clock has its dial divided into 24 divisions instead of 12, and the small hand goes round once in 24 hours, the large hand going round once every hour. The 24 hour is noon. Find when the hands are at right angles to one another between 24 and 1, and find also the interval between two successive meetings of the hands.

18. A legacy of £2420 is left to three persons in such proportions that after the payment of a legacy duty of 15 per cent. the first receives twice as much as the second, the second three times as much as the third. What are their respective shares?

19. What does a person gain or lose per cent. by selling butter at $8\frac{1}{2}$*d*. per pound which cost £5. 5*s*. per cwt.?

20. A sum of £385. 17*s*. 6*d*. is due at the end of two years. What is its present worth, reckoning compound interest at 5 per cent.?

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21. Find the value of $\log_{10} \cdot 01$, $\log_7 343$, $\log \sqrt[6]{8\sqrt{3}}$, $\log 2\sqrt[10]{66}$.

Given
$$\log 2 = \cdot 30103.$$
$$\log 3 = \cdot 4771213.$$
$$\log 11 = 1\cdot 0413927.$$
$$\log 3\cdot 0407 = \cdot 4829736.$$
$$\log 3\cdot 0408 = \cdot 4829879.$$

22. State and explain the rule for equation of payments.

A has to pay £100 per annum half-yearly for two years: find the equated time.

III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Multiply $\frac{3x^3}{2} + 3x^2 - \frac{5x}{4} - \frac{23}{2}$ by $\frac{x^2}{6} - \frac{x}{3} + \frac{1}{2}$, and divide $32x + y^{\frac{5}{2}}$ by $2x^{\frac{1}{6}} + y^{\frac{1}{2}}$.

2. Find the highest common factor of
$$2x^3 - x^2 - x - 3 \text{ and } x^5 - x^3 - 4x^2 - 3x - 2,$$
and write down the L.C.M. of
$$9x^2 - 4,\ 4x^2 - 36,\ 3x^2 - 7x - 6,\ 3x^2 + 7x - 6.$$

3. Find the value of $x^3 + 2y^3 + 2z^3 + 6xyz$ when $x = y + z = \sqrt[3]{4}$, and if $\frac{x^2}{y^2} + \frac{y^2}{x^2} = 3$, prove that the value of $\frac{x^3}{y^3} - \frac{y^3}{x^3}$ will be 4 or -4.

4. Simplify

(1) $\dfrac{1}{x-1} + \dfrac{2x+1}{x^2+x+1} - \dfrac{3}{x}$.

(2) $\left(\dfrac{x}{x-2} + \dfrac{5}{x-8}\right) \times \left(\dfrac{x-3}{3x-8} - \dfrac{2}{x+2}\right)$.

(3) $\dfrac{(x-y)^4 - xy(x-y)^2 - 2x^2y^2}{(x-y)(x^3-y^3) + 2x^2y^2}$.

5. Extract the square root of $47 - 12\sqrt{15}$, and find the value, when $x = \sqrt{3}$, of the expression
$$\frac{2x-1}{(x-1)^2} - \frac{2x+1}{(x+1)^2}.$$

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6. Solve the equations

(1) $\dfrac{x-\frac{1}{2}}{x-1}-\dfrac{3}{5}\left(\dfrac{1}{x-1}-\dfrac{1}{3}\right)=\dfrac{23}{10(x-1)}$.

(2) $ax-by=2ab,\ 2bx+2ay=3b^2-a^2$.

(3) $y^2+xy=4,\ x^2+2y^2-xy=8$.

7. The perimeter of a right-angled triangle is six times as long as the shortest side. What is the ratio of the two sides containing the right angle?

8. Prove that p is the sum and q the product of the roots of the equation $x^2-px+q=0$.

Form an equation whose roots shall be the square of the sum, and the square of the difference of the roots of the equation $3x^2+3ax+a^2=0$.

9. The time during which a body will slide down a smooth inclined plane varies directly as the length and inversely as the square root of the vertical height of the plane. If the time of descent is one second when the height is 4 feet and length 8 feet, what is the height of a plane 1 yard long, down which a body will slide in half a second?

10. The 1st and 2nd terms of an harmonic series are 5 and 3 respectively. Find the next five terms.

A Geometric series, whose common ratio is $\frac{1}{n}$, has the same 1st and 2nd terms as an Harmonic series. Prove that the 3rd term of the former series will be equal to the $\overline{n+2}|$th of the latter.

11. How many different sets of 12 can be chosen from a group of 15 men?

If 12 be selected from each of two groups in how many different ways can the 24 men be arranged in a line so that 4 men, and not more than 4 men from the same group, shall always stand together?

12. In what scale of notation will the decimal fraction ·1 be expressed by ·02222.....?

13. Write down the general term in the expansion of $(a-x)^{2n}$, and show that terms equidistant from the beginning and the end have the same numerical coefficient.

Also find the 11th term in the expansion of

$$\left(4x-\frac{1}{2\sqrt{x}}\right)^{15}.$$

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IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Explain the different methods of measuring an angle by means of degrees and circular measure, and write down the relations between these two measures of an angle.

Find the number of degrees, and the circular measure of an angle of a regular octagon.

2. Define the cotangent and cosecant of an angle, and prove by geometrical constructions the formulae:

$$\operatorname{cosec}(180^\circ - A) = \operatorname{cosec} A, \quad \cot(180^\circ - A) = -\cot A,$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

3. Find a general expression for all the angles which have the same tangent as a given angle.

Solve the equation:

$$\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0.$$

4. If $A+B$ be less than a right angle, prove by means of geometrical figures the formulae,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B,$$
$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B).$$

5. Prove that

(1) $\tan\dfrac{\theta}{2} = \sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}.$

(2) $\dfrac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta \cos 4\theta.$

(3) $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\frac{77}{85}.$

6. Prove that

$$\sin\frac{A}{2} + \cos\frac{A}{2} = \pm\sqrt{1+\sin A},$$

and

$$\sin\frac{A}{2} - \cos\frac{A}{2} = \pm\sqrt{1-\sin A},$$

and find the proper signs when the angle A lies between 990° and 1080°.

Show by a geometrical figure that $\sin\dfrac{A}{2}$, when expressed in terms of $\sin A$, has four values.

7. Prove that in a triangle the sides are proportional to the sines of the opposite angles.

If, in a triangle ABC, $a\cos A = b\cos B$, prove that the triangle is isosceles or right angled.

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8. The lengths of two of the sides of a triangle are 1 foot and $\sqrt{2}$ feet respectively, and the angle opposite to the shorter side is 30°. Prove that there are two triangles which satisfy these conditions; find their angles, and show that their areas are in the ratio $\sqrt{3}+1 : \sqrt{3}-1$.

9. Prove that, in a triangle ABC

$$\cot\frac{C}{2}=\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}.$$

If $a=32$, $b=40$, $c=66$, find the angle C, having given

$$\log 207=2{\cdot}3159703,$$
$$\log 1073=3{\cdot}0305997,$$
$$\mathrm{L}\cot 66^\circ\ 18'=9{\cdot}6424342,$$

and that the difference for 1′ is 3431.

10. A ship sailing due north observes two lighthouses bearing respectively N.E. and N.N.E. After sailing 20 miles the lighthouses are seen to be in a line due east.

Having given

$$\log 2=\ {\cdot}3010300, \qquad \log 11{\cdot}715=1{\cdot}0687423,$$
$$\mathrm{L}\tan 22^\circ\ 30'=9{\cdot}6172243, \qquad \log 11{\cdot}716=1{\cdot}0687794,$$

find the distance in miles between the lighthouses accurately to four places of decimals.

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[*Full marks may be gained by doing about three-fourths of this paper. Great importance will be attached to accuracy in results.*]

1. Draw a straight line perpendicular to a given plane from a given point without it.

What is the locus of the points which are equidistant from two given planes?

2. If two straight lines be cut by parallel planes, they shall be cut in the same ratio.

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If a tetrahedron be cut by a plane parallel to two edges which do not meet, prove that the section is a parallelogram, and that its area is greatest when it bisects the other edges.

3. Define a pyramid and a regular pyramid.

How many regular pyramids can be constructed, the faces of which are equal equilateral triangles?

4. Prove that the section of a right circular cylinder by a plane is either a circle, an ellipse, or two parallel straight lines. If the section be an ellipse, prove that its foci are the points of contact of the two spheres which can be inscribed in the cylinder so as to touch the plane of the section.

5. If PN be the ordinate of a point P of a parabola, of which A is the vertex, and S the focus, prove that

$$PN^2 = 4AS \,.\, AN.$$

If QQ' be a double ordinate of a parabola, and if PD be a straight line drawn parallel to the axis from a point P of the curve, and meeting QQ' in D, prove that

$$QD \,.\, DQ' = 4AS \,.\, PD.$$

6. If the tangent at a point P of an ellipse meet the axis major in T and the axis minor in t, prove that

$$CN \,.\, CT = AC^2, \text{ and } PN \,.\, Ct = BC^2.$$

7. There are two equal fixed circles in a plane, which do not intersect each other. Prove that the locus of the centres of the circles which can be drawn touching the two circles consists of a straight line and a hyperbola, the transverse axis of which is equal to the diameter of either circle, and that its eccentricity is the ratio of the distance between the centres to the diameter.

8. If (x, y), (x', y') be the coordinates of two points, find an expression for the distance between the points (1) when the axes are rectangular, (2) when the axes are inclined to each other at the angle ω.

Find the position of the centre of the circle circumscribing the triangle, the coordinates of whose angular points are (2, 3), (3, 4), (6, 8), the axes being rectangular.

9. Find the polar equation of a circle, the pole being a point on the circumference. O is one of the points of intersection of two fixed circles, and OPQ is a straight line through O, meeting one circle in P and the other in Q; find the locus of the middle point of the line PQ.

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10. Find the equation of the normal at any point of a parabola, and show that it can be put into the form

$$y = mx - 2am - am^3.$$

11. Define the eccentric angle at a point of an ellipse, and write down the equation of the tangent at the point in terms of the eccentric angle.

If CP, CD be conjugate semi-diameters of an ellipse, and if ϕ be the eccentric angle of the point P, find the equation of PD.

If ψ be the inclination of PD to the major axis, prove that

$$a \tan \psi = b \tan \left(\phi - \frac{\pi}{4}\right).$$

12. Trace the curve represented by the equation $x^2 - y^2 = a^2$, and find the value of c in order that the line, $y = mx + c$, may be a tangent to the curve.

Prove that the locus of the point of intersection of tangents to the curve, which are inclined to each other at the angle of 45°, is the curve

$$(x^2 + y^2)^2 = 4a^2 (a^2 + y^2 - x^2).$$

13. Find the condition that the general equation of the second degree may represent a parabola, and interpret the equation

$$\sqrt{x} + \sqrt{y} = \sqrt{a}.$$

Find the locus of the middle points of the chords of this curve which are parallel to the line $x \sin \theta = y \cos \theta$.

VI. PURE MATHEMATICS (2).

[*Great importance will be attached to accuracy in results.*]

1. Find the coefficient of the middle term of $(1 + x + x^2)^n$, and prove that if each term of the coefficients be squared, the result will be the coefficient of the middle term of $(1 + x + x^2)^{2n}$.

2. Show that if m be any positive integer,

$$\left(1 + \frac{x}{m}\right)^m \text{ is} < \left(1 + \frac{x}{m+1}\right)^{m+1}, \text{ and that } 2^m - 1 \text{ is} > m \,.\, 2^{\frac{m-1}{2}}.$$

3. If n Arithmetic and n Harmonic means be inserted between a and b, and a series of n terms be formed by dividing each Arithmetic by the corresponding Harmonic mean, prove that the sum of the series is

$$n \left\{1 + \frac{n+2}{n+1} \frac{(a-b)^2}{6ab}\right\}.$$

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4. Prove that every convergent is nearer to the continued fraction than any of the preceding convergents.

If $\frac{P}{Q}, \frac{P'}{Q'}$ have the same first convergents $\frac{p_1}{q_1} \ldots\ldots \frac{p_n}{q_n}$, the difference

$$\frac{P}{Q} - \frac{P'}{Q'} \text{ is } < \frac{1}{q_n^2}.$$

5. Define, and find the sum of n terms of a recurring series.

Sum the series $x + 3x^2 + 5x^3 +$ &c., to n terms.

6. Prove that

(1) $\sin^4\theta + 2\cos\alpha \sin^2\theta \cos^2\theta + \cos^4\theta = 1 - \left\{\sin\frac{\alpha}{2}\sin 2\theta\right\}^2$.

(2) If $u = \cot^{-1}\sqrt{\cos\alpha} - \tan^{-1}\sqrt{\cos\alpha}$ prove that $\sin u = \tan^2\frac{\alpha}{2}$, and eliminate θ from the equations

$$x = a\cos\theta + b\cos 2\theta,$$
$$y = a\sin\theta + b\sin 2\theta.$$

7. Expand the sine and cosine of an angle in terms of its circular measure. If α, β, γ are the angles of a triangle, and a, b, c the sides opposite to them, and γ nearly $= \pi$, prove that $a + b = \frac{c}{1 + \alpha\beta}$ nearly.

8. Expand θ in powers of $\tan\theta$, and hence derive a rapidly converging series for determining the value of π.

9. If $\cos p\theta + \cos q\theta = 0$, prove that the different values of θ form two Arithmetical Progressions in which the common differences are $\frac{2\pi}{p+q}$ and $\frac{2\pi}{p-q}$ respectively.

10. State the relations between the coefficients and the roots of an algebraic equation. If the roots of the equation $x^3 + px^2 + qx + r = 0$ be a, b, and c, form the equation of which the roots are $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$.

11. Two roots of the following equation are equal, find all the roots:

$$x^3 - 7x^2 + 16x - 12 = 0.$$

12. Prove that the numerically greatest negative coefficient increased by unity is a superior limit of the positive roots of an equation which is in its simplest form.

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VII. PURE MATHEMATICS (3).

[*Full marks will be given for correct answers to about* two-thirds *of this paper. Great importance will be attached to accuracy in results.*]

1. Differentiate $\left(\frac{1-x}{1+x}\right)^n$.

2. Prove the formula for differentiating $y=uv$, where u, v are given functions of x: and apply it to find the differential coefficient of x^n when n is an integer.

3. Find the nth differential coefficient of $\log x$.

4. Expand $\log(1+e^x)$ by powers of x as far as x^2.

5. How may it be ascertained whether a given function $f(x)$ continually increases, as x increases from a given value a to a given value b?

Show that the function

$$x \sin x + \cos x + \cos^2 x$$

continually diminishes as x increases from 0 to 90°.

6. Find the least possible value of $\frac{\sqrt{a^2x^2+b^2c^2}}{x+c}$.

7. Trace the curve $y=x-x^3$. Has it a point of inflexion? If a line be drawn from the origin to an infinitely distant point on the curve, what is the direction of this line?

8. Find the radius of curvature of the curve

$$y=x-\sin x$$

at the origin; also at the point where $x=\frac{\pi}{2}$.

9. Find the equation of the tangent at a given point on the curve

$$x^4+y^4=a^4.$$

What relation will hold between α, β, the portions intercepted on the axes by this tangent, whatever be the point of contact?

10. Integrate $\frac{dx}{1+x+x^2}$; $\cos^2 x\,dx$; $e^{ax}\,dx$.

11. Integrate $\sin^{-1} x\,dx$.

12. Reduce the integral $\int x^n \cos x\,dx$ to the form $\int x^{n-2} \cos x\,dx$.

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13. Find the limit of the series

$$\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots\ldots\frac{1}{2n}$$

when n increases indefinitely.

14. Show that in the curve

$$2y=e^{x}+e^{-x},$$

the area between the ordinates at any two points A, B varies as the arc AB.

VIII. STATICS.

[*Great importance will be attached to accuracy.*]

1. *State* the proposition known as the parallelogram of forces. If the forces are incommensurable, give a demonstration to prove the proposition so far as the *direction* of the resultant is concerned.

A uniform plane lamina in the form of a rhombus, one of whose angles is 120°, is supported by two forces applied at the centre in the direction of the diagonals, so that one side of the rhombus is horizontal; show that if P and Q be the forces, P being the greater, $P^2=3Q^2$.

2. If any number of forces acting on a particle can be represented in magnitude and direction by the sides of a polygon taken in order they will keep the particle in equilibrium.

Show that if a particle, placed in the centre of a regular polygon, be acted on by forces represented by the lines drawn from the particle to each of the angles it will be at rest.

3. Any system of forces acting in one plane on a rigid body may in general be reduced to a single couple and a single force acting at a given point of the plane, the given point being also the extremity of the arm of the resultant couple.

A square is acted on by forces 2, 4, 6, 8, taken in order along its sides; find the resultant and the resultant couple of these forces, if the centre of the square be taken for the given point.

4. Show that the centre of gravity of the perimeter of a given triangle is the centre of the circle inscribed in the triangle formed by joining the middle points of the given triangle.

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5. In a system of three movable pulleys in equilibrium, in which the strings are parallel and each is attached to the weight (usually called the 3rd system), find the tension of the string which passes over the fixed pulley, the weights of the pulleys, which are all equal, being taken into account.

If the weights of the pulleys are neglected and each string is attached to a bar which keeps horizontal, find the point in the bar from which the weight is suspended.

6. Give examples of stable and unstable equilibrium. A cone and a hemisphere of the same material are cemented together at their common circular base: if they are placed on a horizontal plane, the hemisphere being in contact with the plane, find the height of the cone that the equilibrium may be neutral, it being given that the centre of gravity of a hemisphere divides a radius in the ratio of 3 to 5.

7. If any number of forces act in one plane on a rigid body, *state* the three conditions of equilibrium.

A beam whose weight is (W) rests with its ends on two inclined planes whose angles of inclination are (α) and (β); prove that the sum of the pressures on the planes is

$$W.\frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}.$$

8. On what hypotheses is the relation between (P) and (W) usually obtained on a screw considered as a mechanical power? If the screw be rough, and (R) the sum of the normal resistances on the thread of the screw, (W) the weight supported, (ϕ) the angle of resistance, and (α) the pitch of the screw, obtain the equation $R=\frac{W\cos\phi}{\cos(\alpha+\phi)}$, where ($P$) is just on the point of prevailing over (W).

9. The same force acting parallel to two inclined planes of 30° and 60° inclination can just move a given weight up the plane of less inclination, and can just prevent a weight twice as great from moving down the plane of greater inclination; prove that the coefficient of friction which is the same for both planes is nearly $\frac{13}{20}$.

10. State the principle of "virtual velocities." If a weight (W) be supported in equilibrium on a smooth inclined plane by a weight (P) hanging vertically and passing over a fixed pulley placed in the prolongation of the height of the plane, prove that the principle of virtual velocities holds good, and show that for a small displacement the centre of gravity of (P) and (W) will neither ascend nor descend.

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11. A heavy ladder is placed in a given position, between a vertical wall and the horizontal ground, both being considered equally rough; a workman of given weight ascends the ladder with a given load, show how to determine by a *geometrical construction* whether the ladder will slip.

IX. DYNAMICS.

[*Great importance will be attached to accuracy.*]

1. The measure of a certain velocity in feet per second is v; what is it in miles per hour?

2. Two trains, each 200 feet long, are moving towards each other with velocities of 20 and 30 miles per hour respectively. Find the time which elapses from the instant when they first meet till they have completely cleared each other.

3. State exactly what is meant by saying that the accelerating force of gravity is 32.

In the equation $F=mf$, if a foot and a second are the units of space and time, and a weight of one pound the unit of force, what is the unit of mass?

4. Two bodies which weigh 9 lbs. and 16 lbs. respectively at the earth's surface, are placed in space at a distance asunder of 100 feet, no force acting on either. If they were now to attract each other with a constant force equal to 1 lb. at all distances, find after what time they would meet.

5. A body slides down a rough inclined plane 100 feet long, the sine of whose inclination $=0{\cdot}6$, and coefficient of friction $=\frac{1}{2}$: find its velocity at the bottom.

If projected up the plane with a velocity which just carries it to the top, find what height it would reach if thrown vertically upwards with that velocity.

6. Two perfectly elastic balls whose masses are M, M', moving in the same direction, strike each other. If the hindmost ball is reduced to rest by the blow, show that its velocity must have been more than double that of the other.

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7. A train runs from rest down an incline of 1 in 100 for a distance of 1 mile (no engine attached): it then runs up an equal gradient with its acquired velocity for a distance of 500 yards before stopping. Assuming the principle of work, find the total resistance, frictional or other, in pounds per ton, which has been opposing its motion.

8. A body is projected horizontally with a given velocity. Prove that it describes a parabola, and determine the position of its focus.

9. Two bodies are projected from the same point, one later than the other by T seconds, so as to describe the same parabola. Show that they are nearest to each other when in the same horizontal line (if that is possible) and that this occurs at the interval of time $\frac{V \sin \alpha}{g} - \frac{T}{2}$ after the second body was projected. Explain what circumstances as to the *data* are required, if this be possible.

10. Two weights, P, Q, are connected by a light string passing over a smooth fixed peg. Find the acceleration of the system. Mention any experimental use which has been made of this contrivance.

If two equal weights, P, P, are in equilibrium, connected in this way, and a third weight, P, is laid on one of them, find by how much the pressure on the peg is increased.

11. A particle starting with a velocity u, falls down a smooth vertical curve of any form. State what its velocity is when it has arrived at any given point of the curve.

A particle falls down a vertical circle, starting from rest at the highest point. If, when at any point, its velocity be resolved into two components, one passing through the centre, the other through the lowest point of the circle, prove that the latter is of constant magnitude.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1885.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. Define a *right angle*, and show how to draw a straight line at right angles to a given straight line, from a given point in the same.

If one diagonal of a quadrilateral figure bisect the two angles at its extremities, it will bisect the other diagonal at right angles.

2. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

The longer sides of a parallelogram are twice as long as the shorter sides. Show that the straight lines joining the middle point of one of the longer sides with the ends of the opposite side, are perpendicular to each other.

3. If a parallelogram and a triangle be on the same base and between the same parallels, the parallelogram shall be double of the triangle.

4. If a straight line be divided into two equal and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

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If any point D be taken in the base BC of an isosceles triangle ABC, the rectangle under BC, CD will be equal to the difference between the squares on AB, AD.

5. If any point be taken without a circle, and straight lines be drawn from it to the circumference, one of which passes through the centre, of those which fall on the concave circumference, the greatest is that which passes through the centre, and of the rest, that which is nearer to the one passing through the centre is always greater than one more remote; but of those which fall on the convex circumference, the least is that between the point without the circle and the diameter; and of the rest, that which is nearer to the least is always less than one more remote.

Prove that the two lines which are equally inclined to the shortest line are equal.

6. Define a *segment of a circle*, and shew how to bisect its circumference.

Prove that the straight lines joining the ends of the base of a segment of a circle with any point on its circumference, are equally inclined to the straight line passing through that point and the point of bisection of the circumference.

7. About a given circle describe a triangle equiangular to a given triangle.

Prove that the equilateral triangle inscribed in a circle is one-fourth of the equilateral triangle described about the same circle.

8. Inscribe an equilateral and equiangular pentagon in a given circle.

9. The sides about the equal angles of triangles which are equiangular to one another are proportionals.

Prove that the diameters of the circles circumscribing the two triangles formed by joining the vertex with any point in the base of a given triangle are proportional to the sides of that triangle.

10. If four straight lines be proportionals the similar rectilineal figures similarly described on them shall also be proportionals; and if the similar rectilineal figures similarly described on four straight lines be proportionals, those straight lines shall be proportionals.

11. If the vertical angle of a triangle be bisected by a straight line which also cuts the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square on the straight line which bisects the angle.

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II. ARITHMETIC.

(Including the use of Common Logarithms.)

[*Great importance will be attached to accuracy.*]

1. Multiply five million forty-six thousand and one, by four thousand and eight; and divide ·0064 by ·51863 to seven places of decimals.

2. When is the decimal equivalent to a vulgar fraction a terminating decimal, and when a circulating decimal? Multiply together, ·4, ·4, and ·0004.

3. Find a mean proportional between 36 and 2401, and extract the cube root of 128·024064.

4. Find a servant's wages for five months three weeks six days at one pound seven shillings and five pence per month, reckoning seven days to a week, and four weeks to a month.

5. If 2 cwts. 2 qrs. of sugar cost £3. 10*s.* 0*d.*, what will be the cost of 28 lbs. 4 ozs.?

6. Find how many flagstones, each 5·76 ft. long and 4·15 ft. wide, are requisite for paving a cloister which encloses a rectangular court 45·77 yds. long and 41·93 yds. wide: the cloister being 12·45 ft. wide.

7. If the carriage of 17 cwts. 3 qrs. for 7½ miles cost £1. 0*s.* 8½*d.*, what weight should be carried 20 miles for 16*s.* 4*d.*?

8. Reduce 1 r. 14 p. to the decimal of an acre; and find the value of ·05854 of a guinea.

9. What was the average price of a quantity of cattle, of which 60 were bought for £10. 18*s.* 6*d.* each, 60 for £10. 16*s.* 9*d.* each, 50 for £10. 15*s.* 3*d.* each, and 30 for £10. 12*s.* 6*d.* each?

10. In how many months will £1550. 12*s.* 6*d.* amount to £1580. 15*s.* 6½*d.* at 3⅓ per cent. per annum simple interest?

11. What is the difference between the simple and compound interest on £1750 for 3 years, at 4 per cent. per annum?

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12. Explain the difference between true discount and banker's discount, and find the difference between the two for £610 due in 4 months at 5 per cent.

13. What was the cost price of an article which was sold for £1. 4*s.* 6*d.*, and on which there was a profit of $16\frac{2}{3}$ per cent.?

14. Which of the following stocks is the best for investment, and which the worst: 3 per cents. at $83\frac{1}{4}$, $3\frac{1}{2}$ per cents. at $98\frac{7}{8}$, or 4 per cents. at $107\frac{1}{2}$? Find the yield of £100 in each.

15. If £1000 3 per cents. be sold at $91\frac{3}{4}$, and with the proceeds 5 per cent. railway stock be bought at 125: find the increase of income.

16. Find (by duodecimals) the cost of cultivating a garden 109 ft. 6 in. long and 58 ft. 6 in. wide, at $4\frac{3}{4}$*d.* per square yard.

17. Two boats start for a race at 3 o'clock. The race is over at $6\frac{3}{8}$ minutes after 3, the losing boat being 40 yards behind at the finish: at 4 minutes past 3 this boat was 700 yards from the winning post. Find the length of the course, and the speed (supposed uniform) of each boat in miles per hour.

18. If 7 rix dollars are worth 2 ducats and 9 ducats worth 4 moidores, and 20 moidores worth £27: how many rix dollars are there in £72?

19. A cistern is 6 ft. long and 5 ft. broad, and it is of such a depth that it would take 1440 bricks, each 9 inches long, 3 inches deep, and 4 inches broad, to fill it: how many gallons would the cistern hold, if a pint of water contains 72 cubic inches?

20. Define a logarithm, and state the chief uses of logarithms.

Given log 86750 = 4·947519, write down log 867·50, log 8·6750, log ·086750.

21. Given log 2 = ·3010300, log 3 = ·4771213, find log 9, log 128, log $2^{\frac{1}{100}}$, log 2·4.

22. If the logarithm of a number to base 4 is ·35184, what is its logarithm to base 8?

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III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Divide $2a^2 - \frac{55ab}{12} + \frac{29ac}{9} + \frac{21b^2}{8} - \frac{15bc}{4} + \frac{c^2}{3}$ by $\frac{2a}{3} - \frac{3b}{4} + c$.

2. Find the highest common factor of

$$4x^4 - 9x^2 + 6x - 1 \text{ and } 6x^3 - 7x^2 + 1.$$

When of two algebraical expressions the H.C.F. has been found, what is the rule for determining their L.C.M.?

3. Simplify:

(i) $\frac{b}{a+b} - \frac{ab}{(a+b)^2} - \frac{ab^2}{(a+b)^3}$;

(ii) $\left(\frac{x}{x-y} - \frac{y}{x+y}\right)\left(x^2 + 2xy - y^2\right) \div \left(\frac{x}{x-y} + \frac{y}{x+y}\right)$;

(iii) $\frac{a^4 + x^4 + ax(a^2 + x^2) + a^2x^2}{a^5 - x^5} \div \frac{a^2 + x^2 + ax}{a^3 - x^3}$.

4. Find the square root of

(i) $\frac{9a^{2m}c^2}{4b^{12}} - \frac{3a^{m+n}c}{b^3} + a^{2n}b^6 - \frac{2^3a^mc}{b^6} + \frac{2^9a^nb^3}{3} + \frac{2^{16}}{9}$;

(ii) $87 - 12\sqrt{42}$.

5. Find the value of

(i) $(2^{\frac{1}{4}}a + 3^{\frac{1}{4}}b)(3^{\frac{1}{4}}a - 2^{\frac{1}{4}}b) - 6^{\frac{1}{4}}(a^2 - b^2) + 2^{\frac{1}{2}}ab$;

(ii) $(35\sqrt{10} + 77\sqrt{2} + 63\sqrt{3} + 28\sqrt{15}) \times (\sqrt{10} - \sqrt{2} - \sqrt{3})$.

6. Solve the equations

(i) $\frac{3\{ab - x(a+b)\}}{a+b} + \frac{(2a+b)b^2x}{a(a+b)^2} = \frac{bx}{a} - \frac{a^2b^2}{(a+b)^3}$;

(ii) $\begin{cases} (x+5)(y+7) = (x+27)(y+\frac{5}{7}), \\ xy = 1; \end{cases}$

(iii) $\begin{cases} y + z = 3xyz, \\ z + x = 2xyz, \\ x + y = \frac{5}{2}xyz. \end{cases}$

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7. Divide 11 into three parts so that the 1st may be to the 2nd as $\frac{1}{2}$ to $\frac{1}{3}$, and the 2nd to the 3rd as $\frac{1}{4}$ to $\frac{1}{5}$.

8. Show that p and q represent respectively the sum and product of the roots of the equation

$$x^2 - px + q = 0.$$

The area of an oblong room is 328 square feet, and its perimeter is 73 feet; write down and solve the quadratic equation which gives the lengths of the sides of the room.

9. If $a : b :: c : d$, prove that

$$4a^6 + 5b^6 : 4c^6 + 5d^6 :: a^3b^3 : c^3d^3.$$

10. Prove the rule for expressing a given integer in any proposed scale.

758·t83 cubic feet being the volume (expressed in the duodenary scale) of a cube; find the number of feet and inches in an edge of the cube.

11. If the first and last terms of a geometrical progression of 8 terms be respectively α^7 and β^7; find the sum of the series.

Prove the rule for finding the value of a recurring decimal in which all the figures recur.

12. Find the number of permutations (p_r) of n things, taken r at a time.

Show that

$$(p_3 - p_2)(p_4 - p_3)\ldots(p_{n-1} - p_{n-2}) = \frac{p_2 p_3 \ldots p_{n-1} p_n}{p_3 p_{n-1}}.$$

13. Assuming the Binomial Theorem when the exponent is a positive integer, prove it when the exponent is any positive quantity.

Find the 13th term in the expansion of

$$(2^8 + 2^6 x)^{\frac{11}{2}}.$$

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IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Define the sine and cosine of an angle, and find the values of the sines and cosines of 30°, of 210°, and of 930°.

2. Trace the changes in sign and magnitude of the expressions

(1) $\cos x - \sin x$.

(2) $\sin(\pi \sin x)$, as x increases from 0 to 2π.

3. If $\cos x = \cos a$, find a general expression for all the values of x.

Solve the equations—

$$\cos(2x+3y)=\tfrac{1}{2}, \quad \cos(3x+2y)=\frac{\sqrt{3}}{2}.$$

4. Prove the formulae

(1) $\cos(A+B)=\cos A\cos B-\sin A\sin B$.

(2) $\cos(A+B)\sin B-\cos(A+C)\sin C=\sin(A+B)\cos B$
$-\sin(A+C)\cos C$.

(3) $4\sin 20^\circ \sin 40^\circ \sin 80^\circ = \sin 60^\circ$.

5. Prove that in any triangle ABC,

(1) $c=a\cos B+b\cos A$.

(2) $c^2=a^2+b^2-2ab\cos C$.

6. If R be the radius of the circle circumscribing a triangle ABC prove that

$$2R=\frac{a}{\sin A}=\frac{a+b+c}{\sin A+\sin B+\sin C}.$$

If O be the centre of this circle, and R_1 the radius of the circle circumscribing the triangle BOC, prove that

$$2R_1\cos A=R.$$

7. Prove the formulae

(1) $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\dfrac{x+y}{1-xy}$.

(2) $\tan^{-1}\tfrac{1}{3}+\sin^{-1}\dfrac{1}{\sqrt{5}}=45^\circ$.

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8. If perpendiculars be drawn from the angular points of a triangle ABC on the opposite sides, prove that the lengths of them intercepted between the angular points and the orthocentre (*i.e.*, the point of intersection of the perpendiculars), are proportional respectively to $\cos A$, $\cos B$, and $\cos C$, and that the lengths of them intercepted between the orthocentre and the sides are proportional to $\sec A$, $\sec B$, and $\sec C$.

9. Prove that the area of a triangle ABC is equal to $\frac{1}{2}bc \sin A$.

If $B=45°$, $C=60°$, and if $a=2(\sqrt{3}+1)$ inches, prove that the area of the triangle is $6+2\sqrt{3}$ square inches.

10. From each of two ships a mile apart the angle is observed which is subtended by the other ship and a beacon on shore; these angles are found to be $52° 25' 15''$ and $75° 9' 30''$ respectively.

Having given

$$L \sin 75° 9' 30'' = 9{\cdot}9852635$$
$$L \sin 52° 25' 15'' = 9{\cdot}8990055$$
$$\log 1{\cdot}2197 = {\cdot}0862530$$
$$\log 1{\cdot}2198 = {\cdot}0862886,$$

find the distances of the beacon from each of the ships, one distance exact, and the other to five places of decimals.

11. A man who is walking on a level plain towards a tower observes at a certain point that the elevation of the top of the tower is 10°, and, after going 50 yards nearer to the tower, that the elevation is 15°.

Having given

$$L \sin 15° = 9{\cdot}4129962, \qquad \log 25{\cdot}783 = 1{\cdot}4113334$$
$$L \cos 5° = 9{\cdot}9983442, \qquad \log 25{\cdot}784 = 1{\cdot}4113503,$$

find the height of the tower in yards to four places of decimals.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[*Full marks may be gained by doing about* three-fourths *of this paper.*]
[*Great importance will be attached to accuracy in results.*]

1. Draw a straight line perpendicular to a given plane from a given point without it.

Find the locus of a point in a plane such that the straight lines which join it to two given points at equal altitudes above the plane may be equally inclined to the plane.

2. If two planes which cut one another are each of them perpendicular to a third plane, their line of intersection is also perpendicular to the same plane.

3. Assuming that pyramids with equal bases and of equal altitudes are equal to one another, prove that a triangular prism may be divided into three equal triangular pyramids.

Show that the altitude of a regular tetrahedron is to the length of each of its edges as $\sqrt{2} : \sqrt{3}$.

4. The curve formed by the intersection of the surface of a right circular cone with a plane, not passing through the vertex, is a parabola, if the inclination of the cutting plane to the axis of the cone is equal to the constant angle between the generating line and the axis.

5. Draw a pair of tangents to a given parabola from a given external point.

If O is the external point, P, Q the points of contact, and O' the angle opposite to O of the parallelogram constructed on OP, OQ: OO' is parallel to the axis of the parabola.

6. In an ellipse or hyperbola, the perpendiculars from the foci on the tangent intersect it in points which lie on the circumference of the circle described on the major or transverse axis.

Find the foci of an ellipse, whose major axis is a diameter of a given circle, and which touches a given chord of the circle at a given point in it.

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7. Find the length of the perpendicular from the origin upon the straight line which joins the points $(a \cos \alpha, a \sin \alpha)$, $(a \cos \beta, a \sin \beta)$, and show that the perpendicular bisects the distance between these points.

8. Find the equation of the tangent to the circle

$$x^2+y^2+Kx+Ly+M=0,$$

at a given point (x', y').

The equations of two circles are

$$x^2+y^2+Ax+By=0, \quad x^2+y^2+Cx+Dy=0.$$

Find the locus of the point of intersection of the circles, other than the origin, when $A-C$, $B-D$ vary proportionally.

9. Find the polar equation of the tangent to a parabola at a given point, the focus being the pole.

If the tangents at P, Q to a parabola of which the focus is S meet in T, show that $ST^2=SP \cdot SQ$.

10. Express the focal distances of any point on an ellipse, in terms of the abscissa; and if the normal at P meets the axis major in G, show that $PG^2 = \frac{b^2}{a^2} rr'$; a, b being the semi-axes; r, r' the focal distances of P.

11. A hyperbola being defined by the equations $x=a \sec \theta$, $y=b \tan \theta$, show that if 2α and 2β are the sum and difference of the values of θ corresponding to two points on the curve, the equation of the chord joining the points is $a^{-1} \cos \beta \cdot x - b^{-1} \sin \alpha \cdot y = \cos \alpha$.

Prove that a chord which joins the extremities of a pair of conjugate diameters of a hyperbola is parallel to one of the asymptotes.

VI. PURE MATHEMATICS (2).

[*Great importance will be attached to accuracy.*]

1. Expand $\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$ according to the powers of x.

2. Find the sum of the products, two and two, of every pair of the natural numbers $1, 2, 3 \ldots\ldots n$.

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3. In how many ways may n prizes be distributed among n boys, if every boy is to have a prize?

Prove that if $n+1$ prizes be given away to n boys, and every boy is to have at least one, the number of ways in which they may be given is

$$\tfrac{1}{2}n(n+1)\,\underline{|n}.$$

4. Find x, y, z from the equations

$$x+y\alpha+z\beta+2=0,$$
$$x+y\alpha^2+z\beta^2+3=0,$$
$$x+y\alpha^3+z\beta^3+5=0,$$

where α, β are the roots of the equation $u^2=u+1$.

5. How is the logarithm of a number to any base, a, to be found from an ordinary table of logarithms?

If a number $a+\delta$ exceed the base a by a very small quantity δ, prove that its logarithm is nearly $1+\dfrac{\delta}{a\log_e a}$.

6. Given one root, α, of the equation $x^4+1=0$, express the three remaining roots in terms of α.

7. If α, β, γ be the roots of the cubic $x^3+ax^2+bx+c=0$, find the value of the product

$$(\alpha^2+\beta^2)(\alpha^2+\gamma^2)(\beta^2+\gamma^2).$$

8. State Des Cartes' rule of signs. What inference may be deduced from it as to the roots of the equation $x^3+2x^2+3=0$?

9. If $2\cos\alpha=x+\dfrac{1}{x}$, prove that $2\cos r\alpha=x^r+\dfrac{1}{x^r}$.

10. Through a fixed point O within a circle any chord AOB is drawn: find the relation between the angles which AO, OB subtend at the centre.

11. What value should be given to the constant m in order that the formula

$$\alpha=\frac{\tan\alpha+m\sin\alpha}{1+m}$$

shall be the best approximation to the circular measure of a small angle α, in terms of its sine and tangent?

12. Find a symmetrical relation between the cosines of three angles, A, B, C whose sum is $180°$.

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VII. PURE MATHEMATICS (3).

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance will be attached to accuracy in results.*]

1. Differentiate with regard to x the expressions;

$$\frac{2x}{1-x^2},\quad \tan^{-1}\frac{2x}{1-x^2},\quad \text{and}\quad \log\frac{4x^2}{(1-x^2)^2}.$$

2. If $y=e^{3x}\cos 4x$, find $\frac{dy}{dx}$, and prove that, if $\tan\phi=\frac{4}{3}$,

$$\frac{d^ry}{dx^r}=5^re^{3x}\cos(4x+r\phi).$$

Find the general term in the expansion, by Maclaurin's theorem, of $e^{3x}\cos 4x$ in powers of x.

3. Find the maximum value and the minimum value of the expression,

$$x^3-12x+24.$$

Find also the values of x which make the expression,

$$(2-2x)\sin\alpha-\cos(2x-\alpha),$$

a maximum or a minimum.

4. Find the equations of the tangent and normal at the point (x, y) of the curve

$$x^3+y^3=2c^3.$$

If α and β be the distances from the origin of the points where the tangent cuts the axes of x and y, prove that

$$\alpha^{-\frac{3}{2}}+\beta^{-\frac{3}{2}}=2^{-\frac{1}{2}}c^{-\frac{3}{2}}.$$

Prove also that the normal at the point (c, c) passes through the origin.

5. Prove that, at a point of inflexion on a curve, $\frac{d^2y}{dx^2}$ changes sign.

Find the points of inflexion of the curve

$$y=\frac{x^2}{2}-\frac{9}{10}(x-1)^{\frac{5}{3}}.$$

6. If ϕ be the angle between the radius vector to any point of a curve, $r=f(\theta)$, and the tangent at the point, prove that $\tan\phi=\frac{rd\theta}{dr}$.

If the curve be $r=a(1-\cos\theta)$, prove that $2\phi=\theta$, and that, if p be the perpendicular from the origin upon the tangent, $r^3=2ap^2$.

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7. Trace the curves

(1) $y=(x-1)(x^2-4)$.

(2) $r=a\sin 3\theta$.

8. Find the radius of curvature of the curve

$$ay^2=x^3$$

at the origin, and also at the point (a, a).

9. Integrate with regard to x the expressions

$$(1)\ \frac{1}{x+1},\quad (2)\ \frac{1}{(x+1)(x+2)},\quad (3)\ \frac{1}{\sqrt{x^2+1}}.$$

10. Explain the method of integration by parts, and integrate with regard to x the expressions

$$e^x\cos x \text{ and } x\log(x+1).$$

11. Find the limiting value of the sum of the series

$$\frac{1}{n}\left\{\sin\frac{\pi}{2n}+\sin\frac{2\pi}{2n}+\sin\frac{3\pi}{2n}+\ldots\ldots+\sin\frac{(n-1)\pi}{2n}\right\}$$

when n is increased indefinitely.

12. Find the areas enclosed by the curves;

$$(1)\ ay=x\sqrt{a^2-x^2},\quad (2)\ r=a\sin 3\theta.$$

VIII. STATICS.

[*Full marks may be gained by correctly answering* three-quarters *of this paper. Great importance is attached to accuracy.*]

1. Three forces acting at a point make equilibrium. If they make angles of 120° with each other, prove that they are equal. If the angles are 60°, 150°, 150°, in what proportions are the forces?

2. Two given forces meet at a point. Find in what direction a third force of given magnitude must act at the point, if the resultant of the three is the greatest possible.

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3. *State* the conditions necessary for equilibrium for any system of forces acting in one plane on a rigid body.

If the rigid body have one point fixed, what is necessary for equilibrium? What, if it have two points fixed?

4. The door of a room is open: determine (as far as possible) the forces which it exerts upon its two hinges; given all the dimensions, and its weight.

5. Two given forces act at two given points: if they are turned round those points in the same direction through any two equal angles, show that their resultant will always pass through a fixed point.

Extend this theorem to any number of forces.

6. A uniform bar AB 10 feet long and weighing 50 lbs. rests on the ground. If a weight of 100 lbs. be laid on it at a point distant 3 feet from B, find what vertical force applied at the end A will just begin to lift that end.

7. Prove in any way that a couple cannot be balanced by a single force. If it be balanced by *two* forces, what is necessary with regard to them?

8. A heavy horizontal circular ring rests on three supports at the points A, B, C of its circumference. Given its weight, and the sides and angles of the triangle ABC, find the pressure on the supports.

9. Two inclined planes have a common vertex and equal slopes; one is smooth and the other rough. A given weight W is supported on one by a string passing over the vertex and attached to another weight resting on the other plane; this weight being just sufficient to prevent W descending. Prove that the weight required is less when W is on the rough plane, than when it is on the smooth one.

10. Four equal heavy bars are jointed so as to form a rhombus $ABCD$: A and C are joined by a string. The whole is suspended from the angle A; find the tension of the string, by means of the principle of Virtual Velocities (or otherwise).

11. If the resultant of any systems of parallel forces in one plane acts outside the two extreme forces of the system, show that some must act in an opposite sense to others.

A heavy bar lies on a rough floor: a horizontal force, too small to move it, is applied at one end, at right angles to the bar; describe *generally* the forces which have been called into operation to prevent motion ensuing.

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12. A circle is pushed over an equal circle till it completely covers it. Find the centre of gravity of the crescent-shaped space left uncovered just before they coincide.

13. A heavy rectangular block $ABCD$ rests with AB on the ground; a rope is attached to the corner C and pulled round a fixed pulley P vertically over D till motion ensues. Give a *geometrical construction* determining the least height for the pulley P, if the block is to begin to revolve round A without slipping along the ground (given all particulars).

IX. DYNAMICS.

[*Great importance will be attached to accuracy.*]
[*If needed, the measure of the force of gravity may be taken as* 32 *feet.*]

1. A body moves in any direction in a vertical plane: given the horizontal and vertical velocities at any instant, show how to find the actual velocity of the body, and the direction in which it is moving.

A body describes uniformly a vertical circle, whose radius is 12 inches, in 6 seconds, and the centre of the circle moves uniformly in a horizontal line with the same velocity: show how to find the true velocity of the body whenever its direction is inclined at 45° to the horizon.

2. State the first law of motion, and the evidence on which it is accepted.

In an observed case of a body in uniform motion, are we to infer that no force acts on the body? Illustrate this by example.

3. How is uniform force measured? If a body be acted upon by a uniform force, and be initially projected in the direction of that force with a velocity (u), and if it acquire a velocity (v) in (t) seconds, prove that (s) the space described may be determined by the equation $s=t\left(u+\frac{v}{2}\right)$.

A train at the foot of an incline is moving at the rate of 60 miles an hour; the steam is suddenly turned off, and it then runs on the horizontal plane for $3\frac{1}{3}$ miles before it stops; if friction be the constant retarding force, find its measure. Determine also the space the train describes in three minutes from the instant the steam is turned off.

4. A body slides down a rough inclined plane, show that the acceleration is $g\frac{\sin\overline{\alpha-\phi}}{\cos\phi}$ where (ϕ) is the limiting angle of resistance.

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If the height of the inclined plane be 12 feet, the base 16 feet, find how far a body will move on the horizontal plane after sliding down from rest the length of the inclined plane, supposing it to pass from one plane to the other without loss of velocity, the coefficient for both planes being $\frac{1}{8}$.

5. In the case of a projectile in vacuo, find the direction of the projectile with respect to the horizon at any point of its course.

If (θ) be the angle of elevation at which a projectile strikes a given mark, (α) the angle of projection, and ($45°$) the angle which the given mark subtends at point of projection, prove $\tan\alpha + \tan\theta = 2$.

6. State the law of motion that connects the statical and dynamical measures of force. Obtain the equation from which that connexion is shown, and explain the units of reference in that equation.

A body weighing 36 lbs. is moved by a constant pressure, which generates in it a velocity of 8 feet per second, find the statical measure of that pressure.

7. A ball is projected vertically upwards with a velocity of 160 feet per second, when it has reached its greatest height it is met in direct impact by an equal ball which has fallen through 64 feet; find the times from the instant of impact in which the balls reach the ground, the elasticity between them being $\frac{1}{2}$.

8. Assuming the expression for the time of a small oscillation of a pendulum in a circular arc, calculate to two decimal places the length of a second's pendulum in inches.

A simple pendulum beating seconds is lengthened by $\frac{1}{20}$ of an inch; find the number of seconds it will lose in 24 hours.

9. A string 4 feet long can just support a weight of 9 lbs. without breaking, a weight of 8 lbs. fixed to one end of the string describes a circle uniformly round the other end, which is fixed on a smooth horizontal table; determine the greatest number of revolutions the revolving weight can make in a minute so as just not to break the string.

10. Determine the time of a small oscillation in a cycloid.

11. Show how to find the work accumulated in a moving body.

A bullet with an initial velocity of 1500 feet strikes a target 1200 yards distant with a velocity of 900 feet in a second, the range of the bullet being assumed to be horizontal; compare the mean resistance of the air with the weight of the bullet.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JULY, 1886.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.)

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. Any two sides of a triangle are together greater than the third.

Prove that the shortest line, which can be drawn with its ends upon the circumferences of two concentric circles, will, when produced, pass through the centre.

2. Prove one case of the following proposition :—

If two triangles have two angles of the one equal to two angles of the other, each to each; and one side equal to one side, viz., either the sides adjacent to the equal angles in each, or sides which are opposite to equal angles in each; then shall the other sides be equal, each to each; and also the third angle of the one equal to the third angle of the other.

The vertical angle A of the isosceles triangle ABC is half a right angle, and the perpendiculars AD, BE let fall from A, B upon the opposite sides intersect in F. Show that FE is equal to EC.

3. Describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given angle.

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4. If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts is equal to the rectangle contained by the two parts, together with the square on the aforesaid part.

5. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

Describe a right-angled triangle such that the rectangle contained by the hypotenuse and one of the sides containing the right angle may be equal to the square on the other side.

6. Find the centre of a given circle.

Describe a circle of given radius passing through a given point, and touching a given straight line.

When is this construction impossible?

7. In equal circles, the arcs on which equal angles stand are equal to one another, whether the angles be at the centres or circumferences.

The tangent at A to the circle circumscribing the triangle ABC cuts BC produced in D, and AC produced cuts the circumference of the circle circumscribing the triangle ABD in E. Prove that AD is equal to DE.

8. Inscribe an equilateral and equiangular hexagon in a given circle.

If $ABCDEF$ be a regular hexagon, and AC, BE intersect in G, show that EG is three times as long as BG.

9. Find a mean proportional between two given straight lines.

The straight line bisecting at right angles AC, one of the equal sides of an isosceles triangle, meets the base AB produced in D. Show that AC is a mean proportional between AB, AD.

10. Define *similar polygons*, and prove that they can be divided into the same number of similar triangles, having the same ratio to one another that the polygons have.

11. If the vertical angle of a triangle be bisected by a straight line which also cuts the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square on the straight line which bisects the angle.

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II. ARITHMETIC.

[*Great importance is attached to accuracy.*]

1. Multiply one million one thousand and one by one hundred thousand one hundred, and express the result in words.

2. Divide 99·99 by ·0011, and reduce $\frac{\cdot\dot{9}+\cdot 9}{1\cdot 9}$ to its simplest form.

3. Add together £175. 17*s*. 6*d*., five hundred guineas, eighty-seven half-crowns, and 1,143 four-penny pieces.

4. If a person buys 1 cwt. 2 qrs. 15 lbs. of goods for £2. 5*s*. 4½*d*. per cwt., what must he sell it for per cwt. in order to gain £6·88125 on the transaction?

5. Find the value of

$$\cdot\dot{5}4 \text{ of } 8s.\ 3d. + \cdot 0\dot{2}\dot{7} \text{ of } £2.\ 15s. + \cdot 3125 \text{ of } £2.\ 2s.$$

6. If the wages of 45 women amount to £207 in 48 days, how many men must work 16 days to receive £76. 13*s*. 4*d*., the daily wages of a man being double those of a woman?

7. Find the square root of ·02010724 and the cube root of 24137569.

8. What sum will amount to £39. 12*s*. 6*d*. in 16 months at 4½ per cent. per annum simple interest?

9. Find the discount on £663. 17*s*. 0*d*. due six months hence at 4 per cent.

10. What principal lent out at compound interest for 2 years at 5 per cent. will amount to £7425. 6*s*. 9*d*.?

11. A person sells his property for £27,118, the rental of which is £900; he invests £4818 in 3 per cents. at 100⅜, £2450 in Greek securities at 50, 5 per cent., and £19,850 in New Zealand 4 per cent. at 99¼; find the increase of his income.

12. Find, by means of duodecimals, the number of feet of glass in a window which measures 7 feet 10¾ inches by 4 feet 7½ inches.

13. Find the number of gallons in a cylindrical vessel whose diameter is 6 feet, and height 7 feet. Given that the area of a circle equals $\frac{22}{7}$ times the square of the radius, and that a gallon is equal to 277·2 cubic inches.

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14. An empty cistern has three pipes, *A*, *B* and *C*. *A* and *B* can fill it in three and four hours respectively, and *C* can empty it in one hour. If these pipes be opened in order at 1, 2 and 3 o'clock, find when the cistern will be empty.

15. A grocer mixes 17 lbs. of tea worth 4*s.* with 25 lbs. worth 4*s.* 8*d.*, and sells the whole at 5*s.* 4*d.* per lb.; what is his total gain and his profit per cent.?

16. A rectangular piece of ground 6 furlongs long and 4 furlongs broad is divided into four smaller rectangles by a belt 200 feet wide round the outside boundary, the inner part being divided by a road 60 feet wide in the direction of the length, and a cross-road 41 feet wide in the direction of the breadth. Find the area of the four rectangles in acres.

17. Reduce 2 lbs. 7 oz. 3 dwt. 15 grains troy to the decimal of a lb., and express 3 miles 7 furlongs 12 poles in French metres. A metre equals 39·371 inches.

18. Why must the decimal equivalent to $\frac{5}{7}$ recur? Find it, and the value of 1·683 lb. of gold, when it is worth £4·0099 per ounce.

19. In Réaumur's thermometer the freezing point is zero, and the boiling point 80; and in Fahrenheit's the former is 32 and the latter 212. What degree of Fahrenheit's corresponds to 9 of Réaumur?

20. What would be the logarithm of 180 if the base were 12?

$$\log 2 = \cdot 30103 \text{ and } \log 3 = \cdot 4771213.$$

21. Given the mantissae of the logarithms of the following numbers:—2289, 343, 1092, 854, 855677 to be ·3596458, ·5352941, ·0382226, ·9314579, ·9323102. Find the value of

$$\sqrt{2289 \times 343 \times 1092 \times 854}.$$

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III. ALGEBRA.

[*Great importance is attached to accuracy.*]

1. Reduce each of the following expressions to its simplest form :—

(i) $$\frac{x}{x^2+y^2-xy}+\frac{1}{x+y}+\frac{2xy-x^2}{x^3+y^3}.$$

(ii) $$\frac{a+b}{ab+c^2-ac-bc}+\frac{a+c}{ac+b^2-ab-bc}+\frac{b+c}{bc+a^2-ab-ac}$$

(observing that the denominators of these fractions may be written in factors).

(iii) $$\frac{x^2-1-\dfrac{6}{x^3}}{x^2-2x+3-\dfrac{4}{x}+\dfrac{2}{x^2}}.$$

2. Divide

x^4+y^4 by $x^2+y^2-\sqrt{2}\,.\,xy$; and $x^3+y^3+3xy-1$ by $x+y-1$.

3. If $z^2=x^2+y^2$ find the simplest form of

$$\sqrt{(x+y+z)(x+y-z)(z+x-y)(y+z-x)}.$$

4. Solve the equations

(i) $$\sqrt{a-x}+\sqrt{b-x}+\sqrt{c-x}=\sqrt{a+b+c-x}.$$

(ii) $$\frac{x(x-1)}{(x+1)(x+2)}=\frac{(x+3)(x+4)}{(x+5)(x+6)}.$$

5. If α and β be the roots of the equation

$$ax^2+bx+c=0,$$

determine the equation whose roots are

$$\frac{\alpha}{\beta} \text{ and } \frac{\beta}{\alpha}.$$

6. The sum of two numbers is 12, and the sum of their fourth powers is 3026; find the numbers.

7. If $x' : y' : z' = xy : x^2 : yz$, prove that

$$x : y : z = x'y' : x'^2 : y'z'.$$

8. A rectangular enclosure is half an acre in area, and its perimeter is 201 yards; find the lengths of its sides.

9. The sum of 10 terms of an arithmetical series is 145, and the sum of its fourth and ninth terms is five times the third term; determine the series.

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10. If a series of numbers be in arithmetical progression prove that their reciprocals are in harmonical progression.

The first two terms of a harmonic series are 4 and $\frac{8}{5}$, find its ninth term.

11. Find the coefficient of x^6 in the expression

$$\left[1-\frac{x^2}{1.2}+\frac{x^4}{1.2.3.4}-\frac{x^6}{1.2.3.4.5.6}\right]^2 + \left[x-\frac{x^3}{1.2.3}+\frac{x^5}{1.2.3.4.5}\right]^2,$$

and the coefficient of x in the expansion of $(1+x)^{-\frac{1}{2}}$ by the Binomial theorem.

12. Find how many arrangements of letters can be formed with 8 unlike consonants and 4 unlike vowels, if each arrangement contains 3 consonants and 2 vowels; and prove the general theorem on which the result depends.

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. State the greatest and least values of (1) the cosine (2) the secant of an angle; also to what angles these values respectively belong.

Show how to draw geometrically an angle whose secant $=3$.

2. State and prove the formula expressing $\sin A+\sin B$ as a product; and show that the arithmetical mean of the sines of two angles is very nearly equal to the sine of the arithmetical mean of the angles, if the angles are nearly equal to each other.

3. Find an angle whose sine $=-\sin A$, and whose cosine $=-\cos A$.

4. Find the greatest possible value of $\sin x+\cos x$.

5. Prove that $\tan^{-1}t+\tan^{-1}\frac{2t}{1-t^2}=\tan^{-1}\frac{3t-t^3}{1-3t^2}$.

6. Prove the formula for a plane triangle

$$\cos\tfrac{1}{2}C=\sqrt{\frac{s(s-c)}{ab}}.$$

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7. Given in a triangle $a=250$, $b=240$, $A=72^\circ\ 4'\ 48''$, find the angles B, C. State whether they can have more than one value each.

Given	$\log 2{\cdot}5={\cdot}3979400$	$\log \sin 72^\circ\ 4'=9{\cdot}9783702$
	$\log 2{\cdot}4={\cdot}3802112$	$\log \sin 72^\circ\ 5'=9{\cdot}9784111$
		$\log \sin 65^\circ\ 59'=9{\cdot}9606739.$

8. Prove that in a triangle

$$\frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)}=\frac{a+b}{c}.$$

9. If in a triangle the bisector of the base, c, is perpendicular to the side b, prove that $2 \tan A+\tan C=0$.

10. If pq are the perpendiculars from A, B on any arbitrary line drawn through the vertex C of a triangle, prove that

$$a^2p^2+b^2q^2-2abpq \cos C=a^2b^2 \sin^2 C.$$

11. Explain what measurements have to be made at two stations A and B, in order to find the distance CD between two inaccessible objects, $ABCD$ being in one plane: and state clearly the steps of the calculation by which the distance is to be found therefrom.

12. If in a triangle $C=60^\circ$, prove that

$$\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}.$$

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FURTHER EXAMINATION.

V. PURE MATHEMATICS. (1.)

[*In answering the questions on Geometry ordinary abbreviations may be employed, but the method must be geometrical. Full marks may be gained by answering correctly about three-fourths of this paper. Great importance will be attached to accuracy.*]

1. If a straight line be at right angles to a plane, every plane which passes through it shall be at right angles to the plane.

If CD be at right angles to a plane which it meets in D, AB be any straight line in that plane, DM in the same plane perpendicular to AB, then any line drawn from M to the line CD is perpendicular to AB.

2. Prove that every solid angle is contained by plane angles which are together less than four right angles. Hence show that there are only five ways of forming a solid angle by the plane angles of the same regular rectilineal figure.

3. If a perpendicular be drawn from an angle of a regular tetrahedron upon the opposite triangular base, show that the foot of the perpendicular will divide either of the lines drawn from an angle of the base to the point of bisection of the opposite side in the ratio of 2 to 1.

4. If a right cone be cut by a plane, examine when the intersection of the cutting plane with the surface of the cone will be an ellipse, and find the positions of the foci of the ellipse for a given section.

5. Show when $ay^2+2cxy+bx^2=0$ represents two real, and different straight lines, and point out when it fails to do so. Show that the equations to the straight lines bisecting the angles between the lines represented by the above given equation may be expressed thus

$$\frac{x^2-y^2}{b-a}=\frac{xy}{c}.$$

6. Show how to change the axes of rectangular coordinates to another rectangular system inclined at a given angle to the former and having the same origin. Show how the curve represented by $3x^2+2xy+3y^2=1$ may be determined by turning the axes through $45°$ in the same plane.

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7. If from a given point without a parabola tangents be drawn to the parabola, show that these tangents will subtend equal angles at the focus. Prove also that the directrix touches all circles described on focal chords as diameters.

8. In an ellipse prove that the normal at any point bisects the interior angle between the focal distances at that point, and that the focal distances make equal angles with the tangent.

If S and H be the foci of an ellipse, P any point in the curve, show that the ratio of $\tan\frac{PSH}{2} : \tan\frac{PHS}{2}$ is independent of the eccentricity of the ellipse, supposing the abscissa of P and major axis unchanged.

9. Define the polar of a point with respect to a circle, ellipse, or hyperbola; find its equation with respect to an ellipse when the point is without the curve, and show that the equation obtained represents the locus of the intersection of tangents drawn at the extremities of chords passing through a given point.

10. Assuming the ordinary rectangular equation to an hyperbola, find its equation referred to its asymptotes as axes. Prove that the portion of the tangent drawn at any point of an hyperbola intercepted by the asymptotes is bisected at the point of contact.

11. An angle of a cube is cut off by a plane which intersects the edges at distances (a) (b) (c) from the angle; if (α) be the angle of inclination of the cutting plane to the face of the cube, which is perpendicular to c, prove $\tan\alpha = \frac{c}{ab}\sqrt{a^2+b^2}$. Hence prove also

$$(\cos\alpha)^2 + (\cos\beta)^2 + (\cos\gamma)^2 = 1,$$

where (β) and (γ) are the angles which the cutting plane makes with the other two plane faces of the cube respectively.

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VI. PURE MATHEMATICS. (2.)

[*Great importance will be attached to accuracy. Full marks may be obtained by answering* three-fourths *of this paper.*]

1. Prove that, if x is any real quantity, $x+\frac{1}{x}$ is never less than 2, and find the least value of $x+\frac{1}{x-2}$.

2. If a, b, and c be positive quantities in harmonic progression, prove that $a+c$ is greater than $2b$, and that a^n+c^n is greater than $2b^n$.

3. Expand $\frac{1}{(1-2x)(1-3x)}$ in ascending powers of x, and find the coefficient of x^n.

4. Find the sums of the series

(1) $1 \cdot 2+2 \cdot 3+3 \cdot 4+\ldots\ldots,$

(2) $\frac{1}{1 \cdot 2}+\frac{1}{2 \cdot 3}+\frac{1}{3 \cdot 4}+\ldots\ldots,$

each of n terms, and (2) to infinity.

5. Find all the positive integral solutions of the equation

$$9x+11y=455.$$

6. Reckoning compound interest, find an expression for the present value of a sum of M pounds due n years hence, at a given rate of interest.

Having given $\log 2=\cdot 3010300$ and $\log 103=2\cdot 0128372$, find how many years will elapse before a sum of money doubles itself at 3 per cent. per annum compound interest.

7. If n_r represent the number of combinations of n things taken r together, prove that

$$(n+1)_r=n_r+n_{r-1}.$$

8. Find the number of different throws which can be made (1) with two ordinary dice, (2) with n ordinary dice.

9. Prove that, for all values of n,

$$(\cos\theta+\sqrt{-1}\sin\theta)^n=\cos n\theta+\sqrt{-1}\sin n\theta.$$

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If $x_r = \cos\frac{\pi}{2^r}, + \sqrt{-1}\sin\frac{\pi}{2^r}$, prove that, the product being continued to infinity,

$$x_1 \cdot x_2 \cdot x_3 \cdot x_4 \ldots = \cos\pi.$$

10. Find the exponential expressions for the sine and cosine of an angle.

If $\log(1 + \sqrt{-1}\tan a) = A + B\sqrt{-1}$, prove that $A = \log\sec a$, and find B.

11. Prove that, in an equation with real coefficients, imaginary roots occur in pairs.

Having given that $2 + \sqrt{-3}$ is a root of the equation

$$x^4 - 21x^3 + 76x - 84 = 0,$$

find all its roots.

12. Explain Cardan's method of solving a cubic equation, and apply it to find, to two places of decimals, an approximate value of the real root of the equation

$$x^3 + 6x - 6 = 0.$$

VII. PURE MATHEMATICS. (3.)

[*Full marks may be obtained for about* three-fourths *of this paper.*]

1. Differentiate the expressions

$$\sin^{-1}\frac{x}{\sqrt{1+x^2}}, \log\sqrt{\frac{1-\cos x}{1+\cos x}}, x^x.$$

2. If $u = \sin^{-1}x$ prove that

$$(1-x^2)\frac{d^3u}{dx^3} = 3x\frac{d^2u}{dx} + \frac{du}{dx}.$$

3. Prove Maclaurin's theorem, and apply it to the expansions of $\sin x$, and of $\cos x$, in ascending powers of x.

4. Find the minimum value of $\frac{26 + 6x + 9x^2}{1 + 3x}$, and the maximum or minimum values of $a\sin x + b\sin 2x$.

5. Find the equation of the tangent at any point on the curve

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}},$$

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and prove that the portion of the tangent intercepted between the coordinate axes is of constant length.

6. If p be the perpendicular from the origin on the tangent to a curve at a point whose radius vector is r, prove that the radius of curvature at the point is $r\dfrac{dr}{dp}$.

Apply this to determine the length of the radius of curvature at any point on a parabola.

7. Trace the curve $r = a\cos 4\theta + b\sin 4\theta$, and find its entire area.

8. Determine the asymptotes to the curve

$$x^3 + 3x^2y + 2xy^2 + 3axy + 2ay^2 - 2a^3 = 0.$$

9. Integrate the expressions

$$\sin^3 x \cos^5 x\, dx, \quad x^2 \log x\, dx, \quad \frac{x^2 dx}{x^6 - 1}.$$

10. Find the values of *any two* of the definite integrals

$$\int_0^\infty \frac{dx}{1 + 2x\cos\alpha + x^2}, \quad \int_0^1 \frac{dx}{\sqrt{x} + \sqrt{x+1}}, \quad \int_0^1 \sin^{-1} x\, dx.$$

11. Prove that the whole area of a cycloid is three times that of its generating circle.

12. Rectify *either* of the curves

(1) $ay^2 = x^3$,

(2) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

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VIII. STATICS.

[*Great importance will be attached to accuracy.*]

1. Two forces are given in magnitude, but may make any angle with each other. How should they be placed, so as to give (1) the greatest, (2) the least possible resultant? Prove your statements.

2. What is meant by a force resolved in a given direction? Show how a force is to be resolved so that its component along a given direction shall have a given value.

3. A force is given in magnitude and line of action. Give a geometrical construction for resolving it into two other forces which shall be equal to one another and shall pass respectively through two fixed points.

4. Prove that any two couples of equal moments and opposite senses balance each other. (It may be assumed that the same couple can be transferred in its own plane in any manner without changing its effect.)

5. Three parallel forces act on a horizontal bar. Each is $=1$ lb.; the right-hand one acts vertically upwards, the two others vertically downwards at distances 2 ft. and 3 ft. respectively from the first; draw their resultant, and state exactly its magnitude and position.

6. Draw *any* system of pulleys by which a weight of 1 lb. can be made to support a weight of 3 lbs., neglecting friction and the weights of the pulleys. Show that, whatever may be your system, the smaller weight will descend through 3 ft. in raising the other through 1 ft.

7. A heavy circular disc is kept at rest on a rough inclined plane by a string parallel to the plane and touching the circle. Show that the disc will slip on the plane if the coefficient of friction is less than $\frac{1}{2}\tan i$, where i=slope of plane.

8. Two equal weights each $=112$ lbs. are joined by a string which is laid over two pulleys A, B in the same horizontal line. If a small weight, say 1 lb., is attached to the string half way between A and B, find in inches the depth to which it descends below the level of AB: supposing $AB=10$ ft.

What would happen if the weight were attached at any other point of the string?

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9. If a portion m of any mass M is moved to any new position, show that the centre of gravity of the entire mass is thereby moved in a direction parallel to the displacement of the centre of gravity of m, and over a distance $= \frac{m}{M} D$, where $D =$ above distance between the two positions of the centre of gravity of m.

A triangular piece of paper is folded across the line bisecting two sides, the vertex being thus brought to lie on the base. Find the centre of gravity of the paper in this position.

10. Three equal particles are placed anywhere on the three sides of a triangle. If they are moved along those sides, in the same sense, and over three spaces which are proportional respectively to the sides, show that the centre of gravity of the particles remains at rest.

11. Assuming the principle of virtual velocities, deduce the relation between the power and weight on the inclined plane; the power being either (1) parallel to the plane or (2) horizontal.

12. Explain what is meant by stable and unstable equilibrium, and give an instance of each.

IX. DYNAMICS.

[*The measure of the acceleration of gravity may be taken to be* 32 *when a foot and a second are units of length and time. Great importance will be attached to accuracy.*]

1. Explain how velocity is measured, and if 22 be the measure of a velocity when a foot and a second are units of length and time, find its measure when a mile and an hour are the units.

A man, 6 feet in height, walks with the velocity of 3 miles an hour, in a straight line along a road, on one side of which there is a lamp-post, the light of the lamp being 9 feet above the ground. Find the velocity of the end of his shadow on the ground.

2. A ship, which is sailing due north at the rate of 3 miles an hour, observes another ship exactly east of it, which is sailing due east at the rate of 4 miles an hour. Find the rate at which each ship is increasing its distance from the other, and determine graphically the direction of motion of each relative to the other.

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3. Define acceleration, and explain how it is measured.

If a point, starting with no velocity, moves with a constant acceleration f in the direction of its motion, and passes over the space s in the time t, and if v is its velocity at the end of that time prove that

$$2s = ft^2, \text{ and } v^2 = 2fs.$$

If the point starts with the velocity u, and moving with the constant acceleration f, passes over the space s, where is the error in the statement that the final velocity is $u + \sqrt{2fs}$?

4. Enunciate and explain Newton's second law of motion.

A string passing over a smooth pulley supports two scale-pans at its ends, the weight of each scale-pan being equal to the weight of one ounce. If a two-ounce weight be placed in one scale-pan, and a four-ounce weight in the other, find the acceleration of the system, the tension of the string, and the pressures between the weights and the scale-pans.

5. Prove that the time of descent of a heavy particle down any chord of a vertical circle, starting from the highest point of the circle, is the same.

Find the line of quickest descent from a given point to a given circle in the same vertical plane.

6. Prove that the path of a projectile is a parabola, and find the greatest possible range on the horizontal plane through the point of projection for a given velocity of projection.

Show that, for any range short of the greatest, if the velocity of projection is given, there are two directions of projection which are equally inclined to the direction giving the greatest range.

7. If a point moves uniformly, with velocity v, in a circle of radius r, find the direction and the measure of its acceleration.

If a particle of mass m moves in the same circle with the same velocity, find the direction and magnitude of the resultant of the forces which are in action on the particle.

A heavy particle, which is suspended from a fixed point by a string one yard in length, is raised until the string (which is kept tight) is inclined 60° to the vertical, and is then projected horizontally, in the direction perpendicular to the vertical plane through the string; find the velocity of projection that the particle may move in a horizontal scale.

8. Find the velocity acquired by a heavy particle in sliding down a smooth curve.

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A heavy particle is placed very near the highest point of a smooth sphere; find where it runs off the sphere, and prove that the latus rectum of the parabola, which it then describes, is eight twenty-sevenths of the diameter of the sphere.

9. Define the potential energy and the kinetic energy of a system, and enunciate the principle of energy.

A straight rod ACB, without weight, has two particles of equal weight fastened to it, one at the end B, and the other at the middle point C, and the rod can swing about the end A. If it be held horizontally, and then allowed to swing, prove that the greatest velocity acquired by the end B will be the same as the velocity acquired by a particle falling freely through a height equal to six-fifths of the length of the rod.

10. A cannon standing on a smooth horizontal plane is pointed horizontally and loaded with a ball, the mass of which is a given fraction of the mass of the gun and its carriage. Having given the velocity with which the ball leaves the gun when it is fired, find the velocity of recoil of the gun.

If the charge of powder be quadrupled, what will be the effect on the velocities?

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1886.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases equal, the angle which is contained by the two sides of the one shall be equal to the angle, which is contained by the two sides, equal to them of the other.

2. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are together equal to two right angles.

The angle contained between the two lines drawn from the vertex of a triangle, one to bisect the base and the other perpendicular to it, is equal to half the difference of the angles at the base.

3. The opposite sides and angles of a parallelogram are equal to one another, and the diameter bisects the parallelogram, that is, divides it into two equal parts.

4. In every triangle the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle, and the acute angle.

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If D be the middle point of the base BC of a triangle ABC, E the foot of the perpendicular drawn to the base from A; show that the rectangle DE, BC is half the difference of the squares on the sides of the triangle.

5. State and prove Euclid's proposition as to the relative lengths of straight lines drawn from any point within a circle to the circumference.

Two triangles ABC, DBC are on the same base BC and have their vertical angles, A and D, equal to one another. The angles at C being both acute, and the angle DCB being greater than the angle ACB, show that DB is greater than AB.

6. The angle at the centre of a circle is double of the angle at the circumference on the same base, that is, on the same arc.

On the base BC of a triangle ABC, which has an obtuse angle at B, describe another triangle equal to ABC and having its vertical angle double of the angle A.

Show that the construction is impossible unless $2C+A$ is less than a right angle.

7. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

8. Describe a square about a given circle.

The area of a regular octagon inscribed in a circle is equal to the rectangle contained by the sides of the inscribed and circumscribed squares.

9. Triangles and parallelograms of the same altitude are to one another as their bases.

10. In any right-angled triangle, any rectilineal figure described on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

$ABCD$ is a parallelogram whose side AB is divided in E so that AE is less than EB. CF (equal to AE) is drawn perpendicular to DC on the side remote from AB; and FG (equal to EB) is drawn to meet DC produced in G. If GE and DA meet, when produced, in H; show that the triangle HDG is equal to the parallelogram $ABCD$.

11. If from the vertical angle of a triangle a straight line be drawn perpendicular to the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the perpendicular and the diameter of the circle described about the triangle.

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II. ARITHMETIC.

[N.B.—*Great importance is attached to accuracy.*]

1. The quotient in a division is 479, the dividend is 3476418, and the remainder is 794; what is the divisor?

2. Find the whole cost of 20 dozen boxes of fruit at 14*s.* $7\frac{1}{2}$*d.* per box, 40 dozen at 13*s.* $9\frac{1}{4}$*d.*, and 60 dozen at 12*s.* 8*d.*

3. Prove from the definition of a fraction that $\frac{1}{3}-\frac{1}{12}=\frac{1}{4}$. State in words the rule that you deduce for the addition or subtraction of fractions with unlike denominators.

4. Express in tons, cwts., &c. the value of

$$116\tfrac{1}{16}\text{ cwts.}+115\tfrac{1}{14}\text{ qrs.}+152\tfrac{1}{32}\text{ lbs.}+111\tfrac{1}{2}\text{ oz.}$$

5. From $5\frac{1}{2}$ of ·9625 subtract $3\frac{3}{5}$ of $1\cdot09\dot{7}$; and express as a decimal in its simplest form the value of $\dfrac{\cdot\dot{1}+\cdot\dot{0}\dot{1}+\cdot\dot{0}0\dot{1}}{\cdot41}\div\dfrac{1}{\cdot037}$.

6. Find by practice the rent of 47 acres 2 roods 36 poles $2\frac{3}{4}$ yards at £20. 3*s.* 4*d.* per acre.

7. If 6 women and 3 boys weed $\frac{7}{12}$ of a field in 4 days, how long will it take 4 women and 1 boy to weed the rest of it, the work of 4 women being equal to that of 5 boys?

8. Find the square root of $12787\frac{121}{144}$; and the cube root of 112678587.

9. Find the simple interest on £1024. 7*s.* 6*d.* for 4 years 9 months, at 4 per cent. per annum.

10. Find the present value of a bill for £174. 1*s.* $10\frac{1}{2}$*d.* due in 128 days, interest being at the rate of 5 per cent. per annum.

11. How much stock at $97\frac{1}{2}$ must I sell out in order to realise £1300? And if, by investing this £1300 in $4\frac{1}{4}$ per cent. stock at 102, I increase my income by £8. 6*s.* 8*d.* per annum, what rate of interest was I receiving on the former stock?

12. Find by duodecimals the cubical content of a cistern whose internal dimensions are 6 ft. $6\frac{1}{2}$ in., 5 ft. $3\frac{1}{4}$ in., and 4 ft. 8 in. respectively; and reduce the answer to cubic yards, feet, and inches.

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13. If a man buys eggs at 10 for a shilling, and sells them at 8 for a shilling, what rate per cent. profit does he make on his outlay?

14. If 4 men can do a piece of work in 7 days, which 5 women can do in 8 days, or 7 boys can do in 10 days; how long will it take 3 men, 2 women, and 3 boys to do the same if they work together?

15. Determine the weight in pounds of a box, with a lid, made of wood $\frac{3}{4}$ of an inch thick and measuring externally 4 ft. by 3 ft. by 3 ft., the weight of a cubic foot of the wood being 38·4 lbs.

16. What length of carpet, 30 inches wide, will be required for a room 17 ft. 4 in. long and 13 ft. 9 in. wide? And, if the cost of the carpet is £5. 19*s*. 2*d*., what is the price of it per yard?

17. If the annual increase in the population of a state is 25 per thousand, and the present number of inhabitants is 2,624,000; what will the population be in 3 years time? And what was it a year ago?

18. A man having 3 sons, left £9656 to be divided among them in proportion to their ages at the time of his death; when he died, their ages were 25, 22, and 21 years respectively; what was the share of each? And what difference would it have made to each of the sons if the father had lived a year longer?

19. Find to the nearest farthing the price per lb. in English money of tea which costs in France 3·50 francs per kilogramme; £1 being equivalent to 25 francs, and 1 kilogramme to 2·204 lbs.

20. Reduce $\frac{5}{11}$ of a pole (linear) to the decimal of $\frac{13}{88}$ of a furlong; and find the value of ·01481 of a ton, at 4*s*. 6*d*. per lb.

21. (1) Find from the tables supplied log 637 and log 13; from these two logarithms determine log 7.

(2) Find from the tables supplied log 12 and log 18; from these two logarithms determine log 2 and log 3.

22. Find by aid of the tables the value of

$$\frac{87{\cdot}327 \times 784{\cdot}55 \times {\cdot}020868}{{\cdot}61659 \times 58{\cdot}844}.$$

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III. ALGEBRA.

[N.B.—*Great importance is attached to accuracy.*]

1. Divide $1+4a^4b^4$ by $2a^2b^2-2ab+1$: and

$$x^4+x^{-4}+4(x^2+x^{-2})+6 \text{ by } x^2+x^{-2}+2.$$

2. Express in their simplest forms

(i) $\left(1-\dfrac{2xy}{x^2+y^2}\right)\div\left(\dfrac{x^3-y^3}{x-y}-3xy\right)$.

(ii) $(x-y+z)(x+y-z)-(x+y+z)(x-y-z)-4yz$.

(iii) $\dfrac{x^3-2ax^2-5a^2x-12a^3}{x^3-7ax^2+13a^2x-4a^3}$.

3. If $a^2=bc$, prove that $b(c^2-ab)^2+c(b^2-ca)^2+2a(b^2-ca)(c^2-ab)=0$.

4. Solve the equations

(i) $\sqrt{1+\dfrac{bx}{a^2}}+\sqrt{1-\dfrac{bx}{a^2}}=1\frac{2}{5}$.

(ii) $\left.\begin{array}{l} x^2+y^2=xy+7 \\ x^2-y^2=xy-1 \end{array}\right\}$.

5. 1120 square feet of paper will just cover the four walls of a room which is 8 feet longer than it is wide. If the room were 4 feet higher the same quantity of paper would just cover the two smaller and one of the larger walls. What are the dimensions of the room?

6. If α and β are the roots of the equation $x^2+px+q=0$, and $\alpha-\gamma$, $\beta-\gamma$, those of $x^2+Px+Q=0$; show that $4q-p^2=4Q-P^2$.

7. Two cyclists travel, one from A to B, the other from B to A, by the same road, and at uniform speeds. They start at the same moment. One reaches B $2\frac{1}{2}$ hours, the other reaches A 3 hours 36 minutes after they meet. How long was each on the journey?

8. Explain and justify the process ordinarily used for finding the square root of an Algebraic expression.

Show how the method is applied to numerical examples.

9. Show how to find the sum of n terms of a given Geometrical Progression. If x, y, z be in G.P., prove that

$$x^2y^2z^2(x^{-3}+y^{-3}+z^{-3})=x^3+y^3+z^3.$$

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10. The arithmetical mean between two numbers is to the geometric mean as 5 to 4, and the difference of their geometrical and harmonical means is $3\frac{1}{8}$; find the numbers.

11. If m and n are positive integers, show that the coefficient of x^m in the expansion of $(1+x)^{m+n}$ is equal to that of x^n.

Find the coefficients of x^3 and x^5 in the expansion of $(1-2x-3x^2)^{-2}$.

12. Assuming the formula for the number of permutations of n different things taken all together, show how the number may be found when the things are not all different.

Hence find how many different numbers may be composed with the figures in 111223, each consisting of 6 figures.

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Explain what is meant by the *circular measure* of an angle, and show that the circular measure of an acute angle is intermediate in value between the sine and the tangent of the angle.

The perimeter of a certain sector of a circle is equal to the length of the arc of a semicircle having the same radius. Express the angle of the sector in degrees, minutes, and seconds.

2. Prove that the secant of any angle will be either greater than $+1$ or less than -1.

Show that $\sec A = \pm\sqrt{1+\tan^2 A}$, and explain the appearance of the double sign.

3. Express $\sin(A-B)$ and $\cos(A-B)$ in terms of the sines and cosines of A and B.

Find the values of $\sin 15°$, $\sin 105°$, $\cos 165°$, and $\tan 195°$.

4. Obtain a formula including all angles which have a given tangent.

If $\tan A + \tan 2A = \tan 3A$, prove that A must be a multiple of 60° or 90°.

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5. A ring, 10 inches in diameter, is suspended from a point 1 foot above its centre by six equal strings attached to its circumference at equal intervals. Find the cosine of the angle between two consecutive strings.

6. Prove that

(1) $\sin 2A = \dfrac{2 \tan A}{1 + \tan^2 A}$.

(2) $\cos^2 A - \cos A \cos (60^\circ + A) + \sin^2 (30^\circ - A) = \frac{3}{4}$.

7. State and prove the rule for finding, by inspection, the characteristic of the logarithm of a fraction.

Find from the tables supplied an approximate value of the seventh root of ·000026751.

8. Express the cosine of any angle of a triangle in terms of the three sides.

Prove that

$$\sin (B - C) : \sin (B + C) :: b^2 - c^2 : a^2.$$

9. The sides of a triangle are 17, 20, and 27. Find from the tables supplied all the angles.

10. A man travelling due west along a straight road observes that when he is due south of a certain windmill the straight line drawn to a distant tower makes an angle of 30° with the direction of the road. A mile further on the bearings of the windmill and tower are N.E. and N.W. respectively. Find the distances of the tower from the windmill, and from the nearest point of the road.

11. Find the area of a regular polygon of n sides inscribed in a circle of given radius.

If a regular pentagon and a regular decagon have the same perimeter, prove that their areas are as $2 : \sqrt{5}$.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[*In answering the questions on Geometry ordinary abbreviations may be employed, but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If three straight lines meet all at one point, and a straight line stand at right angles to each of them at that point, the three straight lines shall be in one and the same plane.

2. If a plane A be parallel to a plane B, and a plane C to a plane D, show that the straight line which is the intersection of A and C is parallel to the straight line which is the intersection of B and D.

3. Prove that the line of intersection of two spheres is a circle. If any other sphere includes this circle upon its surface, find the locus of its centre.

4. Through a point P on a parabola the tangent, normal, and ordinate are drawn. Show that the portion of the axis intercepted between the tangent and normal is twice the portion of the axis intercepted between the ordinate and the directrix.

5. A parabola is drawn to touch two sides of a triangle at the angular points upon the third side. Compare (with proof) the areas of the portions into which the triangle is divided.

6. Being given four tangents to a parabola in position, determine its focus and directrix by geometrical construction.

7. Define an *asymptote*. Prove that the area of all triangles formed by the asymptotes and a tangent to one branch of a given hyperbola is the same.

From the points B, C, in which a tangent to an hyperbola meets the asymptotes, CH, BG, are drawn parallel to one another to meet the asymptotes in H and G. If AFD is a straight line parallel to CH and meeting the asymptote HB in A and the asymptote CG in D and the tangent in F, show that it also will be a tangent to the hyperbola when the triangle AFG is equal to the triangle HFD.

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8. Find the condition that the straight lines $Ax+By=1$ and $ax+by=1$ may be (1) perpendicular; (2) parallel. In the latter case find the distance between the straight lines.

9. The four angular points of a quadrilateral referred to oblique coordinates are (x_1y_1), (x_2y_2), (x_3y_3), (x_4y_4). Find the conditions that it may be (1) a rhombus, (2) a square.

10. If $\alpha=0$, $\beta=0$, $\gamma=0$ represent, in abridged notation, the three sides of a triangle, show that any right line can be represented by

$$l\alpha+m\beta+n\gamma=0.$$

If a, b, c, are the sides of the triangle, and $l : m : n :: a : b : c$, comment on the above equation.

11. Find the equations to the tangent and normal to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

at a point of which the eccentric angle is θ. What value must be assigned to the co-efficient m in $x^2-y^2=mxy$, in order that this equation may represent straight lines through the origin parallel to the normal and tangent at the point of which the eccentric angle is θ?

12. Examine the nature of the following curves, and trace them :—

(i) $(x+a)^2=c+(y+b)^2$.

(ii) $(x+y-7)(x-y-3)=21(x-1)^2$.

(iii) $x^3-y^3=(y-a)(x^2-y^2)$.

VI. PURE MATHEMATICS (2).

1. If x and y are real quantities, prove that

$$(xy-1)^2>(x^2-1)(y^2-1).$$

2. If x and y are both positive, and if

$$x<\frac{1-y}{1+y}, \text{ prove } y<\frac{1-x}{1+x}.$$

3. If a number N is divisible by 11, prove that

$$a-b+c-d+\&c.$$

(if not 0) is divisible by 11; a, b, c, d... being the digits composing N, beginning from the right.

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4. In how many different orders may 6 people be formed into a ring?

A party of n ladies and n gentlemen sit round a table. How many different arrangements are possible if each lady is to sit between two gentlemen?

5. Show that

$$\phi(x) = a + \frac{\log(\log x)}{\log r}$$

is a function of x which satisfies the condition

$$\phi(x^r) \equiv 1 + \phi(x).$$

6. Find a quadratic whose roots are the squares of the roots of

$$x^2 - 2x\cos\theta + 1 = 0.$$

7. Find in any manner the three roots of the cubic

$$4x^3 - 27(x-1)^2 = 0.$$

8. Expand $(1 + x + x^2 + x^3)^{-1}$ according to powers of x.

9. What is the trigonometrical value of $e^{\theta\sqrt{-1}}$?

Show that, if $\theta < 90°$,

$$\sqrt{-1}\cos^{-1}(\cos\theta + \sin\theta) \text{ is real.}$$

10. If a point Z be taken in the base AB of a triangle ABC, such that

$$AZ : ZB :: AC^2 : BC^2,$$

prove that ZC divides the angle C into two parts, which are equal to those into which C is divided by the bisector of the base.

11. To a person going northward, when at A, two objects B, C, appear in a straight line, pointing N.W. After advancing a distance a the object C still seems almost due N.W. of him, while the angle which B and C subtend at his eye is found to be θ. Find approximately the distance AB.

12. In the equation

$$x^n + ax^{n-1} + bx^{n-2} + \ldots\ldots\ldots = 0,$$

the coefficients a, b, c..... are integers. Prove that it cannot have a fractional root.

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VII. PURE MATHEMATICS (3).

[*Full marks may be obtained for about* three-quarters *of this paper.*]

1. Explain briefly the doctrine of limits on which the differential calculus depends.

Find the differential coefficient of $\tan x$, assuming, if it be needed, that the limit, when $x=0$, of $\frac{\tan x}{x}$ is 1.

2. What is understood by different orders of infinitesimals? Show that if one side of a right-angled triangle be regarded as an infinitesimal of the first order, the difference between the hypothenuse and the remaining side is an infinitesimal of the second order.

Two houses A and B lie north and south of one another at a distance of 100 yards apart. A third house C lies 2 miles from B in a south-west direction. A postman calling at C can either proceed directly to A and thence to B, or he can take the opposite course. How much shorter (approximately) is one way than the other?

3. Obtain an expansion for $\sin^{-1} x$ in terms of x as far as the term involving x^5, and write down the general term of the series. From three terms of this series, obtain an approximate value of π.

4. If $y=f(x)$, show generally that the values of x which are derived from the equation $\frac{dy}{dx}=0$ give the maxima and minima values of (y), and show how to distinguish the maxima from the minima.

AB, AC are two straight lines meeting in A; M is a point between them: draw through M a straight line PMQ meeting AB and AC in P and Q respectively, so that the area of the triangle PAQ may be a minimum.

5. In a plane curve determine the lengths of the portions of the tangent and normal respectively intercepted between the point of contact and the axis of (x).

In an ellipse find the points in the curve at which these lengths of the tangent and normal are equal.

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6. Find an expression for the radius of curvature of a plane curve in rectangular coordinates; determine it at any point of the catenary whose equation is $y=\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right)$, and write down its value at the lowest point of the curve.

7. If (p) be the perpendicular from the pole of a curve referred to polar coordinates, prove that $\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.

In a parabola, whose polar equation is $r=\frac{2m}{1+\cos\theta}$, obtain, from the above, the equation between (p) and (r).

8. Trace the curve $y^2=x^2\cdot\frac{x-a}{x-2a}$; draw its asymptotes, and determine if the ordinates have maxima or minima values.

9. There are certain elementary forms of differentials whose integrals are known from inspection, give the integrals of any four such differentials.

Integrate

$$(1)\quad \frac{xdx}{x^2-7x+12}; \qquad (2)\quad \sqrt{a^2-x^2}\,.\,dx.$$

10. Prove that

$$\int_0^{\frac{\pi}{2}}(\sin x)^6\,dx=\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{\pi}{2}.$$

11. Find the differential expression for determining the area of a plane curve in polar coordinates.

Find the area of the loop of the curve whose equation is $r^2=a^2\cos 2\theta$.

12. Find the content of a given sphere.

Prove that the content of a sphere is to the content of the greatest cone that can be inscribed in it as 27 to 8.

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VIII. STATICS.

1. How are forces measured in Statics? Can three forces, proportional to 9, 5, 3 respectively, acting in one plane on a particle, be so arranged as to be in equilibrium? State and prove any proposition that involves the answer to the question.

2. Two equal forces inclined to each other at a given angle act on a fixed point; find the pressure on the point.

Two equal weights (W) are attached to the extremities of a thin string which passes over three tacks in a wall arranged in the form of an isosceles triangle with the base horizontal, the vertical angle at the upper tack being 120°; find the pressure on each tack.

3. If a straight uniform rod is suspended by a thin string fastened to its middle point, and be kept in equilibrium by two weights on the opposite sides of the middle point, find the tension of the string by which the rod is suspended, and show how the weights are related to each other.

A straight lever two feet long is moveable about a hinge at one end, and is kept in a horizontal position by a thin vertical string attached to the lever at a distance of 8 inches from the hinge, and fastened to a fixed point above the lever; if the string can just support a weight of 9 ounces without breaking, find the greatest weight that may be suspended from the other end of the lever.

4. Show that any system of forces acting on a rigid body in one plane may be reduced to a single force and a single couple. A rod is placed in *any* given position with one end on a smooth floor, and the other end against a smooth wall. Find a single force and a single couple which together will keep it at rest in that position.

5. If a right cone be cut by a plane bisecting its axis, find the distance of the vertex of the cone from the centre of gravity of the frustum thus cut off.

6. In that system of three pulleys (usually called the third system) where each string is attached to the weight and the weights of the pulleys are all equal, find the relation of the power to the weight, when equilibrium is established. If each pulley weighs 2 ozs. what weight would be supported by the pulleys alone?

If the weight supported be 25 bs. and the power 3 lbs., find what must be the weight of each pulley.

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7. Find the force acting up and parallel to a given rough inclined plane that is just able to move a weight up the plane.

Two equal weights are attached to a string that is laid over the top of two inclined planes having the same altitude and placed back to back, the angles of inclination of the planes being 30° and 60° respectively, show that the weights will be on the point of moving if the coefficient of friction between each plane and weight be $\frac{1}{2+\sqrt{3}}$.

8. State the principle of virtual velocities. Assuming the principle true, deduce from it the condition of equilibrium on a bent lever of unequal arms when acted on by weights suspended at the extremities of the arms, and show that for an indefinitely small displacement the centre of gravity of the weights will neither ascend nor descend.

9. A lamina in the form of an isosceles triangle, whose vertical angle is α, is poised upon a sphere, radius r, so that its plane is vertical and one of its equal sides (a) is upon the surface of the sphere; show that the equilibrium will be stable in the plane of the triangle if $\sin\alpha$ be less than $\frac{3r}{a}$.

10. Define the unit of work, and explain how it varies with the units of length, mass, and time.

A chain weighing 8 lbs. per foot is wound up from a shaft by the expenditure of four million units of work; find the length of the chain.

11. A cylindrical shaft has to be sunk to a depth of 100 fathoms through chalk whose specific gravity is 2·3; the diameter of the shaft being 10 feet. What horse-power is required to lift out the material in 12 working days of 8 hours each? [The weight of a cubic foot of water is 62·5 pounds, and one horse power is 33,000 foot-pounds a minute.]

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IX. DYNAMICS.

[*The measure of the acceleration of gravity may be taken to be* 32 *when a foot and a second are units of length and time.*]

[*Great importance will be attached to accuracy.*]

1. State the theorem known as the *parallelogram of velocities.*

A stream runs with a velocity of $1\frac{1}{2}$ miles an hour; find in what direction a swimmer, whose velocity is $2\frac{1}{2}$ miles an hour, should start, in order to cross the stream perpendicularly.

What direction should he take to cross in the shortest *time*?

2. A person walking along a road at a rate v, sees a tower a mile distant from his eye; the nearest distance of the tower from the road is half a mile. Find the rate at which he is approaching the tower. Does this rate alter as he advances?

3. A sphere at rest is struck directly by another in motion. Describe in a few words the essential points of the phenomenon of impact, (1) when the spheres have no elasticity, (2) when they are perfectly elastic.

4. A particle is shot vertically upwards with velocity u; find the time of ascent, and shew that it is equal to the time of descent.

5. If a number of particles are let fall at the same instant down a number of smooth inclined planes having a common vertex, prove that at any moment they all lie on one circle.

Prove also that if all the planes are equally rough, the particles will be on one circle.

6. A ball is discharged with the initial velocity of 1100 feet; find in miles the greatest range it can give on a horizontal plane.

Could this range be reached in practice?

7. A projectile is thrown with given velocity and elevation, give the horizontal and vertical components of its velocity after a given time.

If after a given time a second projectile is thrown from the same point, in the same trajectory, describe what appearance its motion would present, as seen from the first one.

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8. One end of a string is fixed; it then passes under a moveable pulley to which a weight, W, is attached. The string then passes over a fixed pulley, and a (smaller) weight, P, is attached to its other end, all three sections of the string being vertical. Show that, neglecting the masses of the pulleys, the acceleration with which W descends is

$$f = \frac{W - 2P}{W + 4P} g.$$

Verify this result when P is small compared to W; and when W is small compared to P.

9. A bead is strung on a thread whose two extremities are fixed, and on which it can move without friction; show that if the bead be started so as to move in the elliptic arc to which it is constrained, its velocity will be uniform (gravity not acting).

Show also that the tension of the thread, for different positions of the bead, varies inversely as the product of the two focal radii vectores.

10. Define the *work* done by a force when it continues to act while its point of application moves over a given space in the direction of the force. If that point moves at an angle with the force, what is the work done?

A mass m is moving with velocity u. A constant force acts on it in the direction of the motion until its velocity is increased to v. Prove that the work done by the force is

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2.$$

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1887.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. Upon the same base and on the same side of it there cannot be two triangles which have their sides that are terminated at each extremity of the base equal to one another.

2. If a side of a triangle is produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

In the triangle ABC, AD and AE are drawn from the vertex A to the base BC, making the angle BAD equal to C, and the angle CAE equal to B: prove that the perpendicular from A upon BC bisects DE.

3. Describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle.

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4. If a straight line is divided into two equal parts, and also into two unequal parts, the rectangle contained by the unequal parts, together with the square of the part between the points of section, is equal to the square of half the line.

Show that the greatest right-angled triangle, which has the sum of the sides containing the right angle equal to a given straight line, is isosceles.

5. If a straight line touches a circle, the straight line which joins the centre to the point of contact is perpendicular to the touching line.

$ABCD$ is a straight line; circles are described on AB and CD as diameters; and a common tangent to the circles is drawn, meeting them in E and F. Prove that the triangles AEB and CFD are equiangular to one another.

6. The angle in a semi-circle is a right angle, and the angle in a segment greater than a semi-circle is less than a right angle, and the angle in a segment less than a semi-circle is greater than a right angle.

7. If two straight lines cut one another within a circle, the rectangle contained by the segments of one of them is equal to the rectangle contained by the segments of the other. Prove this in the one case only in which one of the straight lines passes through the centre and cuts the other, which does not pass through the centre, not at right angles.

AB, AC are equal chords of a circle; and AED is any chord through A, cutting BC in E; prove that the rectangle $AD.AE$ is equal to the square of AB.

8. Inscribe an equilateral and equiangular pentagon in a given circle.

The five lines, other than the sides of the figure, which can be drawn connecting the angular points of an equilateral and equiangular pentagon include between them a pentagon which is itself equilateral and equiangular.

9. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or these produced, proportionally; and, conversely, if two sides, or these produced, be cut proportionally, the straight line which joins the points of section shall be parallel to the remaining side of the triangle.

Straight lines AOB, COD, intersect in O, and $AO : OB :: CO : OD$; if P, Q, are the middle points of AB, CD, prove that PQ is parallel to AC and BD.

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10. Equal triangles, which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional; and, conversely, triangles which have one angle of the one equal to one angle of the other, and their sides about the equal angles reciprocally proportional, are equal to one another.

11. From the vertex A of a triangle ABC, AD and AE are drawn to the base, making the angle BAD equal to the angle CAE: prove that the rectangle $BD \cdot BE$: the rectangle $CE \cdot CD :: AB^2 : AC^2$.

12. Describe a rectilineal figure which shall be equal to one and similar to another given rectilineal figure.

II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance is attached to accuracy.*]

1. What is the smallest number which, when subtracted from 99099, will make it exactly divisible by 909?

2. What will $37\frac{1}{2}$ lbs. of nutmegs cost at 1*s.* $6\frac{1}{2}$*d.* per oz.?

3. What is meant by (*a*) the numerator, (*b*) the denominator of a fraction? Prove that $\frac{3}{4}=\frac{6}{8}$, and add together $\frac{3}{10}+\frac{7}{12}+\frac{6}{24}$.

4. If 13 yards of cloth cost £2. 0*s.* 1*d.*, what must be given for 75 yards? Do this by Rule of Three, and explain the principle on which your statement is made.

5. If £150 gain £3. 7*s.* 6*d.* in 9 months, what sum will gain £3 in 12 months?

6. Find by Practice the price of 22 cwt. 3 qrs. 21 lbs. at £2. 9*s.* 6*d.* per cwt.

7. Multiply ·0047 by ·00035, and prove the correctness of your result; and divide ·00918 by ·018.

8. Find the value of ·05854 of a guinea, and reduce 3 weeks 4 days 5 hours 6 minutes 6 seconds to the decimal of a month.

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9. Extract the square root of 63409369, and the cube root of 66430·125.

10. The interest of £25 for $3\frac{1}{2}$ years at simple interest was found to be £3. 18*s.* 9*d.*; what was the rate per cent.?

11. What is the difference between the compound interest on £100 for 2 years, according as the interest is paid yearly or half-yearly, 4 per cent.?

12. Define Present Worth and Discount: and find the proportion between the interest of £100 for 4 years, at $6\frac{1}{4}$ per cent., and the discount of the same sum payable at the end of 4 years, at the same rate of interest.

13. A person sells an estate worth £1200 per annum for 25 years' purchase, and after deducting $1\frac{1}{2}$ per cent. for expenses invests the remainder in North-Eastern Consols at 155: allowing 2*s.* 6*d.* per cent. for brokerage, find the amount of stock he will receive. If he gets 7 per cent. on his investment what will be the difference of his income, supposing the management of his estate to have cost him 10 per cent. of the rental?

14. What are meant by duodecimal fractions? By means of duodecimals, find the area of a rectangle 2 ft. 3 in. by 5 ft. 7 in., and express your answer in square feet and inches. And find the cost of glazing a window containing 60 panes, each of which is 1 ft. 2′ 3″ by 11′ 5″, at 3*s.* 8*d.* per square foot.

15. A tradesman forms a mixture of tea by adding 1 lb. at 3*s.* and $\frac{1}{2}$ lb. at 4*s.* 6*d.* to every 2 lb. at 2*s.*; what must he sell it at per lb. in order to gain 10 per cent.?

16. A rectangular wooden box *exactly* contains 6 iron balls, each 10 inches in diameter, packed in sawdust: the wood is 1 inch thick. Find the weight of the whole; given that a cubic foot of wood or sawdust weighs 38·4 lbs.; a cubic inch of iron weighs 4·56 oz.; and that the volume of a sphere is ·52 times the cube of its diameter.

17. What fractions produce circulating decimals? What kind of a decimal is $\sqrt{2}$ equivalent to? Convert $\cdot 0\dot{5}4$ into a fraction, and explain your method of doing so. Multiply together ·4, ·4, ·0004, and find the result to the end of the first recurring period.

18. Two clocks are together at 12 o'clock: one loses 7″ and the other gains 8″ in 12 hours: when will one be half-an-hour before the other, and what o'clock will it then show?

19. Find the value of $16\sqrt{3}+10\sqrt[3]{4}-4\sqrt{12}-3\sqrt[3]{108}$ to 2 places of decimals.

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20. Given $\log 2 = \cdot 3010300$, $\log 3 = \cdot 4771213$, $\log 1\cdot 38 = \cdot 1398791$, find $\log \frac{8}{9}$, $\log 2\cdot 16$, $\log \cdot 006$, and the value of $\sqrt{\dfrac{3\sqrt[3]{138}}{\sqrt[5]{\cdot 01}}}$. Log $6\cdot 240325 = \cdot 7952071$. Between what powers of 10 does 2^{64} lie?

21. If the side of a cube be 8, find the side of another cube exactly double the volume of the former. Given $\log 11 = 1\cdot 04139$.

III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance is attached to accuracy.*]

1. Multiply

$$ax^2 + ax + \frac{1}{a} \text{ by } \frac{x^2}{a} - ax + a,$$

and divide

$$x^4 + 5ax^3 - (n^2 + n - 7)\,a^2x^2 - 5na^3x - 6a^4 \text{ by } x^2 - (n-2)\,ax - 2a^2.$$

2. Find the Highest Common Factor of

$$x^5 - x^3 + 4x^2 - 3x + 2 \text{ and } 5x^4 - 3x^2 + 8x - 3.$$

3. Simplify

(1) $$\frac{1}{axy} - \frac{1}{a(x-a)(y-a)} + \frac{1}{y(x-y)(y-a)}.$$

(2) $$\left\{1 + \frac{2b^2}{a(a-3b)}\right\} \cdot \left\{1 + \frac{b}{2b-a}\right\} - \left(\frac{a^2}{b^2} + \frac{b}{a}\right)\left(\frac{a^2 - ab}{a^2 - ab + b^2} - 1\right).$$

4. Express $\{17 - 3\sqrt{21}\}^{\frac{1}{2}}$ as the difference of two surds, and find the value of

$$\frac{\sqrt{x} + \sqrt{x - \frac{1}{x}}}{\sqrt{x} - \sqrt{x - \frac{1}{x}}} \text{ when } 2x = \sqrt{a} + \frac{1}{\sqrt{a}}.$$

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5. Find the condition that the roots of the equation $ax^2 - bx + c = 0$ may be real and unequal.

Also show that, if one root is the square of the other,

$$b = \sqrt[3]{a^2c} + \sqrt[3]{ac^2}.$$

6. Solve the equations

(1) $$\begin{cases} \dfrac{x+4}{7} - \dfrac{x-y-1}{4} = 2x - 4. \\ 2y - 4 - \dfrac{3x-2y}{3} = 3x. \end{cases}$$

(2) $$\frac{15}{x+2} - \frac{8}{x-1} = \frac{3}{x-4}.$$

(3) $$ax + b\sqrt{x^2 + bx + ab} = a^2.$$

7. If

$$a : b :: b : c :: c : d,$$

prove that

$$a^2 : d^2 :: a^3 : c^3.$$

8. Show how to insert any number of arithmetic means between two given quantities.

$2n$ arithmetic means are inserted between a and b: find the middle pair of means.

9. Sum to $2n$ terms each of the series

$$1 - 3 + 9 - 27 + \&c.,$$

$$1 - 3 + 5 - 7 + \&c.,$$

and write down the last term of each series.

10. How many different selections of five young trees can be made from a plantation containing 45?

Also, if five trees be selected from each of six different plantations, in how many ways can they be planted in six rows, each consisting of trees from the same plantation?

11. State the Binomial Theorem, and, assuming it to be true for any positive index, show that it will also be true for any negative index.

Find the coefficients of x^6 in $(1+x)^{-3}$ and $(1+x+x^2)^{-3}$.

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12. A man's net income is 10 per cent. less than his gross income, which is derived partly from land and partly from personal property. He pays income tax at 8*d.* in the £ on the whole, and also local rates on the income from land. If his whole income had been derived from land, his net income would have been one-ninth less. What are the local rates per £, and in what proportion is his gross income derived from the two kinds of property?

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Give a definition of the tangent of an angle which will apply to angles of all magnitudes; and trace the changes in the value of the tangent as the angle changes from 0° to 360°.

If $\tan A = \frac{3}{4}$, find all the other trigonometrical functions of A.

2. Prove by means of a figure that

$$\cos(A-B) = \cos A \cos B + \sin A \sin B;$$

the angle A being between 90° and 135°, and B between 45° and 90°.

3. If $\tan A = \frac{\sqrt{3}}{4-\sqrt{3}}$, and $\tan B = \frac{\sqrt{3}}{4+\sqrt{3}}$;

prove that $$\tan(A-B) = \cdot 375.$$

4. Find the values of sin 60°; sin 165°; sin 18°.

5. Prove that

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}.$$

6. Prove that in *any* triangle

(i) $a \sin B = b \sin A$.

(ii) $a = c \cos B + b \cos C$.

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7. Assuming $$\cos A = \frac{b^2+c^2-a^2}{2bc},$$

prove that $$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

If $a=40$, $b=51$, $c=43$; find the value of A, having given

$$\log 128 = 2{\cdot}107210.$$
$$\log 603 = 2{\cdot}780317.$$
$$\log \tan 24^\circ\, 44'\, 16'' = 9{\cdot}6634465.$$

8. Given $\log 2 = {\cdot}30103$, and $\log 3 = {\cdot}47712$; find $\log \sin 60^\circ$ and $\log \tan 30^\circ$.

9. The sides of a triangle are a feet, b feet, and $\sqrt{a^2+ab+b^2}$ feet in length; find its greatest angle.

10. In any triangle, if $\tan\frac{A}{2} = \frac{5}{6}$, and $\tan\frac{B}{2} = \frac{20}{37}$; find $\tan C$.

Show also that, in such a triangle, $a+c=2b$.

11. DE is a tower on a horizontal plane. $ABCD$ is a straight line in the plane. The height of the tower subtends an angle θ at A, 2θ at B, and 3θ at C. If $AB=50$ feet, and $BC=20$ feet, find the height of the tower and the distance CD.

12. Prove the formula $r = \dfrac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}$; where r is the radius of a circle inscribed in the triangle ABC.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

(Analytical Geometry, Conic Sections, and Solid Geometry.)

[*Full marks may be obtained for about* three-fourths *of this paper.*]
[*Great importance is attached to accuracy.*]

1. Points A, B, C are taken on three conterminous edges of a cube: prove that the angles of the triangle ABC are all acute.

2. A number of planes pass through a given line: find the locus of the feet of the perpendiculars on them drawn from a given point.

3. Prove that the volume of any pyramid is one-third of the volume of a prism having the same base and altitude.

4. Find the value of c in order that the equation

$$12x^2+xy-6y^2-29x+8y=c$$

should represent a pair of right lines.

5. If ω be the angle which a focal chord of a parabola makes with its axis, prove that the length of the chord is $p \operatorname{cosec}^2 \omega$, where p is the parameter of the curve.

6. Being given two tangents to an ellipse, and one of its foci, find the locus of its centre.

7. From any point P on an ellipse PN is drawn perpendicular to its axis major, and produced until NQ is equal to PF, where F is a focus of the ellipse: find the locus of Q.

8. Show that the equation

$$(\alpha^2+\beta^2-r^2)(x^2+y^2-r^2)=(\alpha x+\beta y-r^2)^2$$

represents a pair of right lines which pass through the point α, β, and touch the circle $x^2+y^2=r^2$.

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9. In an ellipse prove that the parallelogram formed by joining the extremities of a pair of conjugate diameters has a constant area.

10. Find the equation to the normal at any point on the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

If X, Y be the coordinates of the middle point of the portion of the normal which is intercepted between the axes of the ellipse, prove that

$$a^2X^2+b^2Y^2=\tfrac{1}{4}(a^2-b^2)^2.$$

11. Define the polar of a point with respect to a conic, and prove that any right line drawn through a point is divided harmonically by the point, the curve, and the polar of the point.

12. Being given three points on a conic, and one of its foci, find the position of its centre and of its vertices by geometrical construction.

13. Prove that any ellipse can be projected orthogonally into a circle.

Hence show that any triangle can be orthogonally projected into an equilateral triangle.

VI. PURE MATHEMATICS (2).

(Further Questions and Problems on the Subjects of the Qualifying Examination and Theory of Equations.)

[*Full marks may be obtained for about* four-fifths *of this paper.*]

[*Great importance is attached to accuracy.*]

1. If $x+\frac{1}{x}=y$, express $x^6+\frac{1}{x^6}$ in terms of y.

2. If $\frac{x^2+2}{x^4+1}\equiv\frac{A+Bx}{x^2+x\sqrt{2}+1}+\frac{C+Dx}{x^2-x\sqrt{2}+1}$ for all values of x, find the values of A, B, C, D.

3. Find the sum of the series—

(1) $1\cdot3+3\cdot5+5\cdot7+\ldots$

(2) $1+3x+5x^2+7x^3+\ldots$

each to n terms.

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4. Find, by aid of the Binomial theorem, or otherwise, the limiting value of $\sqrt{x^4+ax^2+bx+c}-\sqrt{x^4+a'x^2+b'x+c'}$ when $x=\infty$.

5. In what time will a given sum of money quadruple itself at 4 per cent. compound interest? (Given $\log 2=\cdot 30103$, $\log 1\cdot 3=\cdot 11394$.)

6. If $a+b+c=0$, and $x+y+z=0$, prove that

$$(a^2+x^2)(bc-yz)+(b^2+y^2)(ca-zx)+(c^2+z^2)(ab-xy)=0.$$

7. There are 10 tickets, of which 5 are blanks, and the others are marked 1, 2, 3, 4, 5; find the probability of drawing 9 in three trials, the tickets not being replaced.

8. Find the sum of the series

$$\cos a+\cos 3a+\cos 5a+\ldots+\cos(2n-1)a.$$

9. In a plane triangle prove that—

$$a^3\sin(B-C)+b^3\sin(C-A)+c^3\sin(A-B)=0.$$

10. Find the real and imaginary parts of the expressions

$\sin(\alpha+\beta\sqrt{-1})$ and $\sec(\alpha+\beta\sqrt{-1})$, where α and β are real.

11. Show that the equation $x^3-4x^2-3x+12=0$ has two roots which are equal, with opposite signs: and solve the equation.

12. Prove that a root of the equation $f'(x)=0$ lies between each adjacent pair of real roots of $f(x)=0$.

Solve the equation $x^4-3x^3+x^2+4=0$, which has a pair of equal roots.

13. If α, β, γ be the roots of the cubic

$$x^3+px^2+qx+r=0,$$

form the equation whose roots are

$$\alpha\beta+\frac{2}{\gamma},\quad \gamma\alpha+\frac{2}{\beta},\quad \beta\gamma+\frac{2}{\alpha}.$$

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VII. PURE MATHEMATICS (3).

(Differential and Integral Calculus.)

[*Full marks may be obtained for about* three-quarters *of this paper.*]

[*Great importance is attached to accuracy.*]

1. Differentiate the functions

$$\log\left(\frac{x^2-1}{x^2+1}\right)^{\frac{1}{2}},\quad \frac{x}{\sqrt{a^2-x^2}},\quad (\sin x)^x.$$

2. Find the third differential coefficient of y^2; y being a given function of x.

3. The base angles A, B, of an isosceles triangle ACB are infinitely small. Show that in the limit the triangle is equal to $\frac{3}{4}$ of the circular segment ACB.

If a is infinitesimal, show that $2\sin a+\tan a-3a$ is an infinitesimal of the fifth order.

4. State Maclaurin's theorem for the expansion of a function.

Expand $\int\left(\frac{x}{1-x}\right)$ by powers of x as far as x^3.

5. Find the value of the vanishing fraction $\frac{x^a-a^x}{x-a}$ when $x=a$.

6. If m, n are positive, and if

$$mx^2+ny^2=a^2,$$

find by any method the greatest possible value of $x+y$.

7. Find the equation of the tangent to the curve

$$x^m y^n=a^{m+n}$$

at any point; and show that the portion of the tangent intercepted by the axes is divided in a constant ratio at the point of contact.

8. Find the integrals

$$\int\left(\frac{1-x}{1+x}\right)^2 dx;\quad \int\frac{dx}{\sqrt{1+x\pm x^2}};\quad \int x^{r-1}\log x\,dx.$$

In the second, either of the signs $\pm$ may be chosen.

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9. Find a formula for reducing the integral $\int \cos^n x\, dx$, where n is an integer. When may the integral be easily found without using this formula? Modify the formula so as to give one for the reduction of

$$\int \frac{dx}{\cos^n x}.$$

10. If $z = x^{-r} \int x^{r-1} u\, dx$, prove that

$$x \frac{dz}{dx} + rz = u.$$

11. In the curve $x^m y^n = 1$ the whole area included between the curve and one of its asymptotes [beginning from the point (1, 1)] is infinite, and that between it and the other is finite. Find the value of the latter area.

12. Find the radius of curvature of an ellipse, and show that it is equal to $\frac{b'^3}{ab}$, where b' is the semi-diameter conjugate to that through the point.

VIII. STATICS.

1. Explain what is meant by Graphical Statics. If forces of 5 lbs. and 6 lbs. be represented by the base and altitude of an isosceles triangle, what forces will be represented by the sides?

2. Two weights W_1, W_2, are connected by a string which hangs over a smooth peg, and motion is prevented by means of a second string, shorter than the first, attached to each weight. Find the tensions of the strings.

3. Enunciate the proposition known as the Parallelogram of Forces, and, assuming its truth for the direction, prove it for the magnitude of the resultant.

A weight of 10 lbs. is suspended by two strings, 7 and 24 inches long, their other ends being fastened to the extremities of a rod whose length is 25 inches. If the rod be held so that the weight hangs immediately below its middle point, find the tensions of the strings.

4. Three forces $5P$, $10P$, and $13P$, act in one plane on a particle, the angles between any two of their directions being 120°. Find the magnitude of their resultant.

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5. Show how to find the resultant of two like parallel forces. A rod 16 inches long rests on two pegs, 9 inches apart, with its centre midway between them. The greatest weights which can be suspended from the two ends of the rod without disturbing the equilibrium are 4 lbs. and 5 lbs. respectively. Find the weight of the rod, and the position of its centre of gravity.

6. Prove that any system of co-planar forces can be reduced to a single force or a single couple.

7. Find the centre of gravity of a triangular lamina.

The base of an isosceles triangle is 4 inches, and the equal sides are each 7 inches: find the distances of its centre of gravity from the angular points of the triangle.

8. Find the magnitude of the least force which will draw a weight W up a rough inclined plane, the coefficient of friction being μ, and the force being supposed to act parallel to the plane.

How much work is done in drawing a load of 6 cwt. up a rough inclined plane whose height is 3 feet and base 20 feet, the coefficient of friction being $\frac{2}{15}$?

9. Assuming the principle of Virtual Velocities, deduce the relation between the power and the weight in the case of the wheel and axle.

10. Find the force required to raise a bar weighing 20 lbs. by means of a system of one fixed and three movable pulleys; each string being attached to the bar, and the weight of the pulleys being neglected. Draw carefully a figure showing this system of pulleys.

11. Prove that, if a body be placed on a horizontal plane, it will stand or fall according as the vertical line drawn through its centre of gravity passes within or without the base.

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IX. DYNAMICS.

[*The measure of the acceleration of gravity may be taken to be* 32 *when a foot and a second are units of length and time.*]

[*Great importance will be attached to accuracy.*]

1. What is meant by *uniform velocity*, and how is it measured? What by *uniform* acceleration, and how measured?

A body starting with a velocity of 10 feet describes 11 feet in one second. If the acceleration on it be uniform, find its value.

2. A steamer is going due N. with velocity v; the smoke from the chimney points θ degrees South of E. If the wind is due W., find its velocity.

3. If the measure of an acceleration is f, what change in this measure is produced by taking double the unit of time as the new unit? Prove the result.

If the measure of gravity-acceleration be 1, what is necessary as to the units of space and time?

4. Two bodies start together from rest, from one point, in two diverging lines, each moving with a given acceleration. Show that their velocity of separation is uniformly accelerated during the motion.

5. How is *angular velocity* measured when uniform? A body revolves round an axis with a uniform angular velocity ω; how many complete revolutions does it make per minute?

A point moves so that its angular velocities round two fixed points are always equal and in the same sense; find the path it describes.

6. A perfectly elastic ball strikes another equal ball moving in the same direction. Show from the principles of impact, and without assuming any formulae, that the balls exchange velocities.

7. Given the initial velocity and elevation of a projectile, construct geometrically its position, and the direction in which it is moving, at any given moment.

From your construction (or otherwise) show that if any number of projectiles are thrown together from one point, whatever be their initial motions, their directions of motion at any instant all pass through one point.

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8. A vertical hollow cylinder, 3 feet in circumference, is revolving uniformly round its axis, making one turn per second. A heavy particle is let drop from a point at the mouth of the cylinder, so as just to touch the inside surface during its fall and leave a trace upon it. If the cylinder be afterwards unrolled, what form will the trace present?

9. A body moves uniformly with given velocity in a circle. Is there necessarily any force acting on the body? If so, state its magnitude and direction at any moment.

A smooth hollow sphere is revolving with uniform angular velocity ω round a vertical diameter. Show that a heavy particle placed inside will only remain, resting against the side of the sphere, at one particular level.

If the angular velocity ω be less than a certain limit, show that the particle will remain at the lowest point of the sphere.

10. State the principle of work with regard to the motion (1) of a particle acted on by force, (2) of a rigid body. If a body is at rest, and any forces are applied to it, and it moves under the action of those forces, until it arrives at another position where it is also at rest, what can be inferred as to the action of the forces?

A uniform rectangular block $ABCD$ stands on its base AD on a rough floor. It is pulled at C by a horizontal force just great enough to begin to turn it round the corner D. If this same force continues to pull it horizontally at C till the block has turned through an angle θ, and then ceases, prove that the block will have acquired sufficient momentum to cause it just to overturn round D, provided $\sin\theta = \tan\frac{1}{2}\alpha$, where $\alpha < BDC$.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1887.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. The angles which one straight line makes with another straight line on one side of it, either are two right angles, or are together equal to two right angles.

The straight lines bisecting the angles at the base BC of an isosceles triangle ABC intersect in D. Show that AD produced bisects the angle BDC.

2. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

Describe an isoceles triangle in which each of the angles at the base is one-fourth of the vertical angle.

3. Distinguish between a *Problem* and a *Theorem*, and explain what is meant by the *converse* of a Theorem.

Enunciate carefully, and prove, the converse of the following theorem: "Triangles on equal bases and between the same parallels are equal."

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4. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the two parts.

Prove that, of all right-angled parallelograms having the same perimeter, the square has the shortest diagonals.

5. In every triangle the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle and the acute angle.

6. The diameter is the greatest straight line in a circle; and, of all others, that which is nearer to the centre is always greater than one more remote; and the greater is nearer to the centre than the less.

7. In a circle the angle in a semicircle is a right angle, but the angle in a segment greater than a semicircle is less than a right angle, and the angle in a segment less than a semicircle is greater than a right angle.

Two circles, whose centres are O, O', touch each other in the point A, and are met by a common tangent in the points B, C, respectively. BOD, $CO'E$, are diameters of the two circles. Show that BE, CD, intersect in A.

8. Inscribe a circle in a given square.

Two squares are described, one circumscribing and the other inscribed in a given circle. Prove that the diameter of the circle inscribed in the smaller square is equal to the radius of the circle described about the larger square.

9. How does Euclid test whether four magnitudes are proportionals?

Find a fourth proportional to three given straight lines.

10. If four straight lines be proportionals, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the means be equal to the rectangle contained by the extremes, the four straight lines are proportionals.

Describe a rectangle so that its sides are in a given ratio, and its area is equal to a given square.

11. Parallelograms about the diameter of any parallelogram are similar to the whole parallelogram and to one another.

Show that either of the complements is a mean proportional between the two parallelograms about the diameter.

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II. ARITHMETIC.

(Including the Use of Common Logarithms.)

[N.B.—*Great importance is attached to accuracy. Candidates are expected to use Arithmetical methods of solution.*]

1. Multiply 909 by 990, and explain your method of work.

2. What is the value of 6 oz. 12 dwts. of gold when 1 oz. 13 dwts. cost £4. 18*s*. 10½*d*.?

3. State and explain the rule for division of fractions. Divide $33\frac{3}{7}$ by $7\frac{34}{55}$, and bring the result to its lowest terms.

4. If the sum of the digits of a number be divisible by 9, prove that the number itself is also divisible by 9.

5. Divide ·0053 by 2·5, and 53 by ·0025. Explain your rule for pointing in the quotient in each case.

6. Reduce £2. 15*s*. $6\frac{3}{8}$*d*. to the decimal of a pound: and find the difference between ·625 and ·0625 of a cwt.

7. Reduce $\cdot 007\dot{6}4\dot{8}$ to a vulgar fraction; and add together $\cdot 12\frac{5}{6}$, $4\cdot\dot{1}6\dot{3}$, $1\cdot\dot{7}14\dot{3}$, and $2\cdot\dot{5}\dot{4}$ without reducing them to fractions.

8. If the carriage of 150 feet of wood, that weighs 3 stone per foot, costs £3 for 40 miles, how much will the carriage of 54 feet of marble, weighing 8 stone per foot, cost for 25 miles?

9. Find, by Practice, the cost of 92 tons 7 cwt. 3 qrs. 12 lbs. at £1. 12*s*. 7*d*. per ton.

10. State and explain the rule of Pointing in the extraction of a square root. Extract the square root of 197·96492 to 7 places of decimals.

11. Find the solid content of a block of marble 3 ft. 8 in. 4′ long, 1 ft. 7 in. 6′ broad, and 1 ft. 4 in. 9′ thick (by duodecimals). Express your result in cubic feet and inches.

12. In how many years will £768. 17*s*. 6*d*. amount to £1230. 4*s*. at 5 per cent. simple interest?

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13. A person invests £1842. 15*s.* in 4 per cents. at $102\frac{3}{8}$: he afterwards sells out at 105, and re-invests at 126 in 5 per cents.: find his change of income.

14. Two watches are together at 12 o'clock. One gains 75″ per hour, and the other loses 45″. When will they be together again at 12?

15. If *A*, *B*, and *C* could reap a field in 18 days; *B*, *C*, and *D* in 20 days; *C*, *D*, and *A* in 24 days; and *D*, *A*, and *B* in 27 days: in what time would it be reaped by them all together?

16. If a grocer gains 10 per cent. by selling tea at 2*s.* 3*d.* per lb., what will he gain per cent. by selling it at 2*s.* 9*d.*?

17. Define a Logarithm. Find the log of 81 to base 3, and of 3 to base 81. Given log 234 = 2·3692159, write down log 234000, log 23·4, log ·0234.

18. Find, with the aid of the logarithm tables supplied, a 4th proportional to 1·3046, ·01042, and 2·375.

19. Given log 367200 = 5·5649027, and log 367100 = 5·5647844: find the number whose log is 5·5648815.

20. Find, with the aid of the tables, the amount of £1,000 in 100 years at 5 per cent.—(1) Simple interest. (2) Compound interest.

21. A hare starts to run, when a dog is 44 yards off, at 12 miles per hour. After half a minute the dog sees her, and pursues at 16 miles per hour. How soon will he catch her?

22. Find the present value of an annuity of £300 payable every year for 3 years at 5 per cent. per annum simple interest.

[*Tables of logarithms to seven figures are supplied.*]

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III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorom.)

[N.B.—*Great importance is attached to accuracy.*]

1. Divide $x^3(a+1)-xy(x-y)(a+b)-y^3(b-1)$ by x^2-xy+y^2.

2. Prove that, if

$$y-z=ax,$$
$$z-x=by,$$
$$x-y=cz,$$

then
$$abc+a+b+c=0.$$

3. Prove the rule for finding the Least Common Multiple of two algebraic expressions.

Find the Greatest Common Measure of

$$x^4+4x^3y-21x^2y^2+10xy^3-y^4 \text{ and}$$
$$x^4+12x^3y+33x^2y^2-12xy^3+y^4.$$

4. Simplify the following expressions:

(1) $\left(\frac{x^2}{y^2}-1\right)\left(\frac{x}{x-y}-1\right)+\left(\frac{x^2}{y^2}-1\right)\left(\frac{x^2+xy}{x^2+xy+y^2}-1\right).$

(2) $\frac{x^2-7xy+12y^2}{4x^2-11xy-3y^2}-\frac{2x^2+7xy-4y^2}{8x^2-6xy+y^2}.$

5. Prove that the greater of the two fractions

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}, \qquad \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}-\sqrt{2-\sqrt{3}}},$$

exceeds the less by $2-\sqrt{2}$.

6. Extract the square root of

$$4\left(\sqrt[3]{\frac{a^2}{c^2}}+\sqrt[3]{\frac{c^2}{a^2}}-\sqrt[3]{\frac{a}{c}}-\sqrt[3]{\frac{c}{a}}\right)+9.$$

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7. If α, β, be the roots of the quadratic $x^2+px+q=0$, prove that the equation whose roots are $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$ is

$$qx^2+p(1+q)x+(1+q)^2=0.$$

8. Solve the equations:

$$(1)\quad \frac{2x-y}{10a+3b}=\frac{x-3y}{4b}=\frac{y+b}{2a}.$$

$$(2)\quad \frac{1}{2x-5a}+\frac{5}{2x-a}=\frac{2}{a}.$$

$$(3)\quad x-\sqrt{x^2-1}=\frac{1}{2(x+1)}.$$

9. A and B run a race; B has 50 yards start, but A runs 20 yards while B runs 19: what must be the length of the course, that A may come in a yard ahead of B?

10. What is meant by the statement that one quantity varies directly as another?

If the number of passengers by an excursion train vary directly as the square of the number of degrees of temperature above 50° Fahrenheit, and if there be 405 passengers when the temperature is 59°, how many will there be when the temperature is 56°?

11. Assuming the formula for the number of arrangements of n different things taken r together, prove the formula for the number of combinations of n things taken r together.

A man has 10 friends: how many different parties could he make by inviting them six together?

12. Show that in the expansion of $(1+x)^n$, where n is a positive integer, the coefficients of terms equidistant from the beginning and end are equal.

Write down the middle term in the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ and the coefficient of x^r in the expansion of

$$(1+2x+3x^2+\ldots\ldots)^2.$$

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IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

[*Logarithm tables are supplied.*]

1. Define the sine, cosine, and tangent of an angle, and express each of the three trigonometrical ratios in terms of the other two.

Find the sines and cosines of all the angles in the first four quadrants whose tangent is equal to cos 135°.

2. Prove that $\sin(A+B)=\sin A\cos B+\cos A\sin B$, and deduce from it corresponding formulæ for $\cos(A-B)$ and $\cos 2A$.

Prove that $\sin 50^\circ-\sin 70^\circ+\sin 10^\circ=0$.

3. Prove that

$$\tan\left(45^\circ+\frac{A}{2}\right)=\left(\frac{1+\sin A}{1-\sin A}\right)^{\frac{1}{2}}$$

and

$$\cos^3 2\theta+3\cos 2\theta=4(\cos^6\theta-\sin^6\theta).$$

4. Find the values of $\tan^{-1}\sqrt{3}$ and cos 105°.

Construct the acute angle whose cosine is equal to its tangent.

5. Prove that

$$\cos^{-1}x=\sin^{-1}\sqrt{\frac{1-x}{2}}+\cos^{-1}\sqrt{\frac{1+x}{2}}.$$

6. Express $\sin 4A$ in terms of $\tan A$; and solve the equation

$$\cos 3\theta+\sin\theta=0.$$

7. Prove that in any triangle $c=a\cos B+b\cos A$.

Hence show that

$$s=a\cos^2\frac{B}{2}+b\cos^2\frac{A}{2}=b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}=c\cos^2\frac{A}{2}+a\cos^2\frac{C}{2};$$

and deduce the equation

$$\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}.$$

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8. The base of a triangle being seven feet, and the base angles $129° 23'$ and $38° 36'$, find the length of the shortest side.

9. Prove that in any triangle

$$\frac{a+c}{b} = \frac{\cos\frac{A-C}{2}}{\sin\frac{B}{2}}.$$

Hence show that, if C is a right angle,

$$\tan\frac{A}{2} = \frac{a-b+c}{a+b+c}.$$

10. A flagstaff a feet high is on a tower $3a$ feet high: prove that, if the observer's eye is on a level with the top of the staff, and the staff and tower subtend equal angles, the observer is at a distance $a\sqrt{2}$ from the top of the staff.

11. If S and S' are the areas of a given triangle and the triangle formed by joining the points of contact of its inscribed circle, prove that

$$\frac{S'}{S} = \frac{2(s-a)(s-b)(s-c)}{abc}.$$

[*Logarithm tables to seven figures are supplied.*]

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FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

(Analytical Geometry, Conic Sections, and Solid Geometry.)

[*Full marks may be obtained for about* three-fourths *of this paper.*]
[*Great importance is attached to accuracy.*]

1. From a point in space perpendiculars are let fall, one on a plane, another on a straight line lying in that plane. Prove that the line joining the feet of the perpendiculars is perpendicular to the straight line.

2. Prove that in a trihedral angle any two of the dihedral angles containing it are together greater than the third.

If A, B, C, D, are four points not in one plane, prove that the four angles of the quadrilateral $ABCD$ are together less than four right angles.

3. A circle in space is projected orthogonally on any plane. Show that the projection is an ellipse, and find its axes.

4. Three lines in space, OA, OB, OC, are mutually at right angles. Their lengths being a, b, c, express the area of the triangle ABC in its simplest form.

5. From a fixed point A on a given line any number of lengths, AP, are measured along the line; and at P a perpendicular PQ is erected, whose length in each case represents the square root of the number represented by AP. Trace the locus of Q, and give the position of its focus.

6. Show that $x+y=\sqrt{a^2+b^2}$ represents a tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

and find the point of contact.

7. A variable circle always passes through two fixed points A, B. Find the locus of a point on it where the tangent is perpendicular to AB.

8. The vertex O of a parabola is joined with any point P on the curve; and PQ is drawn at right angles to OP, meeting the axis in Q. Show that the projection of PQ on the axis is always equal to the latus rectum.

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9. If, in the equation $y=mx+c$, c is given, but not m, how far is the position of the straight line determined?

Give an equation which will include all straight lines passing through the intersection of two given straight lines.

10. Two points (xy), $(x'y')$, subtend a right angle at the origin. What relation exists among their coordinates?

What loci are represented by the equations

(1) $x^2+xy+y^2=0$, (2) $x^2-xy^2=0$, (3) $(x^2-a^2)^2+(y^2-b^2)^2=0$.

11. Prove geometrically that the tangent to a parabola is equally inclined to the focal radius vector and to the diameter.

A parabola touches two given straight lines at two given points. Find its focus by geometrical construction.

12. Assuming that the sum of the focal distances of any point on an ellipse is constant, and that the tangent is equally inclined to them; prove *geometrically* that:—

(1) The locus of the foot of the perpendicular from a focus on any tangent is a circle.

(2) The rectangle of the perpendiculars from the two foci on any tangent is constant.

VI. PURE MATHEMATICS (2).

(Further Questions and Problems on the Subjects of the Qualifying Examination, and Theory of Equations.)

[*Full marks may be obtained for about* four-fifths *of this paper.*]
[*Great importance is attached to accuracy.*]

1. If it is possible to find values for x, y, z, to satisfy the equations

$$x^2+y^2=2axy,$$
$$y^2+z^2=2byz,$$
$$z^2+x^2=2czx,$$

show that $$a^2+b^2+c^2=2abc+1.$$

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2. If x be any positive quantity, and n any integer, prove that

$$\left(\frac{x+1}{n+1}\right)^{n+1} > \left(\frac{x}{n}\right)^{n},$$

being given the theorem that, if a positive number be divided into any given number of parts, the product of the parts is greatest when they are equal.

3. Express $y=\sqrt{1+x^2}$, (1) in a series of ascending powers of x, (2) of descending powers.

Give an approximate value for y when x is a large number.

4. Prove in any manner that n^4-1 is divisible by 5 if n be any integer not divisible by 5.

Show that, n being any integer, n^5-n is divisible by 30.

5. If n straight lines are drawn at random on a plane, how many points of intersection will there be?

If n pairs of lines are drawn, each pair being parallel to one another, how many intersections?

6. Three cards are drawn at random from a pack of 52. Find the chance that all three are from different suits.

7. State the series for $\log(1+x)$ by ascending powers of x.

Prove also that

$$\log(1+x)=\frac{x}{1+x}+\frac{1}{2}\left(\frac{x}{1+x}\right)^2+\frac{1}{3}\left(\frac{x}{1+x}\right)^3+\&c.$$

8. Find the three roots of the cubic $4x^3-15x^2+12x+4=0$.

Prove that, for all positive values of x except 2,

$$x^3+3x+1>\tfrac{15}{4}x^2.$$

9. If $\tan^2\theta=\frac{b}{a}$, find, in terms of a and b, the simplest form for the value of $\frac{a}{\cos\theta}+\frac{b}{\sin\theta}$.

10. What are the factors of $1-2x\cos\theta+x^2$?

Find the sum of the series

$$\sin\theta+x\sin 2\theta+x^2\sin 3\theta+\&c.$$

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11. State the formula for the tangent of the sum of any number of angles.

If n be odd, express by Trigonometry the n roots of the equation

$$nx - \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} x^3 + \frac{n(n-1)(n-2)(n-3)(n-4)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} x^5 - \&c. = 0.$$

12. A long building in front of us is found to subtend an angle θ; on walking a distance d perpendicularly towards it, the length of building subtends an angle ϕ; on continuing a distance e in the same direction, the building is reached. Find its length.

VII. PURE MATHEMATICS (3).

(Differential and Integral Calculus.)

[*Full marks may be obtained for about* three-fourths *of this paper.*]

[*Great importance is attached to accuracy.*]

1. Starting from first principles, find the values of $\frac{du}{dx}$, (1) when $u = x^m$, for all real values of m; (2) when $u = \log x$.

2. Explain what is meant by different orders of infinitesimals. If a side of a regular polygon inscribed in a circle be a very small magnitude of the first order in comparison with the radius, show that the difference between the circumference of the circle and the perimeter of the polygon is a small magnitude of the second order.

3. If $a \tan x = b \tan y$, express $\frac{dx}{dy}$ in terms of y and of the constants a, b.

4. If $p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$, prove that $p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$.

5. Explain Lagrange's theorem on the limits of Taylor's theorem. Give an instance in which the expansion according to Taylor's theorem is shown by Lagrange's theorem to be inapplicable.

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6. Find an expression for the length of the subtangent at any point on the curve

$$xy^2 = a^2(a - x),$$

and find the point at which the subtangent has a maximum length.

7. Define the pedal of a curve with respect to any point.

Find the equation to the pedal of the curve

$$r^m = a^m \sin m\theta.$$

8. In the curve

$$(a^2 - 2ay)(3y^2 - x^2) = 2(x^4 - y^4)$$

find the position of the asymptotes, and also the direction of the tangents at the origin.

9. Find the values of the following integrals:

$$\int \sin^2 x\, dx, \qquad \int \log x\, dx, \qquad \int \frac{dx}{x^2 + x - 6}.$$

10. Find the values of the definite integrals:

$$\int_0^{\frac{\pi}{2}} \cos x\, dx, \qquad \int_0^{\frac{\pi}{2}} x \sin x\, dx, \qquad \int_0^1 \frac{dx}{1 + x + x^2}.$$

11. Show how to express the area of a curve whose equation is given in polar coordinates.

Find the area of a loop of the curve $r = a \cos 3\theta$.

12. Find the length of an arc, measured from the vertex, of the semi-cubical parabola $ay^2 = x^3$.

VIII. STATICS.

1. Enunciate and prove the proposition known as the Triangle of Forces. Construct geometrically the directions of two forces $2P$ and $3P$ which make equilibrium with a force $4P$ whose direction and point of application are given.

2. Forces act in one plane on a particle; show how to find the magnitude and direction of their resultant.

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$ABCD$ is a square; forces of 1 lb., 6 lbs., and 9 lbs. act in the directions AB, AC, and AD respectively: find the magnitude of their resultant correct to two places of decimals.

3. A light triangular frame ABC stands in a vertical plane, C being uppermost, on two supports A, B, in the same horizontal line, and a weight of 18 lbs. is suspended from C. If $AB=AC=6$ feet, and $BC=5$ feet, find the pressures on the supports.

4. If four forces in equilibrium be parallel, two and two, prove that they form two unlike couples of equal moment.

A uniform ladder 13 feet long rests with one end against a smooth vertical wall, and the other on a rough horizontal plane at a point 6 feet from the wall. Find the friction between the ladder and the ground, the weight of the ladder being 56 lbs.

5. Find the centre of gravity of a pyramid, and show that it coincides with that of four equal particles placed at the angular points.

6. A triangular lamina ABC, obtuse-angled at C, stands with its side AC in contact with a table: show that the least weight which, suspended from B, will overturn it $=\frac{1}{8}W\frac{a^2+3b^2-c^2}{c^2-a^2-b^2}$, where W=the weight of the triangle.

Interpret the above if c^2 be $> a^2+3b^2$.

7. A light rod rests wholly within a smooth hemispherical bowl of radius r, and a weight W is clamped on to the rod at a point whose distances from the ends are a, b. Show that, if θ be the inclination of the rod to the horizon in the position of equilibrium, then $\sin\theta=\frac{a-b}{2\sqrt{r^2-ab}}$; and find the pressures between the rod and the bowl.

8. Investigate the relation between the power and the weight in the case of a smooth screw.

What must be the length of the power arm of a screw having six threads to the inch in order that the mechanical efficiency may be 216?

9. State the laws of limiting friction.

The poles supporting a lawn-tennis net are kept in a vertical position by guy ropes, one to each pole, which pass round pegs 2 feet distant from the poles. If the coefficient of limiting friction between the ropes and pegs be $\frac{1}{4}$, show that the inclination of the latter to the vertical must not be less than $\tan^{-1}\frac{9}{11}$, the height of the poles being 4 feet.

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10. Explain the terms *virtual velocity* and *virtual moment* of a force.

A uniform beam AB of weight W, movable in a vertical plane about a hinge at A, is kept in a given position by a force P applied at the end of a string PCB passing over C, a pulley vertically above A, and at a distance $AC = AB$. Show that, if the system be slightly displaced owing to a small change in P—

$$P \times P\text{'s virtual velocity} + W \times W\text{'s virtual velocity} = 0.$$

IX. DYNAMICS.

[*The measure of the acceleration of gravity may be taken to be* 32 *when a foot and a second are the units of length and time.*]

[*Great importance will be attached to accuracy.*]

1. Compare the velocities of the extremities of the hour, minute, and second-hands of a watch, their lengths being ·48, ·8, and ·24 inches respectively.

2. Enunciate and prove the proposition known as the Parallelogram of Velocities.

Two points describe the same circle in such a manner that the line joining them always passes through a fixed point: show that at any moment their velocities are proportional to their distances from the fixed point.

3. Find the space described in time t by a particle projected with velocity u, subject to a constant acceleration f in the direction of its motion.

If a, b, c, be the spaces described in the pth, qth, and rth seconds, prove that $a(q-r)+b(r-p)+c(p-q)=0$.

4. If the acceleration due to gravity be taken as the unit acceleration, and the velocity generated in one minute as the unit of velocity, find the unit of length.

5. Two weights, P, Q, are connected by a weightless string which passes over a smooth pulley: find the acceleration and the tension of the string.

Two strings pass over a smooth pulley; on one side they are each attached to a weight of 5 lbs., and on the other to weights of 3 lbs. and 4 lbs. respectively: find their tensions.

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6. A body is projected in a given direction with a given velocity: find the range on a given inclined plane passing through the point of projection.

The angular elevation of an enemy's position on a hill h feet high is β: show that, in order to shell it, the initial velocity of the projectile must not be less than $\sqrt{gh(1+\operatorname{cosec}\beta)}$.

7. A ball of mass m moving with velocity u impinges directly upon a ball of mass m' moving with velocity u': find the velocities of the balls after impact, e being the coefficient of elasticity.

Two equal marbles A, B, lie in a horizontal circular groove at opposite ends of a diameter. A is projected along the groove, and after a time t impinges on B: show that a second impact will take place after a further interval $\frac{2t}{e}$.

8. A particle slides down a smooth curve in a vertical plane under the action of gravity: find the velocity of the particle in any position.

A bead slides on a wire bent into the form of a parabola whose axis is vertical and vertex upwards; if the bead be just displaced from its position of equilibrium, then at any subsequent time its velocity will vary as its distance from the axis.

9. A particle weighing $\frac{1}{4}$ oz. rests on a horizontal disc and is attached by two strings 4 feet long to the extremities of a diameter. If the disc be made to revolve 100 times a minute about a vertical axis through its centre, find the tension of each string.

10. A train weighing 200 tons is running at 40 miles an hour down an incline of 1 in 120: find the resistance necessary to stop the train in half a mile.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1888.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases, or third sides, equal; the angle which is contained by the two sides of the one is equal to the angle contained by the two sides, equal to them, of the other.

ACB, ADB are two triangles, on the same base AB and on the same side of it; AC is equal to BD, and AD to BC; and AD, BC intersect in O: prove that the triangles OAB, OCD are each isosceles.

2. If a straight line falls upon two parallel straight lines, it makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle on the same side of the line, and the two interior angles on the same side of it together equal to two right angles.

AB and CD are any two diameters of a circle; prove that, if C and D be joined to B, the straight lines CB, DB will bisect the angles made with AB by a straight line through B parallel to CD.

3. If a parallelogram and a triangle are on the same base and between the same parallels, the parallelogram is double of the triangle.

Construct the greatest triangle which has two of its sides equal to two given straight lines each to each.

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4. If a straight line is bisected and produced to any point, the rectangle contained by the whole line thus produced and the part produced, together with the square on half the line bisected, is equal to the square on the line which is made up of the half and the part produced.

Find a point D in AB produced so that the rectangle AD, DB shall be equal to the square on a given straight line which is greater than half AB.

5. If a straight line is divided into two equal parts, and also into two unequal parts, the squares on the unequal parts are together double of the square on half the line, and of the square on the part between the points of section.

6. The angles in the same segment of a circle are equal to one another.

If two segments of circles have a common chord AB, and any points P and Q are taken on their arcs, the locus of the point of intersection of the bisectors of the angles PAQ, PBQ is another segment of a circle on the same chord AB.

7. Upon a given straight line describe a segment of a circle, which shall contain an angle equal to a given rectilineal angle.

8. Describe a circle about a given triangle.

Show that, if the centre of the circle described about a triangle coincides with the centre of the circle inscribed in it, the triangle must be equilateral.

9. The sides about the equal angles of equiangular triangles are proportionals, those sides being homologous which are opposite to the equal angles.

A and B are the points of intersection of two circles; CAD is a chord through A; and BC, BD are drawn: prove that BC is to BD as the diameter of the circle ABC is to that of the circle ABD.

10. Similar triangles are to one another in the duplicate ratio of their homologous sides.

Of two equilateral and equiangular hexagons, one is inscribed in a circle and the other is described about the same circle: prove that the areas of the hexagons are to one another in the ratio of 3 : 4.

11. In any right-angled triangle, a rectilineal figure described on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

Explain the term "similarly described."

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II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance is attached to accuracy. Candidates are expected to use Arithmetical methods of solution.*]

1. Divide 10101255 by 2185, and explain the process.

2. Give a definition of Multiplication which shall include the multiplication of one fraction by another.

Simplify the expression $\frac{3\frac{1}{2}}{7\frac{1}{4}} \times \frac{17\frac{2}{3}}{51\frac{8}{11}}$.

3. Find the Greatest Common Measure, and the Least Common Multiple of the numbers 15496 and 12665, and show, by general reasoning, that these four numbers form a proportion.

4. Multiply 4·327615 by ·003248, and divide ·292262016 by 32·7648.

5. Find the rent of a piece of ground covering 14 acres 16 poles 22 sq. yards 6 sq. feet at £3. 0*s*. 6*d*. per acre.

6. Explain what is meant by a Recurring Decimal, and find the vulgar fractions equivalent to the decimals, ·1$\dot{2}\dot{3}$, 11·12$\dot{3}\dot{4}$, and ·00$\dot{3}\dot{4}\dot{5}\dot{6}\dot{8}$.

7. Reduce $\frac{3}{10}$ of 4 guineas to the fraction of £7, and £3. 17*s*. 2½*d*., to the decimal of £4 to eight places of decimals.

8. Find the Square Root of 199·6569.

When a regiment of 962 men is drawn up in a solid square, one man is left out; find the number of men in the face of the square.

9. *A* can do a piece of work in 11 days, *B* in 20 days, and *C* in 55 days; how soon can the work be done if *A* is assisted by *B* and *C* on alternate days?

10. Find the cost of carpet 2 feet 3 inches wide for a room 20 ft. 3 in. long by 13 ft. 4 in. wide, at 5*s*. a yard.

11. If the simple interest on £4373. 6*s*. 8*d*. for 1¼ years be £246, what is the rate per cent.?

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12. Find the discount on £51. 15*s*. 10*d*., due $4\frac{1}{2}$ years hence, at 3 per cent., simple interest.

13. A gravel walk 6 feet wide runs round a grass-plot 60 feet long and 40 feet wide. If gravel is 3*s*. per cubic yard, find the cost of a coat of gravel on the path 3 inches deep.

14. A person transfers £4865 from the $3\frac{1}{2}$ per cents. at $93\frac{3}{4}$ to the 3 per cents. at $87\frac{1}{2}$. Find the change in his income.

15. Explain what is meant by the characteristic of a logarithm, and state the rule for finding the characteristic of the logarithms to base 10 of the numbers less than unity.

16. Find the logarithms to the base 2 of—

$$64,\ \frac{1}{64},\ \text{and}\ \sqrt[3]{32}.$$

17. Find from the tables the logarithms of 35726 and 357·26437.

18. Find from the tables the number whose logarithm is 4·9220534.

19. Employ the tables to find the value of the product of 52·4574 by 3·78472.

20. Employ the tables to find the mean proportional between 33·549 and 44·642.

21. Employ the tables to find the value of the expression—

$$(5{\cdot}7432)^{1{\cdot}246}.$$

22. A tricycle, going at the rate of 5 miles an hour, passes a milestone, and 14 minutes afterwards, a bicycle, going in the same direction at the rate of 12 miles an hour, passes the same milestone; find when and where the bicycle will overtake the tricycle.

III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—*Great importance is attached to accuracy.*]

1. Find the value of

$$x^4 - 5x^3 - 12x^2 - 13x - 7 \text{ when } x = -\frac{1+\sqrt{-3}}{2}.$$

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2. Find the factors of

$$(a+b+c)^2-a^2+b^2-c^2 \text{ and of } x^5-y^5-(x-y)^5.$$

3. Reduce to its lowest terms

$$\frac{2x^4+17x^3+30x^2+8x-5}{x^4+4x^3-18x^2-29x-10}.$$

4. Simplify

(i) $$\frac{6x^2-5xy-6y^2}{14x^2-23xy+3y^2}-\frac{15x^2+8xy-12y^2}{35x^2+47xy+6y^2}.$$

(ii) $$\frac{c}{ab(b-c)(c-a)}+\frac{b}{ac(a-b)(b-c)}+\frac{a}{bc(a-b)(c-a)}.$$

5. Apply the process for extracting the square root to find m and n when $x^4+ax^3+mx^2+cx+n$ is a complete square.

6. If the roots of the equation

$$\left(1-q+\frac{p^2}{2}\right)x^2+p(1+q)x+q(q-1)+\frac{p^2}{2}=0$$

are equal show that $p^2=4q$.

7. Solve the equations

(a) $$\frac{5}{x+10}+\frac{8}{x+4}=\frac{13}{x+7}.$$

(b) $$\sqrt{2x+6}-\sqrt{x-1}=2.$$

(c) $$\frac{a^2(x-b)}{a-b}+\frac{b^2(x-a)}{b-a}=x^2.$$

8. Find the value of

$$\frac{(4+\sqrt{15})^{\frac{3}{2}}+(4-\sqrt{15})^{\frac{3}{2}}}{(6+\sqrt{35})^{\frac{3}{2}}-(6-\sqrt{35})^{\frac{3}{2}}}.$$

9. The metal of a solid sphere, radius r, is made into a hollow sphere whose internal radius is r; required its thickness. [The volume of a sphere, radius r, is $\frac{4}{3}\pi r^3$.]

10. Two rectangular lawns have the same area (a^2), but the perimeter of the one is one-fourth longer than that of the other, which is a square; required its dimensions.

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11. If the sum of the first p terms in an A.P. $=0$ the sum of the next q terms $=\dfrac{-a(p+q)q}{p-1}$.

12. Is the coefficient of x^r in the expansion of $(1-x)^{-n}$ equal to the number of combinations of $(n+r)$ things taken r together?

If not, amend the proposition, and prove it.

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Define the circular measure of an angle.

Taking $\pi=\dfrac{22}{7}$, calculate the number of degrees, minutes, and seconds in the unit of circular measure.

2. Prove geometrically that $\cos 2a=\cos^2 a-\sin^2 a$.

Find the value of $\cos 15^\circ$, and prove that

$$(\sec 15^\circ+\operatorname{cosec} 15^\circ)^2=24.$$

3. Write down a single expression for all the angles which have the same cotangent as the angle a; and give, in a single expression for θ, the complete solution of the equation $3\tan^2 2\theta=1$.

4. Prove the following formulae

$$\cos^2 \tfrac{1}{2}a\,(1+\tan \tfrac{1}{2}a)^2=1+\sin a,$$

$$4\,(\cos^3 10^\circ+\sin^3 20^\circ)=3\,(\cos 10^\circ+\sin 20^\circ),$$

and express $4\cos a\cos\dfrac{a}{2}\cos\dfrac{5a}{2}$ as the sum of four cosines.

5. Determine x from the equation $\sin\cot^{-1}\frac{1}{2}=\tan\cos^{-1}\sqrt{x}$; and prove that if $a=\tan^{-1}\frac{1}{7}$, $\beta=\tan^{-1}\frac{1}{3}$, then $\cos 2a=\sin 4\beta$.

6. ABC is an isosceles triangle, right angled at C. D is the middle point of AC. Prove that DB divides the angle B into two parts whose cotangents are as $2:3$.

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7. Prove that in any triangle $c = a \cos B + b \cos A$.

From this and the two corresponding formulæ

$$a = b \cos C + c \cos B, \quad b = c \cos A + a \cos C$$

deduce the "Rule of Sines."

8. Prove that in any triangle

$$\frac{b-c}{a} \cos^2 \frac{A}{2} + \frac{c-a}{b} \cos^2 \frac{B}{2} + \frac{a-b}{c} \cos^2 \frac{C}{2} = 0,$$

and if the sines of the angles are as 13 : 14 : 15 find the ratio of the cosines.

9. Obtain the expression for the area of any triangle in terms of the sides.

If the sides of a triangle are in Arithmetical Progression and its area $= \frac{3}{8}$ths of the area of an equilateral triangle of the same perimeter, prove that its sides are in the ratio 3 : 5 : 7.

10. Given two sides of a triangle and the included angle, obtain a formula, adapted to logarithmic computation, which will enable us to determine the other two angles.

Given $a = 2{\cdot}7402$, $b = {\cdot}7401$, $C = 59° 27' 5''$, solve the triangle completely, with the aid of the logarithm tables supplied.

11. The extremity of the shadow of a flagstaff 6 feet high, standing on the top of a regular pyramid on a square base, just reaches a side of the base and is distant 56 feet and 8 feet from the extremities of that side. If the height of the pyramid be 34 feet, find the sun's altitude.

[*Logarithm Tables to seven figures were supplied.*]

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FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

(Analytical Geometry, Conic Sections, and Solid Geometry.)

[*Full marks may be obtained for about* four-fifths *of this paper.*]

[*Great importance is attached to accuracy.*]

1. When are two non-intersecting straight lines said to be at right angles?

If the two edges AB, CD of a pyramid be at right angles, show that

$$AC^2+BD^2=AD^2+BC^2.$$

2. Prove that the plane angles which contain any solid angle are together less than four right angles.

3. Define a right circular cone, and show that if a plane be drawn through any two generating lines which intersect at an angle 2β, it will touch a fixed cone whose vertical angle $=2\cos^{-1}\{\cos\alpha\sec\beta\}$, 2α being the vertical angle of the given cone.

4. Show that any section of a cylinder by a plane not parallel or perpendicular to the axis is an ellipse.

If e_1, e_2 be the eccentricities of two sections at right angles and having a common minor axis, show that

$$e_1^2+e_2^2=1.$$

5. If a right cone be cut by a plane, find under what conditions the section will be a parabola, and give (without proof) a construction for determining the position of its focus.

6. The tangents drawn from any point on the directrix of a conic subtend right angles at the corresponding focus. Prove this, and show that if TP, TQ be the two tangents the conic will be a parabola, ellipse, or hyperbola, according as the angle PTQ is equal to, less or greater than, a right angle. Examine, with particular care, the case of the hyperbola.

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7. In an ellipse, if CD be a semi-diameter conjugate to CP, show that

$$CP^2+CD^2=AC^2+BC^2.$$

8. Investigate an expression for the area of a triangle in terms of the coordinates of its three angular points.

E, F, G, H are points in the four sides of a square $ABCD$, such that $AE=\frac{1}{4}AB$, $BF=\frac{1}{4}BC$, $CG=\frac{1}{4}CD$, $DH=\frac{1}{4}DA$. Show that the area of the quadrilateral $EFGH$ is equal to that of a square whose side $=\frac{3}{4}$ of the side of the given square.

9. Find the angle between the two straight lines whose equations are

$$y=m_1x+c_1$$

and

$$y=m_2x+c_2.$$

Show that $(2, -1)$ $(0, 2)$ $(3, 0)$ $(-1, 1)$ are the coordinates of the angular points of a parallelogram and find the angle between its diagonals.

10. Find the equation of a parabola in the form $y^2=4ax$, the vertex of the curve being the origin.

If PQ be a double ordinate of the parabola, find the locus of its points of trisection.

11. Investigate the equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, at the point whose coordinates are x', y'.

If CP, CD be conjugate semi-diameters, and $CP=\sqrt{ab}$, show that the normal at D touches the circle whose equation is

$$x^2+y^2=(a-b)^2.$$

12. What loci are represented by the equations

(i) $x\{(2x+y)^2-(x+a)(x-a)\}=0$;

(ii) $x^2+y^2+\frac{1}{8}y(x-1)-2x+1=0$;

(iii) $x=a\theta+b\theta^2$,
$y=b\theta+a\theta^2$;

where θ is a variable parameter?

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VI. PURE MATHEMATICS (2).

(Further Questions and Problems on the Subjects of the Qualifying Examination, and Theory of Equations.)

[*Full marks may be obtained for about* four-fifths *of this paper.*]

[*Great importance is attached to accuracy.*]

1. Solve the equations

$$\left.\begin{aligned} ax+cy+bz&=a \\ cx+by+az&=b \\ bx+ay+cz&=c \end{aligned}\right\}$$

and find x, y, z in their simplest forms.

2. Sum the series

(i) $1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \ldots\ldots$ to n terms.

(ii) $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \ldots\ldots$ to infinity.

3. Write down an expression for e^x arranged according to ascending powers of x, and thence deduce a series for $\log(1+x)$.

Obtain the first four terms in the expansion of $\log\left(1 - \frac{2x^2}{1+x}\right)$, according to ascending powers of x.

4. State the law for the formation of the successive convergents to a continued fraction.

Express the continued fraction $2 + \frac{1}{3+}\,\frac{1}{4+}\,\frac{1}{3+}\,\frac{1}{4+} \ldots\ldots$ as a quadratic surd.

5. Prove that the arithmetic mean of n positive quantities is greater than their geometric mean.

Show that $(bc+ca+ab)^2$ is not less than $3abc\,(a+b+c)$.

6. Find the number of permutations of n things taken r at a time.

How many different words of two consonants and one vowel can be formed from the letters in the word *Woolwich?*

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7. Show that the equations

$$x^3+x^2+x=-6 \text{ and } x^4-x^3+x^2+2x=6$$

have two imaginary roots in common, and solve the equations.

8. Solve the equations

(i) $x(x^2-4)=a(a^2-4)$.

(ii) $x^4+x^2+1=\frac{3}{2}x(x^2+1)$.

9. Show that the limit of $\frac{\sin\theta}{\theta}$ when θ is indefinitely diminished is unity.

Find the angles of an isosceles triangle whose equal sides are a and whose base $=a(1+\lambda)$ where λ is a very small fraction.

If $\lambda=\cdot022$, show without tables that the base angles $=59^\circ\cdot16'$ nearly.

10. Investigate an expression for the radius of a circle which touches one side of a triangle and the other two sides produced.

If r_1, r_2, r_3 be the radii of the three escribed circles, and

$$\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right)=2,$$

show that the triangle must be right-angled.

11. Prove Demoivre's Theorem in the case of a positive index, and if

$$2\cos\theta=x+\frac{1}{x}, \text{ show that } 2\cos n\theta=x^n+\frac{1}{x^n}.$$

12. The altitudes of a tower are observed at two points A, B, 120 feet apart, situated in a line with the foot of the tower, to be θ, 2θ, and at a point midway between them to be 45°. Show that the height of the tower $=109{\cdot}37$ feet nearly.

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VII. PURE MATHEMATICS (3).

(Differential and Integral Calculus.)

[*Full marks may be obtained for about* four-fifths *of this paper.*]

[*Great importance is attached to accuracy.*]

1. Differentiate the expressions

$$\tan^{-1}\frac{x}{\sqrt{1-x^2}}, \quad \log(x+\sqrt{x^2-a^2})+\sec^{-1}\frac{x}{a}.$$

2. If $\sin^{-1}y = m\sin^{-1}x$, prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0.$$

3. Expand $\tan^{-1}x$ in a series of ascending powers of x, and show that the expansion is inapplicable when $x > 1$. In this latter case show how to expand the function in an infinite series.

4. Explain the method of determining the maximum or minimum values of a function of a single variable; and find the maximum or minimum values of

$$\frac{2x+9}{x^2+4x+10}, \text{ and of } \frac{\log x}{x}.$$

5. Prove that, in the parabola, $y^2 = 4mx$, the length of the subnormal is constant; and in the logarithmic curve, $y = a^x$, the subtangent is constant.

6. Find the equation of the tangent at any point on the curve $x^m y^n = a^{m+n}$; and show that the portion of the tangent intercepted by the coordinate axes is divided in a constant ratio at the point of contact.

7. Find the length of the radius of curvature at any point on an ellipse.

8. If a curve be referred to polar coordinates, prove that at a point of inflexion we have

$$\frac{d^2\left(\frac{1}{r}\right)}{d\theta^2} + \frac{1}{r} = 0.$$

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9. Find the following integrals—

$$\int \frac{dx}{\sqrt{1+x-x^2}}, \quad \int \sin^3 x\,dx, \quad \int x \log x\,dx.$$

10. Evaluate the definite integrals—

$$\int_0^{\frac{\pi}{2}} \sin^2 x\,dx, \quad \int_0^1 \frac{dx}{\sqrt{x-x^2}}, \quad \int_0^{\pi} \frac{dx}{5+4\cos x}.$$

11. Find the whole area of *either* of the curves—

(1) $a^2y^2 = x^3(2a-x)$.

(2) $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

12. Determine the length of any arc of a parabola measured from its vertex.

VIII. STATICS.

1. Assuming that the resultant of two equal forces bisects the angle between them, prove that the resultant of two forces, one of which is double the other, acts in the diagonal of the parallelogram of which they are sides.

2. A straight uniform lever, weighing 10 lbs., rests on a fulcrum one-third of its length from one end; it is loaded with a weight of 4 lbs. at that end; find what vertical force must act at the other end to keep the lever at rest.

3. Four forces act along the sides of the rectangle $ABCD$, and are measured by those sides; the first three, AB, BC, CD, act in a contrary sense to the fourth, AD. Find their resultant and its line of action.

4. Three forces, PA, PB, PC, diverge from the point P; and three others, AQ, BQ, CQ, converge to the point Q. Show that the resultant of the six is represented in magnitude and direction by $3PQ$, and passes through the centre of gravity of the triangle ABC.

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5. It is required to decompose a force whose magnitude and line of action are given into two equal forces passing through two given points. Give a geometrical construction for solving the problem (1) when the two points are on the same side of the line; (2) on opposite sides.

6. Two uniform rods, AB, BC, jointed together at B, are placed in a horizontal line, ABC, and supported by two props, one at A, the other between B and C. Determine the position of the latter, given the lengths and weights of the two rods.

7. A weight is supported on a smooth inclined plane by a force acting along the plane. Find the force, also the pressure on the plane.

If the plane be rough, prove that this pressure is unaltered; and explain how far the force may be determined.

8. Three given weights are rigidly connected together; prove that the resultant weight always passes through one point, fixed as regards the weights, however the system be displaced in the plane of the weights. Does this hold if the system be displaced out of its original plane?

9. Two weights, joined by a string passing over a pulley, rest on two smooth inclined planes of the same height, and whose highest points coincide at the pulley. Apply the principle of *virtual velocities* to show that the weights are as the lengths of the planes, and that, however they are displaced, their centre of gravity moves horizontally.

Determine by the same principle whether these theorems hold when the pulley is not at the vertex of the planes.

10. Four weightless rods are freely jointed together, forming a quadrilateral $ABCD$. If two equal and opposite forces are applied at two points, one on the rod AB, one on the opposite rod CD, determine the conditions that they shall balance each other.

If, instead of forces, two equal and opposite *couples* be applied to the rods AB, CD, what is necessary for equilibrium?

State generally in each case what will happen if the conditions are not satisfied.

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IX. DYNAMICS.

[*The measure of the acceleration of gravity may be taken to be* 32 *when a foot and a second are units of length and time.*]

[*Great importance will be attached to accuracy.*]

1. Define uniform velocity and uniform acceleration, and state how each is measured.

2. A body whose weight is W lies on a smooth table. What horizontal force applied to it will cause it to move with unit acceleration, a foot and a second being the units of space and time?

3. From a train moving with velocity V, a carriage on a road parallel to the line, at a distance d from it, is observed to move so as to appear always in a line with a more distant fixed object, whose least distance from the railway is D. Find the velocity of the carriage.

4. A particle moving from rest with (unknown) uniform acceleration acquires a velocity v in the time t; find the space described.

5. A particle is projected vertically upwards with a velocity u; find the height it will reach.

Show that the velocities with which it passes through any assigned point, in its ascent and fall, are equal.

6. Find the conditions that two given non-elastic balls, on striking each other, shall both be reduced to rest. If they are perfectly elastic, what will happen after the collision?

Show this directly, without employing general formulas.

7. State (and prove) with what acceleration a particle will fall down a smooth inclined plane. State also (without proof) the acceleration, if the plane be rough: given μ the coefficient of friction.

A heavy slab whose under surface is rough, but the upper smooth, slides down a given inclined plane. Find the acceleration with which a small particle laid on its upper surface will move along the slab.

8. If a train ascends a gradient of 1 in 40 by its own momentum for a distance of 1 mile, the resistance from friction, &c., being 10 lbs. per ton, find its initial velocity.

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9. Find the greatest range which a projectile with an initial velocity of 1,600 ft. per second can attain on a horizontal plane.

Show also that for a small difference of elevation, not exceeding 10′, whether of excess or defect, the range attained will fall short of the maximum by less than $1\frac{1}{2}$ feet.

10. A body of mass m is placed at a distance R from a centre of force O, which attracts it with a force P. Find the velocity and direction with which it must be started so as to describe a circle round O.

If the force of attraction, for different distances, varies inversely as the squares of the distances, and for different bodies, directly as their masses, prove that if several bodies move round O in concentric circles, the squares of the times of revolution are as the cubes of the radii.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1888.

PRELIMINARY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Define a *right angle*, and a *rhombus*.

If, at a point in a straight line, two straight lines on opposite sides of it make the adjacent angles together equal to two right angles, they are in the same straight line.

AOB, COD are two intersecting straight lines, and each of the figures $AOCE$, $BODF$ is a rhombus. Show that the straight line EF passes through O, and that AC, BD are parallel.

2. The opposite sides and angles of a parallelogram are equal, and either diagonal bisects it.

Straight lines drawn through A, C, the extremities of one diagonal of a parallelogram $ABCD$, respectively perpendicular and parallel to the other diagonal, intersect in E. Prove that $BE = CD$.

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3. If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts is equal to the rectangle contained by the two parts, together with the square on the aforesaid part.

The diagonal AC of a square $ABCD$ is produced to E, so that $CE=BC$. Prove that the square on BE is equal to the rectangle under AC, AE.

4. Describe a square equal to a given rectangle.

5. Explain fully how you establish the truth of the proposition that "the straight line drawn at right angles to a diameter of a circle at its extremity touches the circle."

Describe a circle passing through a given point, and also touching a given circle in a given point.

6. The angle in a semicircle is a right angle; the angle in a segment greater than a semicircle is acute; the angle in a segment less than a semicircle is obtuse.

The circle described with centre A and radius AB, cuts the circle circumscribing the rectangle $ABCD$ in the point E. Show that CE is equal to AD, and that DE is parallel to AC.

7. From a given circle, cut off a segment which shall contain an angle equal to a given angle.

8. Describe a circle about a given equilateral and equiangular pentagon.

9. If an angle of a triangle be bisected by a straight line which cuts the base, the segments of the base have the same ratio which the sides of the triangle have to one another.

Prove that the radius of the circle inscribed in any isosceles triangle has the same ratio to the perpendicular altitude as the base of the triangle has to its perimeter.

10. On a given straight line construct a quadrilateral figure similar and similarly described to a given quadrilateral figure.

11. One of the sides containing the right angle of a triangle is double of the other, and circles are described having these two sides as diameters. Show that the length of their common chord is two-fifths of the hypotenuse of the triangle.

12. In equal circles, angles, whether at the centres or at the circumferences, have the same ratio to one another as the arcs on which they stand.

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II. ALGEBRA.

(Up to and including the Binomial Theorem; the theory and use of Logarithms.)

1. Simplify the expression

$$\frac{\{(a+b)(a+b+c)+c^2\}\{(a+b)^2-c^2\}}{\{(a+b)^3-c^3\}\{(a+b+c)\}}.$$

2. Show that $x^3+y^3+z^3-3xyz$ is divisible exactly by $x+y+z$; and hence, or otherwise, show that

$$(b-c)^3+(c-a)^3+(a-b)^3=3(b-c)(c-a)(a-b).$$

3. Find the highest common factor of $16x^4+36x^2+81$ and $8x^3+27$; and write down the least common multiple of $8x^3+27$, $16x^4+36x^2+81$, and $6x^2-5x-6$.

4. Extract the square root of

$$4x^2a^{-2}+16x^{-2}a^2-12\frac{x}{a}-24\frac{a}{x}+25.$$

5. Determine what relation must hold between a, b, and c in order that the roots of the equation $ax^2+bx+c=0$ may be real and different.

If a, b, c, x are all real quantities in the equation

$$(a^2+b^2)x^2-2b(a+c)x+b^2+c^2=0;$$

prove that a, b, c are in G.P., and that x is their common ratio.

6. Solve the equations

(1) $\frac{1}{2}(x-2)-\frac{x-5}{9}+5\frac{x-1}{6}=0;$

(2) $\frac{b}{x-a}+\frac{a}{x-b}-2=0;$

(3) $\left.\begin{array}{l} x+y+z=19 \\ x^2+y^2+z^2=133 \end{array}\right\}$ having given that y is a mean proportional to x and z.

7. Two casks, A and B, are filled with two kinds of sherry, mixed in cask A in the ratio 2 : 7, and in cask B in the ratio 1 : 5; what quantity must be taken from each to form a mixture which shall consist of 2 galls. of the first kind and 9 galls. of the second?

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8. Prove the formula for the sum of n terms of an Arithmetic Progression whose first and last terms are given.

If s_1, s_2, s_3, &c., are the sums of m Arithmetic series, each to n terms, the first terms being 1, 2, 3, &c., respectively, and the differences 1, 3, 5, &c., respectively, show that

$$s_1+s_2+s_3+\ldots s_m=\tfrac{1}{2}mn\,(mn+1).$$

9. The number of combinations of n letters, taken 5 together, in which a, b, c occur is 21. Find the number of combinations of them, taken 6 together, in which a, b, c, d occur.

10. Write down the two middle terms in the expansion of

$$\left(x+\frac{1}{x}\right)^{2m+1};$$

and show that, if x be a small fraction,

$$\frac{\sqrt{1+x}+\sqrt[3]{(1-x)^2}}{1+x+\sqrt{1+x}}$$

is very nearly equal to $1-\frac{5}{6}x$.

11. Prove that the logarithm of the quotient of two numbers is the difference of the logarithms of the numbers.

If the logarithms of a, b, c be respectively p, q, r, prove that $a^{q-r}\,.\,b^{r-p}\,.\,c^{p-q}=1$.

Prove that $\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}=\log 2$.

12. If the number of persons born in any year equals $\frac{1}{45}$th of the whole population at the beginning of the year, and the number who die equals $\frac{1}{60}$th of it, find in how many years the population will be doubled. [Use the logarithm tables supplied.]

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III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy. Questions* 1, 8, 9, 10, 11 *may be worked with the aid of the logarithm tables supplied.*]

1. Prove that, to turn circular measure into seconds, we must multiply by 206265; and, to turn seconds into circular measure, we must multiply by ·000004848, approximately. [$\pi = 3{\cdot}14159265\ldots$]

Determine the circumference and the diameter of the Earth in geographical miles [60 to a degree of latitude], each degree subtending 1° at the centre of the Earth.

2. Define the *sine*, *cosine*, *tangent*, *cotangent*, *secant*, *cosecant*, and *versed sine* of an angle, illustrating by reference to a figure; and express them all in terms of the secant.

Construct a table showing their values for angles of 0°, 30°, 45°, 60°, 90°, 120°, 180°.

3. Prove that

(i) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ (by geometrical construction);

(ii) $\tan\alpha - \tan\beta = \sin(\alpha-\beta)\sec\alpha\sec\beta$;

(iii) $$\sec\theta + \tan\theta = \sqrt{\left(\frac{1+\sin\theta}{1-\sin\theta}\right)} = \tan\left(\tfrac{1}{4}\pi + \tfrac{1}{2}\theta\right).$$

4. Determine, in circular measure, all the angles given by the equation

$$\cos\theta - \sin\theta = \sqrt{2}.$$

Prove that, if $$\cos\theta = \tan\theta,$$

then $$\sin\theta = 2\sin\frac{\pi}{10}.$$

5. Construct a table expressing the *inverse* trigonometrical functions in terms of each other.

Prove that

$$\cos^{-1}\sqrt{\frac{a-x}{a-b}} = \sin^{-1}\sqrt{\frac{x-b}{a-b}} = \cot^{-1}\sqrt{\frac{a-x}{x-b}}$$
$$= \frac{1}{2}\sin^{-1}\frac{\sqrt{\{(a-x)(x-b)\}}}{\frac{1}{2}(a-b)}.$$

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6. Prove that, if

$$\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha,$$

then

$$\frac{x^2}{a^2}-2\frac{xy}{ab}\cos\alpha+\frac{y^2}{b^2}=(\sin\alpha)^2.$$

7. Prove that, in every triangle, $a \operatorname{cosec} A = b \operatorname{cosec} B = c \operatorname{cosec} C =$ the diameter of the circle circumscribing the triangle.

In determining the distance of a far object C, a base AB is measured, and the angles ABC, BAC are observed, each angle being very nearly a right angle. Prove that the distance may be taken to be

$$AB \operatorname{cosec}(180^\circ - ABC - BAC).$$

8. Given the sides a, b, c of a triangle, write down expressions for determining the angles and the area of the triangle, and the radii of the circles touching its sides.

Given, in feet, $a=10$, $b=24$, $c=26$, determine the angles, in degrees and minutes, and the area of the triangle, in square feet.

9. From a boat the angles of elevation of the highest and lowest points of a flagstaff, 30 feet high, on the edge of a cliff are observed to be $46^\circ\ 12'$ and $44^\circ\ 13'$; determine the height of the cliff and its distance.

10. Determine the diameter of a cylindrical gasholder to contain 10 million cubic feet of gas, supposing the height to be made equal to the diameter; and determine in tons the weight of iron plate, weighing $2\frac{1}{2}$ pounds per square foot, required in the construction of the gasholder, supposing it open at the bottom, and closed by a flat top.

11. Determine the number of cubic yards in a bank of earth on a horizontal rectangular base 60 feet long and 20 feet broad, the four sides of the bank sloping up to a ridge at an angle of 40° to the horizon.

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IV. STATICS AND DYNAMICS.

1. Show how to place three forces, represented by the numbers 3, 4, 5, at a point so as to balance each other.

Would it be possible for the forces 2, 3, 5 to make equilibrium?

2. Three equal forces, P, diverge from a point, the middle one being inclined at an angle of 60° to each of the others. Find the resultant of the three.

3. A uniform lever, weighing 30 lbs. and 16 feet long, is loaded at either end with weights of 20 and 30 lbs. respectively. Find where the fulcrum must be placed for equilibrium.

4. Prove that the moments of any two non-parallel forces, about any point on their resultant, are equal and opposite.

If two forces form a couple, show that the algebraical sum of their moments about any point in the plane is constant.

5. Show how a weight of 2 lbs. can be balanced by a weight of 1 lb., by means of pulleys; and show that the former will rise through half the space which the latter descends.

6. State, and prove, what power acting along a smooth inclined plane is required to support a weight resting on the plane.

A heavy string is placed with part of it resting on a given inclined plane, the remaining part hanging vertically over a small pulley at the top of the plane. Find the point of the string which must be placed over the pulley, for equilibrium.

7. Explain, and prove, the Parallelogram of Velocities.

Decompose a given velocity, v, into two components whose directions are each inclined at 30° to its own.

8. What is meant in Dynamics by the *Mass* of a body? What result of observation proves that the Weights of all bodies are proportional to their Masses?

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9. Explain clearly what is meant by saying that $g=32$, where g is the acceleration due to gravity.

If the unit of time were altered to 2 sec., what would be the value of g; and why?

10. In what time will a body fall from rest through 100 ft.? If it be retarded in its fall by the tension of a string attached to it, so as to occupy 5 sec. in the fall, what is the tension of the string, the weight being given?

11. A body falls freely through 100 ft. from rest, with what velocity will it reach the ground?

If, instead of falling from rest, it be projected downwards so as to reach the ground with twice the former velocity, find the initial velocity.

12. A ball is rolled across a smooth table, and leaves it with a given horizontal velocity. What curve does it describe during its fall?

If v be the initial velocity, and h the height of the table, find the horizontal distance from the table of the point where the ball reaches the floor.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[*Full marks may be obtained for about* four-fifths *of this paper.*]

[*Great importance is attached to accuracy.*]

1. Show that, if from the focus of a parabola a perpendicular be drawn to any tangent, the point of intersection lies on the tangent at the vertex.

Having given the extremities of a focal chord of a parabola, and also the focus, find the position of the vertex.

2. Prove that the area of the parallelogram of which two semi-diameters of an ellipse are adjacent sides is invariable, if the diameters be conjugate.

Show that the sum of the lengths of two conjugate diameters of an ellipse is never less than the sum of the lengths of the principal axes.

3. Define a hyperbola by means of the relation between the distances of any point upon it from a given point and a given line, and deduce from the definition the property that the difference between the focal distances of a point on the curve is invariable.

Two circles in the same plane are external to each other. Show that the locus of the centre of a circle touching them externally is a hyperbola, and determine the position of the asymptotes.

4. Prove that the focal radii of a point of a central conic section make equal angles with the tangent at that point.

Show that an ellipse and a hyperbola which have the same foci cut one another at right angles.

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5. Establish the focus and directrix property of the section by a plane of a right circular cone.

Determine the eccentricity (1) of a circle; (2) of a pair of parallel straight lines.

6. Find the rectangular coordinates of a point dividing in a given ratio the straight line joining two points with given coordinates.

Examine the case when the given ratio $= -1$.

7. The equations to two straight lines being given, find the equation to a straight line joining a given point to the point of intersection of the straight lines.

Find the equation to the straight line passing through the origin and the intersection of the straight lines given by

$$\frac{x}{a}+\frac{y}{b}=1, \quad \frac{x}{b}+\frac{y}{a}=1.$$

8. Show how to determine the points where the straight line represented by

$$y=mx+b$$

cuts the locus of the equation

$$(x-\alpha)^2+(y-\beta)^2=k^2.$$

When will the straight line touch the locus?

9. Find the equation to the normal to the parabola given by $y^2=4ax$ at the point whose coordinates are (x', y'). Show that not more than three normals can be drawn to a parabola from a point in its plane.

10. Show how to determine the pair of diameters common to the circle given by $x^2+y^2=k^2$ and the locus of $ax^2+by^2+2hxy+c=0$. Under what conditions will the diameters become coincident?

11. Determine the locus of the intersection of the straight lines given by

$$\frac{tx}{a}-\frac{y}{b}+t=0, \quad \frac{x}{a}+\frac{ty}{b}-1=0,$$

t being a variable parameter.

What connection is there between the angle $2\tan^{-1}t$ and the point on the locus where the two straight lines intersect?

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12. Discuss and trace the loci of the following equations:—

(1) $(3x-y)^2=6x-2y$;

(2) $\frac{a}{x}+\frac{b}{y}=1$;

(3) $x=a\cos(\theta+\alpha)$, $y=b\cos(\theta+\beta)$, where θ is a variable parameter.

VI. PURE MATHEMATICS (2).

[*Full marks may be obtained for about* four-fifths *of this paper.*]

[*Great importance is attached to accuracy.*]

1. Find x in its simplest form from the equation

$$x^4-3x^2+1=0.$$

2. Prove that the greatest and least values of

$$\frac{6x^2-22x+21}{5x^2-18x+17}$$

are $\frac{5}{4}$ and 1, corresponding to the values 1 and 2 of x.

3. Find x, y, and z from the equations

$$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1,$$

$$\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=1,$$

$$\frac{x}{4}+\frac{y}{5}+\frac{z}{6}=1.$$

Exhibit the solution also by means of determinants.

4. Prove that, if A varies as B^m when C is constant, and if A varies as C^n when B is constant, then will A vary as B^mC^n when both B and C vary. Determine the resistance of the air to a projectile 16 inches in

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diameter at a velocity of 1500 feet per second, supposing the resistance of the air varies as the square of the diameter and the cube of the velocity; given that the resistance of the air to a projectile 1 inch in diameter at a velocity of 1900 feet per second is $2\frac{1}{2}$ pounds' weight.

5. Prove that the equations

$$x+y+z=100,$$

$$5x+y+\frac{z}{20}=100,$$

have only one solution in positive integers; and determine this solution.

A man bought 100 animals, consisting of oxen, sheep, and rabbits, for £100; the oxen costing £5, the sheep £1, and the rabbits 1*s.* each; how many of each did he buy?

6. Resolve into partial fractions the expression

$$\frac{x^4}{(x-1)(x-2)(x-3)};$$

and then expand the fractions in positive powers of x, writing down the coefficient of x^n.

7. Explain how the theorems

(i) $a^x \times a^y = a^{x+y}$,

(ii) $a^x \div a^y = a^{x-y}$,

(iii) $(a^x)^r = a^{xr}$,

(iv) $\sqrt[r]{a^x} = a^{x \div r}$,

are established for *all* values of x, y, and r.

Examine the truth of the following:—

(v) $e^{\frac{1}{r}} = \sqrt[r]{e} = e$,

(vi) $\log[\log\{\log e^{e^e}\}] = 1$,

the logarithms being to the base e.

8. Determine x in degrees and minutes from the equation

$$6\cos x + 8\sin x = 9.$$

Give also a graphical method of solution, the sketch being made freehand.

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9. Prove that

$$\theta - n\pi = \tan\theta - \tfrac{1}{3}(\tan\theta)^3 + \tfrac{1}{5}(\tan\theta)^5 - \ldots;$$

and determine the value which must be given to n when θ lies between

$$\frac{11\pi}{4} \text{ and } \frac{13\pi}{4}.$$

10. Prove that, when resolved into real and imaginary parts,

$$\cos(u+\iota v) = \cos u \cosh v - \iota \sin u \sinh v,$$

where $\cosh v$ denotes $\frac{1}{2}(e^v + e^{-v})$, $\sinh v$ denotes $\frac{1}{2}(e^v - e^{-v})$, and $\iota = \sqrt{(-1)}$.

Prove also that

$$\cos \iota v = \cosh v, \qquad \sin \iota v = \iota \sinh v.$$

11. Prove the trigonometrical summations

$$\sum_{r=0}^{r=n} \cos ra = \cos \tfrac{1}{2}na \sin \tfrac{1}{2}(n+1)a \operatorname{cosec} \tfrac{1}{2}a;$$

$$\sum_{r=0}^{r=n} \sin ra = \sin \tfrac{1}{2}na \sin \tfrac{1}{2}(n+1)a \operatorname{cosec} \tfrac{1}{2}a.$$

VII. STATICS AND DYNAMICS.

[*Mathematical instruments must be used for questions requiring a graphical method of solution.*]

1. Find the locus of all points in a plane, such that two forces given in magnitude and position shall have equal moments, and in the same sense, round any one of the points.

2. AE, DF are vertical walls, AB, BC, CD are tight strings supporting two weights. If the weight at B is one pound, find to two places of decimals the weight X at C; and also the tensions of the three strings. $AB = 4\frac{1}{2}$, $BC = 4\frac{5}{12}$, $CD = 6\frac{10}{12}$, $\angle BCD = 90°$.

(This problem had to be solved by graphical construction on a lithographed plan.)

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3. Three forces, 5·05 lbs., 4·24 lbs., 3·85 lbs. in magnitude, act at three given points of a flat disk resting on a smooth table. Place the forces, by geometrical construction, so as to keep the disk in equilibrium; and measure and write down the number of degrees contained in each of the three angles they make with one another.

4. A vertical beam, of weight W, is constrained by guides so as to move only in its own direction, the lower end resting on a smooth floor. If a smooth inclined plane of given slope be pushed under it by a horizontal force acting at the back of the inclined plane, so as to raise the beam, find the force required.

If there is friction between the floor and the inclined plane, but none elsewhere, what must be the least value of μ, if the inclined plane will remain, when left in a given position under the beam, without being forced out; given the weight of the inclined plane?

When there is no friction, verify in this case that what is gained in power is lost in velocity.

5. State the analytical conditions necessary for the equilibrium of any number of forces in a plane, given the forces and the coordinates of their points of application.

Show that these equations are unaltered if the point of application of any force be changed to any other point on its line of action.

6. A particle is projected along a rough horizontal plane with a given velocity; what is its acceleration? If at each point P of its path a vertical PH be drawn, such that its velocity at P is that due to a fall through the height PH, find the locus of H.

Hence, deduce a simple geometrical construction for determining the point at which it will stop.

7. A weight P is descending, drawing upwards a smaller weight Q by means of a string passing over a smooth pulley; if, when they have each moved through a space h from rest, the string is suddenly cut, find by means of the principle of Work, or otherwise, the additional height to which Q will ascend.

8. How is *angular velocity* usually measured? Find the angular velocity of the Earth round its axis.

If a line turns round its extremity O, which is a fixed point within a given circle, with uniform angular velocity, show that the velocity with

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which its intersection P with the circle travels along the circumference varies as PH, which is a line drawn from P through the centre, to meet OH, a perpendicular to OP.

9. Find the equation of the path of a projectile, referred to horizontal and vertical axes.

Show that all projectiles the horizontal component of whose initial velocities is the same describe equal parabolas.

Prove that it is impossible for two projectiles discharged together from the same point to meet each other in their flight.

10. A sphere at rest is struck by an equal sphere moving with given velocity in a line inclined at 30° to the line joining the centres at the instant of contact. Find the motions of both after impact; the elasticity being perfect.

Determine also the motions of their common centre of gravity before and after impact.

11. An elastic string is gradually stretched by an increasing force. When the string has been lengthened 2 inches, the force is found to amount to 10 lbs. What is the work in foot-pounds done by the force, assuming that the force is always proportional to the elongation produced?

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ANSWERS.

July, 1880.

II. ARITHMETIC. 1. $13\frac{183}{264}$. 2. $2\frac{1711}{2278}$. 3. $21\frac{83}{84}$. 4. $6\frac{57}{55}$. 5. 45·168*d*. 6. ·3240643 42. 7. ·0346. 8. 7850. 9. ·28125. 10. $\frac{677}{6750}$. 11. £39. 8*s*. 3*d*. 12. £7; £11. 13*s*. 4*d*.; £16. 6*s*. 8*d*.; £21. 13. $11\frac{1}{2}$ years. 14. £4. 1*s*. 0*d*. 15. 200 : 243. 16. 252; 1081080. 17. 10 ft. 9 in.; 99846. 18. $18\frac{2}{3}$. 19. £715. 20. £7. 1*s*. 9*d*. 21. £6768. 7*s*. $10\frac{1}{2}$*d*. 22. 1·0034606; 9·076226.

III. ALGEBRA. 1. $(a-1)x+a$. 2. 0. 3. $\frac{12}{a^8-5a^4+4}$; $\frac{2a}{\sqrt{x+a}}$. 4. $x+4-8x^{-1}$. 5. $\frac{b^2-2ac}{a^2c^2}$; $x^2+10x-24=0$. 6. (i) $x=7$; (ii) $x=4, -5, \frac{1}{2}\{-1\pm\sqrt{21}\}$; (iii) $x=9, y=4$. 7. 300. 8. $\frac{n}{21}(7n+17)$. 9. $39\frac{117}{256}$; 48; -1 and 1. 11. 10. 13. $1+6x+30x^2+140x^3+630x^4+\ldots$

IV. TRIGONOMETRY. 1. 20 ft.; 2600 grades; $173\frac{1}{3}$ grades. 5. 57·2957795... degrees; ·0002908882...; 3·1416. 8. 115·47 ft.; 100 ft. 9. Read L tan $19^0\,28'\,50''=9{\cdot}5486864$; $74^0\,13'\,50''$; $35^0\,16'\,10''$. 10. 114·412... ft. 11. $-2{\cdot}30103=\bar{3}{\cdot}69897$; $10{\cdot}1030286$; $38^0\,16'\,15''$.

V. PURE MATH. (1). 5. $-3, \frac{3\pm\sqrt{-7}}{2}$.

8. (1) $\frac{1}{2}\sqrt{a^2+d^2}=\frac{1}{2}\sqrt{b^2+c^2}$. (2) $\frac{ac+bd}{\sqrt{a^2+d^2}}$. (3) $\frac{1}{2}(ad+bc)$.

9. $r=2a\sin\theta$.

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VI. PURE MATH. (2). 1. x^5-2x^4. 3. 1·146128, 1·991226, $\bar{1}$·727966. 4. 75, $1\frac{1}{5}$. 5. 25.
8. $-1+5x-7x^2+\ldots+\{1+(-2)^{n+1}\}x^n+\ldots$; $1+2x+x^2+x^4+\ldots$
$+x^{2n}+\{1+(-1)^n\}x^{2n+1}+\ldots$ 9. ·798 ft.

VII. PURE MATH. (3). 1. $-3, 7, 9$. 4. (1) $-\dfrac{1+x}{(1+x^2)^{\frac{3}{2}}}$.

(2) $x^{c-1}\log_a \dfrac{x^c}{e}$. (3) $\dfrac{c\sqrt{e^{-cx}}}{2(1-e^{cx})}$. 6. $m-\frac{4}{3}m^3$.

7. $x^2-\frac{2}{3}x^4+\frac{61}{120}x^6-\frac{2213}{5040}x^8+\frac{702293}{1814400}x^{10}$.

8. $\dfrac{l^2\cot\frac{\alpha}{2}}{1-\frac{e^2}{2}}$, $\dfrac{l^2\sin\alpha}{2\left(1-e\cos\frac{\alpha}{2}\right)^2}$. 10. $y+x=a$.

11. [Read $=a\sin\theta\cos\theta$] $(y^2+x^2)^3=\dfrac{a^2}{2}(y^2-x^2)^2$.

12. (1) $\frac{1}{2}\log\dfrac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}$. (2) $\theta\cos^{-1}\theta-\sqrt{1-\theta^2}$.

(3) $\frac{1}{4}\log\dfrac{(1+x)^2}{1+x^2}+\frac{1}{2}\tan^{-1}x$. 13. ∞.

VIII. STATICS. 1. $\sqrt{7}:2\sqrt{3}$. 3. 40 lbs. 10. 6·2.

IX. DYNAMICS. 1. 22, 30 miles per hr. 2. $2\frac{1}{12}$ f. s., 3 ft., $1\frac{23}{25}$ secs. 10. $Wv^2:W'v'^2$. 11. $\sqrt{2ag}$, $\sqrt{2ag(1+\sin\alpha)}$.
14. 518400 : 516961.

November, 1880.

II. ARITHMETIC. 1. $6\frac{29}{90}$. 2. $6\frac{89}{154}$. 3. $4\frac{2}{3}$. 4. $\frac{7}{9}$.
5. 5·4198 oz. 6. ·0165576736. 7. 1850. 8. ·00545.
9. ·1375. 10. $\frac{56}{125}$ dwt. 11. £2231. 9*s*. $0\frac{3}{4}$*d*. 12. £2338. 2*s*. 0*d*.
13. 139 c. ft. 519 in. 14. $5\frac{1}{3}$ days. 15. 7*s*. 7*d*. 16. 1·0349; 543. 17. £5. 3*s*. 6*d*. 18. $5\frac{3}{5}$ miles. 19. $113\frac{1}{623}$ grs. = 113·0016...
20. 51^0. 21. 46·58847. 22. $\bar{3}$·0625820; 178·141516.

III. ALGEBRA. 1. 94. 2. $2ax-(3b-4c)y$; $2a^6$, $-6a^5$; 0.
3. a^2+xy; $(a^2+xy)(2x+3y)(2x-3y)$. 4. $\dfrac{3n^2}{(3m+2n)(3m+n)(3m-n)}$.

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5. $\frac{x(x+1)}{x^2+4x+1}$. 6. $2x-\frac{1}{2x}$; $\sqrt{2x-3}+\sqrt{x+2}$. 7. $b^2=4ac$; $b^2=ac$.

8. (1) $\frac{p}{4}$ and $\frac{3p}{4}$; (2) $x=8\frac{4}{5}$, $y=-11$; (3) $x=2, =-3, =-2\pm\sqrt{2}$. $y=3, =3, =-1\pm\sqrt{2}$.

9. Up to 14 lbs. 3*d.*; every additional 7 lbs. 2*d.* 11. 0. 13. 1120; 60.

14. $1-2x-x^2-\frac{4}{3}x^3-\frac{7}{3}x^4-\ldots$; $-\frac{2\cdot1\cdot4\cdot7\ldots(3r-5)}{\lfloor r}x^r$.

IV. TRIGONOMETRY. 1. $58\frac{52}{281}$ degrees; $18\frac{279}{281}$ degrees.

8. 30^0, 60^0, 90^0; $\frac{\sqrt{3}}{2}$, $\sqrt{3}$. 9. 6·038932; $\bar{1}$·3979400; 9·6989700; 10·3010300. 10. $C=90^0$. Third side of obtuse-angled triangle is 60·38932.

V. PURE MATH. (1). 2. 100·7639 cu. inches. 5. $4\frac{2}{5}$.

6. $7\frac{1}{2}$, 1·0767. 7. 5; -3, 2; $\sqrt{24}$.

VI. PURE MATH. (2). 2. 1·5726. 3. $7\pm4\sqrt{3}$. 4. 6 cwt.

6. 80·52524 sq. in., ·1986 lbs. 9. ·268 in. 10. $\frac{2\sqrt{2}}{3}$, $\frac{1}{3}$, $2\sqrt{2}$; $\sqrt{\frac{2}{3}}$, $\frac{1}{\sqrt{3}}$, $\sqrt{2}$. 12. $3\sqrt{5}$, $6\sqrt{2}$, 9; $\tan^{-1}2$, $\tan^{-1}3$.

VII. PURE MATH. (3). 5. (1) $-\frac{3x+x^3}{(1+x^2)^{\frac{3}{2}}}$. (2) $\frac{2}{\sqrt{1-x^2}}$.

(3) $x\{1+\tan(\log x)\}^2+2x$. 6. $5+4\tan(\log x)+3\tan^2(\log x)+5\tan^3(\log x)$.

7. 2. 8. $h-2a$, $2\sqrt{a(h-2a)}$. 10. $y+2a=0$, $x-a=0$, $y-x-3a=0$, $y+x-a=0$. 11. Two straight lines.

12. (1) $\frac{1}{2}\left(x-\frac{a}{2}\right)\sqrt{ax-x^2}+\frac{a^2}{8}\sin^{-1}\frac{2x-a}{a}$.

(2) $\frac{\sin x}{2}(1+\tan^2 x)+\frac{1}{4}\log\frac{1+\sin x}{1-\sin x}$.

(3) $\frac{1}{a^2+b^2+c^2-2ab}\left\{\log(x-b)-\frac{1}{2}\log\left(\overline{x-a}^2+c^2\right)+\frac{a-b}{c}\tan^{-1}\frac{x-a}{c}\right\}$.

13. a^2.

VIII. STATICS. 4. $\frac{49}{60}h$. from vertex. 5. $\frac{1}{4}$ length of beam.

7. $\frac{\sin\alpha}{\sin\beta-\sin\alpha}$ tons. 9. 30. 10. $\frac{1}{3}$ length of rod, 3. weight.

11. 47040, 448 lbs., 105 ft.

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IX. DYNAMICS. 1. 60, 10 hrs. 25′, $2166\frac{2}{3}$, $83\frac{1}{3}$. 2. Same as its weight. 3. $\frac{4V^2}{3g}$. 4. It makes angle $\tan^{-1}\frac{675}{503}$ with ship's length. 7. $4\sqrt{l}$. 9. $\frac{1}{3}$, 2 : 1. 11. 48.

June, 1881.

II. ARITHMETIC. 1. 5. 2. 1. 3. $1\frac{1}{3}$. 4. $\frac{3}{4}$. 5. 11·0112. 6. 3·6737066634. 7. ·00013. 8. 2501. 9. ·0416. 10. 6208·9238 oz. 11. 1 lb. 2 oz. 12 dwt. 2 gr. 12. 32 m.; 42 m. 13. £307. 2*s*. 6*d*. 14. £3520. 15. 10*s*. $11\frac{1}{4}$*d*. 16. 39·15; 3·2396... 17. 2 . 3 . 5 . 7 . 11; $2^3 . 3^2 . 7 . 13$; 3 . 5 . 811; $2^3 . 3^2 . 5 . 7 . 11 . 13 . 811 = 292{,}251{,}960$. 18. 105 c. ft. $2' 4'' 3''' 6^{iv} 2^{v}$ = 105 c. ft. 339·5138 c. in. 19. 30,000. 20. 9*s*. $4\frac{1}{2}$*d*., 5*s*. $7\frac{1}{2}$*d*., 1*s*. $10\frac{1}{2}$*d*.; $2\frac{1}{2}$*d*., $1\frac{1}{2}$*d*., 1*d*. 21. 7 hours. 22. $\bar{4}$·8864907, 2·7203018, ·0535353. 23. £11434·105.

III. ALGEBRA. 1. $(2a-3b+4c)(2a+3b-4c) = 4a^2 - 9b^2 + 24bc - 16c^2$.

2. $a^{\frac{1}{2}} - 2a^{\frac{1}{4}}x^{\frac{1}{4}} + x^{\frac{1}{2}}$; 18. 3. x^2+x+1; $2a(4a^2-b^2)(4a^2-9b^2)(4a-3b)$.

5. $\dfrac{4}{(3x+2)(3x-2)(3x-1)}$; $\dfrac{x^2-3x+1}{x^2-4x+1}$.

6. $3\sqrt{5} - 2\sqrt{3}$; $1 - 2a + 3a^2$. 7. $x = \dfrac{a^2+b^2}{a+b}$; $\left.\begin{matrix} x = & 2\frac{1}{2} \\ y = & -1\frac{1}{6} \end{matrix}\right\}$ and $\left.\begin{matrix} -1\frac{3}{4} \\ 1\frac{2}{3} \end{matrix}\right\}$.

9. $x^2 - \dfrac{p^2+4q}{2p}x + q = 0$. 10. 6, 3, $1\frac{1}{2}$. 11. 20 miles.

12. 144. 13. $-6435\,a^3x^7$; $6435\,a^7x^8$.

IV. TRIGONOMETRY. 1. 57·2956; 2094·4.

5. $\frac{1}{2}$; $-\dfrac{1}{\sqrt{2}}$; $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$; $\frac{1}{4}\sqrt{4-\sqrt{6}+\sqrt{2}}$. 6. $2n\pi \pm \dfrac{\pi}{6}$.

9. 9·427172; $74^0\ 29'\ 12''$; ·510544. 10. $80^0\ 46'\ 26{\cdot}5''$, $63^0\ 48'\ 33{\cdot}5''$.

11. 20 ft., 40 ft.

V. PURE MATH. (1). 6. $a(x-a)+by=0$. 7. 2, 1; $\tan^{-1}\frac{7}{17}$.

8. $\dfrac{2ab}{a^2+b^2}$, $(a^2+b^2)(x^2+y^2) - 2ab^2x - 2a^2by = 0$. 11. $\sqrt{\frac{3}{8}}$.

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VI. PURE MATH. (2).

1. $\{x(1-y)+1+y\}\{x(1-y)-1-y\}(x+1)(x-1)$. 2. 13 years.

3. 25 and 16 years, £250, £200. 4. 9 hr. 30 min., 10 o'clock.

8. Face of cube $=\frac{16}{\sqrt{3}}$. hexagon.

9. $\frac{4}{3}+\frac{12}{9}x+\frac{32}{27}x^2+\ldots+2\left(1-\frac{1}{3^{n+1}}\right)x^n+\ldots;$

$$-1-\tfrac{4}{3}x-x^2-\frac{8x^3}{9}-\ldots-x^{2n}-\left(1+\frac{(-1)^n}{3^{n+1}}\right)x^{2n+1}-\ldots$$

13. $2\pm\sqrt{-3}$, $-2\pm\sqrt{-1}$. 15. (1) $\dfrac{\sin\frac{n+1}{2}a\sin\frac{n}{2}a}{\sin\frac{1}{2}a}$.

(2) $2^n\sin\left(\frac{n}{2}+1\right)a\cos^n\frac{1}{2}a$.

VII. PURE MATH. (3).

2. (1) $-(\cos x)^{\cot^2 x}\{2\cot x\operatorname{cosec}^2 x\log\cos x-\cot x\}$.

(2) $-\frac{1}{x}(\cot^{-1}x)^{\frac{1}{x}}\left\{\frac{1}{\cot^{-1}x(1+x^2)}+\frac{\log\cot^{-1}x}{x}\right\}$.

(3) $\left(\frac{x}{n}\right)^{nx}\left\{n\left(\log\frac{x}{n}+1\right)^2+\frac{1}{x}\right\}$. 3. $\frac{e^x}{2^{\frac{n}{2}}}\sin\left(x+\frac{n\pi}{4}\right)$.

4. $\frac{\pi}{2}-x+\frac{1}{3}x^3-\frac{1}{5}x^5+\ldots$ 5. (1) $-\frac{1}{a}$. (2) 1.

7. $y+x=a$, $5y+10x+4a=0$, $5y-15x+a=0$. 10. (1) $\sin^{-1}\frac{x+1}{\sqrt{2}}$.

(2) $\tan^{-1}\frac{x}{a}+\frac{1}{2}\log\frac{x-a}{x+a}$. (3) $\frac{1}{2}\log(\tan x)$.

(4) $a\tan^{-1}\sqrt{\frac{x}{a}}\left(1+\frac{x}{a}\right)-\sqrt{ax}$.

VIII. STATICS. 2. $2\frac{1}{2}$, $3\frac{1}{2}$ cwt. : 1 ft. 5. $\sin^{-1}\frac{1}{4}$, $\cos^{-1}\frac{11}{16}$.

8. 4. 9. $2\tan^{-1}\frac{1}{4\sqrt{3}}$. 10. $8\frac{13}{14}$.

IX. DYNAMICS. 1. $\frac{\pi}{4\sqrt{2}}$, ·011 f. s. 2. 256 f. s., 1024 ft.

6. 144 lbs. 7. $\frac{v^2\sin 2a}{3g}$, $\frac{2v\sin a}{3g}$. 8. $\frac{1}{2}$. 10. 2032·8 lbs.

11. $1\frac{4}{5}$ miles.

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November, 1881.

II. ARITHMETIC. 1. $17\frac{187}{210}$. 2. $\frac{29}{280}$. 3. $1\frac{1}{3}$. 4. $\frac{119}{128}$, $\frac{888}{1107}$. 5. 78·012 lbs. 6. ·023203468 47. 7. 86400. 8. ·0125. 9. 157·6388. 10. ·40625. 11. 2 fur. 3 po. 4 yds. 2 ft. 12. £2342. 16*s*. $6\frac{3}{4}$*d*. 13. $4\frac{1}{2}$. 14. £7173. 6*s*. 8*d*., £8070, £8608, £8966. 13*s*. 4*d*. 15. £15. 16. £40. 17. $31\frac{16}{129}$, 56·7. 18. £$888\frac{24}{67}$, £$444\frac{28}{67}$, £$555\frac{15}{67}$. 19. $\frac{1}{36}$ in. 20. £14,400, $83\frac{1}{3}$. 21. $38\frac{2}{11}$ min. past 10; $24\frac{6}{11}$ miles. 22. $\bar{3}$·3222194, 4·3346348, $\bar{1}$·9451522, 703·116. 23. 8·08147 59...

III. ALGEBRA. 1. 1. 2. $a^3+b^3+c^3-3abc$; $(a+b)(c+d)$. 3. $\dfrac{3x^2-ax+a^2}{2x^2+a^2}$. 5. $\dfrac{3}{x^3-1}$; $\dfrac{6}{3^{\frac{2}{3}}-1}=\frac{3}{4}(3^{\frac{4}{3}}+3^{\frac{2}{3}}+1)$.
6. (1) $x=a+b, y=a-b$; (2) $x=4\frac{1}{2}$, 10, and -1. 7. $x^2+6x+7=0$.
8. Factor $=3^{\frac{5}{2}}-9\,.\,2^{\frac{1}{2}}+3^{\frac{3}{2}}\,.\,2^{\frac{3}{2}}-6+3^{\frac{1}{2}}\,.\,2^{\frac{4}{3}}-2^{\frac{5}{3}}$; 23. 9. 62·48.
10. $a\dfrac{r^n-1}{r^{n-1}(r-1)}$; 1. 12. 10 gallons. 13. $n=\frac{1}{3}$; $\frac{35}{243}$.

IV. TRIGONOMETRY. 1. 17·9049... 3. sine $=\frac{63}{65}$, cos $=-\frac{16}{65}$.
6. $\theta=n\pi$ and $n\pi\pm\dfrac{\pi}{6}$. 8. 9·8494850, 10·0624694, 9·9849438.
11. $35^0\ 5'\ 48\frac{1}{2}''$. 12. $2\frac{1}{2}$ miles.

V. PURE MATH. (1). 9. (1) A point. (2) 2 st. lines. (3) 2 st. lines. 10. $(m-1)y+(m+1)x=mk+h-c$.
13. $(ak+bh)y+(bh-ak)x=ak^2+bh^2+(b-a)hk$.
15. $-\frac{34}{75}a$, $\frac{23}{75}a$; $\tan^{-1}(-2):ae=bd$.

VI. PURE MATH. (2). 2. 24 half-crowns, 2 shillings, 54 fourpenny-pieces. 3. 11 : 24.
6. (1) $1+\frac{7}{6}x+\frac{41}{36}x^2+\ldots+\left(1+\dfrac{1}{2^n}-\dfrac{1}{3^n}\right)x^n+\ldots$
(2) $2x-x^3-x^5+2x^7-x^9-\ldots+2x^{6n+1}-x^{6n+3}-x^{6n+5}+\ldots$
8. $\tan\alpha=\dfrac{c}{a}=-\dfrac{b}{d}$, $\beta=\frac{1}{2}\log\dfrac{c-b}{c+b}$. 10. (1) $\sin(2^n-1)\theta\cos(2^n+1)\theta$.
(2) $\cos 3\left(\alpha+\dfrac{n-1}{2}\right)\beta\sin\dfrac{3n\beta}{2}\Big/4\sin\dfrac{3\beta}{2}+3\cos\left(\alpha+\dfrac{n-1}{2}\beta\right)\sin\dfrac{n\beta}{2}\Big/4\sin\dfrac{\beta}{2}$.
11. $-\frac{1}{2}$, $\frac{3}{4}$, $-\frac{3}{4}$.

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VII. PURE MATH. (3).

2. (1) $x^4 - 2a^2x^2 + 4a^4 / (x^2 - a^2)^{\frac{3}{2}}\sqrt{x^2 - 4a^2}$.

(2) $\left(1 + \frac{1}{x}\right)^x \left\{\log\left(1 + \frac{1}{x}\right) - \frac{1}{1+x}\right\}$. (3) $\frac{1}{x \cdot \log^{n-1} x \log^{n-2} x \ldots \log x}$.

(4) $b\left(x^2 - ay\sec^2\frac{y}{x}\right) \Big/ ax\left(x\sec^2\frac{y}{b} - b\sec^2\frac{y}{x}\right)$.

4. $1 + \sqrt{m^2+n^2} \cdot x\cos\phi + \frac{m^2+n^2}{\lfloor 2} x^2 \cos 2\phi + \ldots + \frac{(m^2+n^2)^{\frac{r}{2}}}{\lfloor r} x^r \cos r\phi + \ldots$

where $\tan\phi = \frac{m}{n}$. 5. (1) 1. (2) 4. 6. $\alpha + \cos^{-1}(e\cos\alpha)$.

8. $5(x+y) = a$. 10. (1) $\frac{1}{2}\log\frac{(x-2a)^3}{x}$. (2) $\log\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}$.

(3) $\frac{1}{2}\sec^{-1} e^{2x}$. (4) $\frac{1}{2}\log(\sec^2 x + 1)$. 11. $\frac{1}{2}a^2 \log 3$.

VIII. STATICS. 7. $10\sqrt{3}$ lbs., (1) $5\sqrt{3}$ lbs., horizontally.

(2) none. 8. $\frac{20}{\sqrt{3}}$ lbs., (1) $\frac{10}{\sqrt{3}}$ lbs., (2) none.

IX. DYNAMICS. 1. 12 in. 2. 5 secs., 64 f. s.

3. $\sqrt{2gh}$; 0 and $\sqrt{2gh}$ (h = vert. height of plane).

4. [Read $\sqrt{3}+1$] 75^0, 60^0, 45^0. 8. 5 oz. 10. 432.

11. 1355200.

June, 1882.

II. ARITHMETIC. 1. $3\frac{10}{11}$. 2. $4\frac{9}{10}$. 3. $16\frac{7}{8}$. 4. $\frac{27}{16}$; $\frac{3}{4}$.

5. 532·746 in. 6. ·08651169225. 7. 54300. 8. $\frac{130}{177} = \cdot 7344$.

9. ·1. 10. ·156875. 11. 25 sq. po. 15 yds. 3 ft. 28 in.

12. £83,000. 13. £7630. 14. $3\frac{9}{11}$ hrs. 15. £200.

16. 9 cwts. 1 qr. 26 lbs. 3 drs. 17. Yes, side of cube is 190·8 ins., and side of hatchway is 194·4 ins.; $14\frac{233}{242}$. 18. $34\frac{142}{377}$.

19. 3*s*. 4*d*. 20. 36 miles. 21. The second, by £10 per annum.

22. $\bar{2}$·5910647; 6·5793741; ·6948141; 812·24. 23. 7.

III. ALGEBRA. 1. $(m^2 - 4n^2)(2x - y)$; 9. 2. $2a - b + 3c$; $16a^2 - 81x^2$. 3. $a - 2b$; $(64x^6 - 729)(3x + 2)$ or $192x^7 + 128x^6 - 2187x - 1458$.

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4. $\dfrac{2}{x(x-2)}$; $\dfrac{5(a+x)}{(2a-x)^2}$. 5. Real, but not rational. 6. $9x^2+2x-9$.
7. (*a*) $x=2\frac{1}{2}$; (*b*) $x=1\frac{7}{26}$ and -1; (*c*) $x=a$, $y=-b$; $x=\frac{25}{9}a$, $y=\frac{7}{9}b$.
8. $8x^2-20a^3x-a^6=0$. 10. 1*s.* 4*d.*, 2*s.* 10*d.* 11. $94\frac{1}{2}$;
$\dfrac{1-r^n}{(1-r)^2}-\dfrac{nr^n}{1-r}$. 13. $\dfrac{(n+1)(2n+1)\ldots(\overline{r-2}\,.\,n+1)}{\underline{|r-1}}\dfrac{1}{x^{2nr-2n+1}}$.

IV. TRIGONOMETRY. 1. $135^0-\dfrac{270^0}{\pi}=49^0{\cdot}056$; $\dfrac{3\pi}{4}-\frac{3}{2}={\cdot}85619$.
2. 120^0 and 240^0. 3. $n\pi+\dfrac{\pi}{4}$, and $n\pi+\dfrac{\pi}{6}$.
8. ·69897; $\bar{3}{\cdot}90309$; $\bar{1}{\cdot}451545$. 10. $74^0\,50'\,38''$; $50^0\,32'\,58''$.
11. 5 ft. 4 in., 6 ft. $5\frac{1}{3}$ in.

V. PURE MATH. (1). 6. $1, -1: y+3x-2=\pm\sqrt{2}(2x+y-1)$.
9. (1) 2 st. lines. (2) 2 coincident pts. (3) parabola.
14. $y^2=a(x-3a)$.

VI. PURE MATH. (2). 2. £9. 7*s.* 6*d.*
3. [Read $x+y+z=d$] $a^4+b^4+c^4-a^2b^2-a^2c^2-a^2d^2-b^2c^2-b^2d^2-c^2d^2=0$.
4. 11, $n-1$. 5. $l=1$, scale $1-x-2x^2$. 6. $\sqrt{\frac{5}{3}}$. 7. -32.
11. 1 and 2, 2 and 3, -1 and -2, -2 and -3. 12. $-1\pm\sqrt{2}$,
$\frac{1}{2}(1\pm\sqrt{-3})$; $1\pm\sqrt{2}$, $\frac{1}{2}(1+\sqrt{2}\pm\sqrt{-1+2\sqrt{2}})$, $\frac{1}{2}(1-\sqrt{2}\pm\sqrt{-1-2\sqrt{2}})$.

VII. PURE MATH. (3).
1. $\dfrac{2(1-x^2)}{x^4+x^2+1}:(\tan x)^{\tan x}\sec^2 x(\log\tan x+1)$.
2. $\dfrac{(-1)^n\,\underline{|n}}{(x+a)^{n+1}}:e^{x\cos\theta}\sin(x\sin\theta+n\theta)$. 4. 82 max., 55 min.
5. $\dfrac{c^4}{\sqrt{(x^6+y^6)}}$. 6. $2a$, $\frac{13}{9}c$. 7. [Index $\frac{3}{2}$ omitted] $4\sqrt{2}\,.\,a$.
8. $r=\dfrac{a}{2\sqrt{3}}\cdot\dfrac{1+\sqrt{3}\tan\theta}{\tan\theta-\sqrt{3}}$. 10. (1) $\dfrac{x}{2}\sin 2x+\frac{1}{4}\cos 2x$.
(2) $\log\dfrac{x+2}{x+3}$. (3) $\log(x^3+2ax^2+a^2x)$. (4) $\frac{1}{2}\tan^{-1}(2\tan x)$.
(5) $\frac{1}{2}e^{2x}(x^2-x+1)$. 11. $\dfrac{4a^2}{3}$.

VIII. STATICS. 2. 12, 8, 4 lbs. 4. $4\frac{1}{3}$, $3\frac{2}{3}$ tons.
7. 10 inches from end. 9. 50 ft. 10. 10560.

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IX. DYNAMICS. 2. 115920, $57\frac{1}{2}$ ft. 3. 30^0; at right angles. 6. 30^0. 8. 10 sec., 2 f. s. 10. Nearly 11 secs. 11. 108.

November, 1882.

II. ARITHMETIC. 1. $6\frac{5}{6}$. 2. 0. 3. $\frac{25}{144}$. 4. $\frac{2}{3}$; 20. 5. 1·87104. 6. 244393·4. 7. 266·6̇. 8. $1\frac{5}{7}=1\cdot\dot{7}1428\dot{5}$. 9. $\frac{3}{11}=\cdot\dot{7}\dot{2}$. 10. $\cdot 3308\dot{3}$. 11. 5 days 23 h. 48 m. 9 s. 12. £2418. 17*s*. $3\frac{1}{2}$*d*. 13. £1882. 14. 1000. 15. £33. 6*s*. 8*d*. 16. 8 : 9. 17. $\frac{3}{125}$. 18. 50 c. ft. 6′ 6″ $10'''$ 10^{iv} 6^{v} = 50 c. ft. $946\frac{7}{8}$ c. in. 19. 15 m. 16 s. 20. £50, increase. 21. £817. 1*s*. 8*d*.; £1129. 3*s*. 4*d*. 22. $\bar{2}\cdot 3802113$; 9·4768640; ·4610087; 1314·56. 23. 5.

III. ALGEBRA. 1. $13a-21b$; 3.
2. $(a-1)x^3-3ax^2-a^2(a-1)x-(a^3+2a^2+2a+1)$; $a^2+b^2+c^2-ab-ac-bc$.
3. $\frac{a}{bc}+\frac{b}{ac}+\frac{c}{ab}$; 3003. 4. $(ab)^{25}$. 5. $3x-7$; $(x-1)(x+2)(x-3)$ $=x^3-2x^2-5x+6$. 6. $\frac{1}{x^2-1}$; $2(x+y+z)$. 7. (1) $x=4$; (2) $x=5$, $y=2$; (3) $x=27$ and $\frac{25}{147}$. 8. $15\frac{3}{4}$ oz., $16\frac{1}{4}$ oz. 10. 4, and 8. 12. 70.

IV. TRIGONOMETRY. 1. 12·0031 ins. 4. $\pm\frac{3}{4}$. 6. 5·6954904; $\bar{4}\cdot 6954817$; 4·6954838. 7. $C=18^0$; $b=\sqrt{5}+1$; $a=\sqrt{5+2\sqrt{5}}+1$. 8. $78^0\ 27'\ 47''$. 11. 800 yds. $200(\sqrt{3}-1)=146\cdot 41$; $200(\sqrt{3}+1)=546\cdot 41$.

V. PURE MATH. (1). 7. $\frac{\sqrt{b^2-4ac}}{a+c}=\tan\alpha$, $c(a+b)=0$.

VI. PURE MATH. (2). 1. 6, 5 : −5, −6 : $\frac{\pm\sqrt{-47}+13}{2}$, $\frac{\pm\sqrt{-47}-13}{2}$: $\frac{\pm\sqrt{-74}-14}{2}$, $\frac{\pm\sqrt{-74}+14}{2}$. 2. $a^2+b^2=c^2$. 4. 3, 2. 6. 2^n-1. 7. $\frac{1}{2}$. 11. $x^2-bx-acx-c^2=0$. 12. $-\frac{1}{2}$, −1, −2, $\frac{1}{2}(-3\pm\sqrt{5})$.

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VII. PURE MATH. (3).

2. $\dfrac{x^2-2x-2}{2(1+x+x^2)\sqrt{1-x^3}}$, $\dfrac{\cos^{-1}x - x\sqrt{1-x^2}}{(1-x^2)^{\frac{3}{2}}}$, $\dfrac{\log(\sin y)+y\tan x}{\log(\cos x)-x\cot y}$.

3. $(-1)^n \underline{|n}\,(x+a)^{-n-1}$, $(-1)^n \underline{|n}\,(b-a)\{(x+a)^{-n-1}-(x+b)^{-n-1}\}$.

5. $\frac{1}{2}$. 9. $y-x=0$. 11. $\frac{1}{2}x\sqrt{a^2-x^2}+\frac{1}{2}a^2\sin^{-1}\dfrac{x}{a}$, $\frac{1}{3}\log\dfrac{1+2x}{2(1-x)}$, $\log\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$, $-\dfrac{1}{2(e^{2x}+1)}$. 12. $\dfrac{4}{15c^2}(3c\sqrt{x}-2)(1+c\sqrt{x})^{\frac{3}{2}}-\dfrac{8}{15c^2}$, where $c=\dfrac{25}{16\sqrt{a}}$.

IX. DYNAMICS. 1. $\frac{11}{720}$ f. s., $\dfrac{11\pi}{21600}$; $13\frac{7}{11}$, $27\frac{3}{11}$ miles per hour. 2. 96 f. s., 0. 3. $\dfrac{1}{\sqrt{2}}$ secs. 4. 20 ft. 6. 12 lbs. 7. 6 lbs. 10. $2117\frac{1}{2}$ ft.

June, 1883.

II. ARITHMETIC. 1. $\frac{1}{20}$. 2. $1\frac{196}{205}$. 3. 63. 4. $\frac{1}{3}$. 5. ·62963 yd. 6. 51·25854371. 7. 11010000. 8. 180. 9. ·007954. 10. ·0431365746. 11. 1 ton 7 cwt. 3 qrs. 27 lbs. 8 oz. 12. £7738. 16*s.* 5*d.* 13. 5. 14. 2181·3046875 c. ins. 15. £56,250. 16. 60, 70, 84, 105. 17. 2 min.; 880 yds. 18. 2·5; 1 ft. $11\frac{1}{2}$ ins. 19. 6 c. ft. 1′ 4″ = 6 c. ft. 192 ins.; 4 yds. 20. $10\frac{1}{8}$ hrs. 21. £97. 4*s.* 0*d.* 22. 2·3222193; $\bar{1}$·1367499; ·5204397; 2·5. 23. 1·06.

III. ALGEBRA. 1. $2(a-3b)$; $2\frac{1}{2}$. 2. $\frac{1}{4}x^5-\frac{5}{3}x^4+\frac{77}{24}x^3-\frac{43}{12}x^2-\frac{11}{6}x+9$. 3. $(x-3)(x-4)$: $3(x+1)(x-3)$: $2(x+1)(x-4)$: $6x^3-36x^2+30x+72$. 4. $x^2-\dfrac{3a^2x}{4b}$. 5. $x-x^{\frac{1}{3}}$; $\frac{7}{2}\sqrt{3}$. 6. $-\dfrac{4(x-2)}{x^4-1}$; $\dfrac{3n-m}{2}$. 7. $x=\frac{5}{3}$; $x=13$, $y=7$; $x=3$, $y=1$ and $x=-\frac{37}{17}$, $y=\frac{39}{17}$. 8. $x=2$; £5. 5*s.* 0*d.* 9. 48, 60, 75. 10. 123456; 444·44. 12. 6286. 13. $-1{,}400{,}000\,a^{12}$; $\dfrac{\underline{|2n}}{\underline{|n}\,.\,\underline{|n}}x^n$.

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IV. TRIGONOMETRY. 1. $\frac{\pi}{6}=\cdot 5236$; 9^0. 2. 60^0; $n\pi\pm\frac{\pi}{6}$.

4. $\frac{1}{\sqrt{3}}$ and $-\sqrt{3}$. 6. $2\cdot 09691$; $\bar{4}\cdot 6989700$; $9\cdot 6989700$; $\cdot 4771212$.

9. $100\sqrt{3}=173\cdot 205$; $C=90^0$. 10. $\sqrt{15{,}000}=122\cdot 474487$.

11. 229·149 yds.

V. PURE MATH. (1). 4. $\sin a$. 10. $\frac{x^2}{a^4}+\frac{y^2}{b^4}=c^2$.

12. $\frac{x^2}{l^2}+\frac{y^2}{m^2}=1$. (rod divided $l:m$.)

VI. PURE MATH. (2). 1. 3, 4, 5. 2. >3 or <1.

3. $\frac{(-1)^r}{\lfloor r}\{a+br+cr(r-1)\}$. 4. $3\frac{7}{51}$. 10. $x^3-23x+4=0$.

11. -7, -2, 3.

VII. PURE MATH. (3). 1. $(2x+4x^3)/3(1+x^2+x^4)^{\frac{3}{2}}$, $\cot x$.

2. $\frac{a}{x\sqrt{x^2-1}}\cdot\sin\left(a\sin^{-1}\frac{1}{x}\right)$, $\frac{\cos x\,.\,e^{\cos^2 x}(1-2\sin^2 x)}{1+\sin^2 x\,.\,e^{2\cos^2 x}}$, $\frac{n\sqrt{a}}{2x\sqrt{a+bx^n}}$.

3. -4. 4. $\sin QOA=\frac{AC}{OC\sqrt{2}}$. 5. $x-\frac{1}{2}x^2+\frac{2}{3}x^3\ldots$

6. $r(\sin\theta+\cos\theta)=\frac{4a}{3}$. 7. For tangent at pt. $a\left(\frac{b}{a+b}\right)^{\frac{3}{2}}$, $b\left(\frac{a}{a+b}\right)^{\frac{3}{2}}$.

8. $\frac{a^4}{x^2}+\frac{b^4}{y^2}=\frac{c^4}{a^2}$. 9. $x^2+y^2-10ax+4ay-3a^2=0$: at pt. $9a$, $-6a$.

10. $r=\frac{a}{\pm\sqrt{15}}$, $\cos\theta=\pm\sqrt{\frac{3}{8}}$ give the co-ordinates of the pts.

11. (1) $\frac{1}{2}\log\frac{1-\cos x}{1+\cos x}$. (2) $\sin^{-1}\frac{2x+1}{\sqrt{5}}$.

(3) $\sqrt{x(x+a)}-a\log(\sqrt{x}+\sqrt{x+a})$. (4) $\frac{e^{ax}}{r^2+a^2}(a\sin rx-r\cos rx)$.

12. $\frac{16a^2}{3}$.

VIII. STATICS. 1. No, 20. 3. 6 inches from end, $1\frac{5}{7}$ inch.

5. $\frac{r}{6}$ from centre. 8. 45°. 10. 12·35...

IX. DYNAMICS. 1. ·104... 6. $3\frac{1}{3}$. 8. ·0105 oz.

9. $6\frac{7}{8}$ tons.

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November, 1883.

II. ARITHMETIC. 1. 1. 2. $1348\frac{4}{5}$. 3. 18. 4. $1\frac{1}{2}$; $19\frac{1}{2}$. 5. 88·875 sq. ft. 6. 2101·12701106. 7. ·0311. 8. ·0275. 9. ·008$\dot{3}$. 10. ·075. 11. 5 yds. 1 ft. 3 in. 12. £5241. 11*s*. $8\frac{1}{2}$*d*. 13. $4\frac{1}{2}$ years. 14. £78. 15*s*. 0*d*. 15. £450. 16. 26 : 19. 17. 640·8386$1\frac{1}{3}$ yds. 18. B; $4\frac{23}{48}$ ft. 19. 61 : 35. 20. £625. 21. ·8562427.
22. 2·2174840; ·5772363; $\frac{3}{4}\log 2+\log 3+\frac{1}{2}\log 7-\frac{1}{2}\log 11$. 23. 7·04.

III. ALGEBRA. 1. $\frac{1}{6}a-2b$; 59. 2. $b^2-a^2+\frac{b^4}{a^2}-\frac{a^4}{b^2}$; $a^2-3ab+b^2$; $2(a-b)(1-ab)$. 3. $2x^2+1$. 5. $x^3+x(y+z)+yz$. 6. $a+b^2+c^3-3a^{\frac{1}{2}}b^{\frac{2}{3}}c$; $a^{\frac{11}{12}}b^2$; 0. 7. $\frac{1}{2x^2-1}$; $\frac{3x-4}{(x+1)(x^2-1)}$. 8. (1) $x=6$; (2) $x=3, y=2$; (3) $x=\left(\frac{a+b}{a-b}\right)^{\pm 1}$. 9. £30 against A, £40 against B, £45 against C. 10. $x^2-10x+9=0$; roots are 1 and 9. 12. 3960. 13. $\frac{n(n+1)\ldots.(n+r-1)}{\lfloor r}x^4$; $8r-4$: 6th and 7th, each $=\frac{5103}{1024}$.

IV. TRIGONOMETRY. 1. $57^0\,16'\,21\frac{9}{11}''$; ·8675595.
3. $(2n+1)\pi-\frac{\pi}{3}$. 4. (1) Read $-1+\sec^2(45^0-A)$. 5. 15^0, 75^0.
7. 2·30103; $\bar{2}$·39794; ·598626; 9·69897; 9·849435.
9. $40^0\,53'\,36''$, $19^0\,6'\,24''$; $c:a=\sqrt{7}:2$.

V. PURE MATH. (1). 4. $1:\sqrt{6}:4$. 8. $\frac{1}{3}$. 10. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$.
11. $2, \sqrt{2}, y\pm x=0$. 12. $x^2+y^2-\frac{2a\sqrt{2}}{1+\sqrt{2}}(x+y)+\frac{2a^2}{3+2\sqrt{2}}=0$.
13. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

VI. PURE MATH. (2). 2. 49. 3. $n-r+1$, $nr+r+2n+1$.
4. $3a, \frac{6a^2+1}{2a}, \frac{36a^2+9a}{12a^2+1}, \frac{72a^4+24a^2+1}{24a^3+4a}, \frac{432a^5+180a^2+15a}{144a^4+36a^2+1}$.
8. $\pm\frac{\sqrt{-1}\pm 1}{\sqrt{2}}$.

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9. $\sin\frac{2n\pi}{2n+1}\Big/2\sin\frac{\pi}{2n+1}$, $\frac{1}{2}\{\tan^2\theta-\frac{1}{2}\tan^4\theta+\frac{1}{3}\tan^6\theta\ldots\}$.
11. $\frac{2}{3}$, -2, -2, -2. 12. -6, $1\frac{1}{2}$, $\frac{2}{3}$.

VII. PURE MATH. (3).

2. $-m\sin y\cos(m\cos y)$, $\frac{1-x\log a}{x}$, $-\frac{\sqrt{b^2-a^2}}{b+a\cos x}$.
5. $\log 2-\frac{1}{4}x^2-\frac{1}{96}x^4\ldots$ 6. [Read $-\sqrt{2}.\sin x$.] (1) 2. (2) e^m.
12. $\sin^{-1}\frac{x}{a}$, $\tan\frac{x}{2}$. 13. $\frac{a^2}{27}$.

VIII. STATICS. 1. $2\sqrt{2}$, S. W. 6. In centre of triangle.
8. $\frac{(r-r')W}{2a}$. 10. $\frac{l}{a}=\frac{1}{\sqrt{\left\{1-\left(\frac{W}{2T}\right)^2\right\}}}+\frac{1}{\sqrt{\left\{1+\left(\frac{W'}{2T}\right)^2\right\}}}$.

IX. DYNAMICS. 2. Same instant. 4. $19\frac{7}{11}$. 7. $\frac{1}{4}g(1-\mu)t^2$.
10. $15\frac{3}{11}$.

July, 1884.

II. ARITHMETIC. 1. $\frac{212}{243}$. 2. £1. 3*s*. 6*d*. 3. $\frac{1}{2}$. 4. $1\frac{13}{14}$.
5. 8·98079072. 6. 2·4008354748. 7. ·037. 8. 6·76̇.
9. 6*d*. 10. ·375. 11. 4 yds. 1 ft. 12. £1590. 0*s*. $2\frac{1}{4}$*d*.
13. $3\frac{1}{2}$. 14. 140; 135; 104. 15. 5 miles.
16. $3\frac{1}{3}$ gallons at 17*s*. 6*d*., and $8\frac{2}{3}$ gallons at 14*s*. 6*d*. 17. $10\frac{6}{7}$.
18. £2500. 19. 3 hrs. $5\frac{80}{271}$ min. 20. 2·0211893; 1·2736442.
21. ·1664843... 22. $x=16·30075$. 23. 2·5157...

III. ALGEBRA. 1. $-1\frac{2}{45}$; $(17b-8a)(b+4a)$. 2. $2ab(4x+a+b)$; x^2+xy+z^2. 4. a^2+a+1; $(5a^2-5a+3)(a^5-a^3-4a^2-3a-2)$.
5. $\frac{3x(1-x^2)}{1+x^2+x^4}$; $\frac{(a-c)(b^2-bc+ac-c^2)}{(a-b)(b-c)^2}$. If the 2nd factor was intended to be $\frac{2x+b+c}{2x+a+c}$, the value would be $\frac{a-c}{c-b}$. 6. $3\sqrt{7}-2\sqrt{3}$; 7; $6p^3$, $2p$.
7. (1) $x=7$. (2) $x=\pm\sqrt{3}$ and $\pm\frac{1}{\sqrt{2}}$. (3) $x=0$, $y=-4$ and $2\pm2\sqrt{-3}$; $x=-2$, and $1\pm\sqrt{-3}$, $y=2$ and $-1\mp\sqrt{-3}$. (4) $x=2a$ and $\frac{72}{25}a$.
8. Distance $=6$ miles, rate of carriage $=8$ miles per hour.
9. $a^{\frac{1}{2}}+b^{\frac{1}{2}}+c^{\frac{1}{2}}=p^{\frac{1}{2}}$. 11. $5\frac{670}{2187}$; -18. 12. 495.
13. $1-\frac{1}{2}x-\frac{1}{8}x^2-\frac{1}{16}x^3-\frac{5}{128}x^4-\ldots$

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IV. TRIGONOMETRY. 1. ·05585...; $45^0\ 50'\ 11''$; 32142·85 feet. 2. $A=2n\pi\pm\frac{\pi}{2}$; $B=n\pi$. 3. $\pi+\alpha$. 4. $n\pi+\frac{\pi}{4}$. 5. $BAE=75^0$, $ABD=45^0$. 6. $\frac{A}{2}$ lies between 135^0 and 225^0. 7. -4; 1·8; $\bar{3}\cdot8326827$; 1·09779. 9. $CD=57\cdot3979$ inches. 11. $r=\frac{1}{2}(a+b-c)$ $r:r'=(b+a)(a+b-c):(b-a)(a+c-b)$.

V. PURE MATH. (1). 3. $\frac{2\sqrt{5}}{3\sqrt[3]{3}}a$, where a^2=area of a triangle.

9. $\frac{130}{17\sqrt{29}}$. 10. $\left(\frac{x}{h}+\frac{y}{k}\right)^2-2\frac{x}{h}-2\frac{y}{k}+1=0$,

$x\left\{\frac{x'}{h^2}+\frac{y'}{hk}-\frac{1}{h}\right\}+y\left\{\frac{y'}{k^2}+\frac{x'}{hk}-\frac{1}{k}\right\}-\frac{x'}{h}-\frac{y'}{k}+1=0.$

12. $-\frac{1}{3}$, $-\frac{1}{3}$: $\frac{2\sqrt{2}}{3}$ and $\frac{2\sqrt{2}}{\sqrt{3}}$.

VI. PURE MATH. (2). 1. $b^3-3bc^2-3ab^2+3ac^2+2a^3=0$.

4. $\frac{x(x^n-1)}{x^{n+1}-1}$. 6. $\frac{m\pi}{n+1}$, $\frac{(2m+1)\pi}{2(n+1)}$.

7. $\frac{4(s-a)(s-b)(s-c)}{kabc}$, ($k$ ratio of sides). 8. $\log\left(\frac{1}{2}\operatorname{cosec}\frac{\theta}{2}\right)$.

9. $\pm\sqrt{-2}$, $\pm1\pm\sqrt{3}$. 11. $x^3+6qx^2+9q^2x+27r^3+4q^3=0$.

12. $\pm\sqrt{-\frac{1}{2}}$, $-1\pm\sqrt{-\frac{3}{2}}$.

VII. PURE MATH. (3). 2. $e^x\left(1+\frac{5}{x}-\frac{10}{x^2}+\frac{20}{x^3}-\frac{30}{x^4}-\frac{24}{x^5}\right)$.

3. $\frac{2(x-na)(x+a)^n}{(x-a)^n}$, $\frac{a^{-\frac{x}{2}}x(\log a)^2}{2(1-x\log a)^{\frac{3}{2}}}$, $\frac{1}{2\sqrt{x(x-a)}}$.

5. $\frac{\pi}{2}-x-\frac{x^3}{6}-\frac{3x^5}{40}\ldots$ 6. (1) $\frac{1}{3}$. (2) e.

8. $8y\pm3\sqrt{3}.x+a=0$, $36y\mp32\sqrt{3}.x-41a=0$. 9. $r=a\tan\theta$.

10. $2a:a$, $\frac{5a}{2}$.

12. $\frac{1}{2}\log\frac{1+\sin x}{1-\sin x}$, $x\sin x+\cos x$, $-1/3(n-1)(a+3x)^{n-1}$. 13. $2a^2$.

VIII. STATICS. 9. $w\operatorname{cosec}\alpha$, $w\cot\alpha$. 10. $w\left(\frac{1}{\mu}\cot\alpha-1\right)$.

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IX. DYNAMICS. 2. 4 f. s. 3. 40000 ft.-lbs. 4. 144 ft. 7. $\frac{u^2}{g}$, $w\sqrt{2}$. 8. $102\frac{3}{32}$ lbs., $153\frac{9}{64}$, $\frac{99\sqrt{3}}{2}$ ft. 9. $\cos^{-1}\frac{54}{125\pi^2}$. 10. 126·44.

November, 1884.

II. ARITHMETIC. 1. $1\frac{19}{200}$. 2. $\frac{4}{5}$. 3. £675. 4. $3\frac{1}{2}$. 5. 18·903296. 6. ·05182069 36. 7. 846; with remd. ·0046875 ft. 8. $\frac{71}{105}$. 9. 3*s.* $8\frac{1}{4}$*d.* 10. ·3140625. 11. *A*, £300: *B*, £98: *C*, £112: *D*, £120. 12. £316. 17*s.* $3\frac{3}{4}$*d.* 13. £836. 14*s.* $2\frac{3}{4}$*d.* $\frac{1}{15}$. 14. 792. 15. £2160. 16. 11 : 80. 17. £3. 4*s.* 0*d.* 18. 7*s.* 6*d.*; 5*s.* 19. £285. 8*s.* 0*d.* 20. $\frac{1}{2}$. 21. $\bar{3}$·0569049; ·2373609. 22. ·00126445. 23. ·563119 foot.

III. ALGEBRA. 1. $2a$. 2. $a^6-2a^5b+8a^3b^3-32ab^5+64b^6$; $4a^2+2ab-2ac+b^2+bc+c^2$. 3. $x-3$; $48a^2b^2(a^2-b^2)(a^6-b^6)$. 4. $\frac{3(a+2b)}{a-6b}$; 0. 6. $1-\frac{3}{2}x-\frac{3}{2}x^2$; $a^4-4a^{\frac{1}{2}}+6-4a^{-\frac{1}{2}}+a^{-1}$. 7. $\frac{1}{4}(2+3\sqrt{2}+\sqrt{6})$; $a^{\frac{1}{2}}$. 8. $8\frac{13}{18}$; 4 and $\frac{10}{11}$; $x=\pm 3$, $y=\pm 2$; $x=\pm\sqrt{2}$, $y=\mp 4\sqrt{2}$. 9. $4x^2-6x+1=0$; between 2 and -2. 10. £5. 14*s.* 0*d.* 11. 24. 12. 151,200. 13. $-252a^5x^5$; $\frac{1\cdot 3\cdot 5\ldots(4n-1)\times 2^{2n}}{\lfloor 2n}$.

IV. TRIGONOMETRY. 1. 1·023, 58^{0}·620. 3. -1. 4. $\frac{1}{8\sqrt{2}}\{(\sqrt{5}-1)(\sqrt{3}+1)-\sqrt{10+2\sqrt{5}}(\sqrt{3}-1)\}$. 6. $2n\pi$ and $2n\pi-\frac{\pi}{2}$. 7. $n\pi+(-1)^n x$; $6n\pi+(-1)^n x$. 9. 28·47717 sq. inches. 11. $\tan^{-1}\left\{\frac{(b\cot\alpha-a\cot\beta)}{a+b}\right\}$ with vertical.

V. PURE MATH. (1). 8. $r(\sin\theta+\cos\theta)+1=0$.

VI. PURE MATH. (2). 1. 1260, 55. 3. £205. 18*s.* 2*d.* 5. $5^{r-1}.r+(-7)^r$. 6. $(2n+1)\pi\pm\alpha$, $2n\pi\pm\cos^{-1}(2\cos\alpha)$. 10. $1, \frac{5\pm\sqrt{45}}{2}$. 11. $1, 1, \frac{1\pm\sqrt{-15}}{4}$. 12. $-1, 4, \frac{-3\pm\sqrt{-31}}{2}$.

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VII. PURE MATH. (3). 1. 1.

3. $\dfrac{3}{x^4\sqrt{1+x^2}}$, $\dfrac{-1}{2\sqrt{a^2-x^2}}$, $x^{\cos x}\left(\dfrac{\cos x}{x}-\sin x\log x\right)$.

4. $\dfrac{\lfloor n}{(1-x)^{n+1}}$, $\dfrac{\lfloor n}{(1-x)^{n+1}}+\dfrac{(-1)^n\lfloor n}{(2+x)^{n+1}}$,

$$\{x^2-2n(n-1)\}\sin\left(x+\frac{n\pi}{2}\right)+2nx\sin\left\{x+\frac{(n-1)\pi}{2}\right\}.$$

6. (1) $\frac{3}{4}$. (2) 0. 7. Coordinates $\pm\dfrac{a}{\sqrt{2}}$, $\pm\dfrac{b}{\sqrt{2}}$.

8. $x-2a=0$, $y+3a=0$, $y=x+5a$.

11. $\dfrac{x\sqrt{a^2-x^2}}{2}+\dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a}$, $\frac{1}{2}(\log x)^2$, $\frac{1}{3}\log\dfrac{x^3-1}{x^3}$.

12. $a(\sqrt{125}-8)$.

VIII. STATICS. 5. $\dfrac{P\sqrt{2}}{\sqrt{3}+1}$, $\dfrac{2P}{\sqrt{3}+1}$. 7. $3\frac{1}{3}$ lbs. 8. *W*.

9. arc$=\dfrac{ab}{2a-b}$.

IX. DYNAMICS. 1. $\frac{1}{2}$ cwt. 4. $\frac{1}{10}$. 5. 2·49 ft. 6. $\dfrac{mv^2}{4}$.

7. $3666\frac{2}{3}$ ft. 8. $266\frac{2}{3}$ lbs. 9. $1\frac{4}{5}$ miles. 10. 2·74 in.

April, 1885.

II. ARITHMETIC. 1. $1\frac{1}{2}$. 2. $\frac{8}{13}$. 3. £7. 8*s*. 4. $\frac{34}{51}$.

5. 59·73286. 6. ·08225424 sq. mètres. 7. 4250.

8. £4. 3*s*. $1\frac{1}{2}$*d*. 9. ·0036. 10. 6·6. 11. £385.

12. £28. 3*s*. $0\frac{1}{4}$*d*. 13. $4\frac{1}{2}$. 14. 165. 15. £3750.

16. $82\frac{7}{24}$. 17. 9*s*. 8*d*.

18. 23·37, 25·87, 25·7, 28·54, 33·34, 26·98. 19. £3. 13*s*. 9*d*.

21. ·7·3180634 : ·3046341. 22. 2·174773. 23. £12151. 16*s*. 9*d*.

III. ALGEBRA. 1. $2x^2-4xz+2z^2-2y^2 : \dfrac{8y+16}{8y-1}$.

2. $3x^2-7x+9$. 3. $(2x+3y+z)(2x-3y-z)(2x+3y-z)$.

4. (i) $\dfrac{x^2+y^2}{(x+y)^2}$: (ii) $\dfrac{1}{(x-1)(x-2)(x-3)}$. 5. a^2+b^2.

7. $a^2-ab^{\frac{2}{3}}+b^{\frac{4}{3}}$, 2. 8. (i) $4\frac{2}{3}$. (ii) $\pm 4, 0 : \pm 2, 0$.

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(iii) $\pm 2, \pm 1$; $\pm 1 \pm 2$. 9. 30, 40. 10. 10, 15 hours.

12. $\sqrt{ab}$. (i) $\dfrac{9}{\sqrt{2(3\sqrt{3}-2)}}$. (ii) $n^2(n+1) - \frac{3}{4}\{1-(-\frac{1}{3})^n\}$.

13. 246480. 14. $1 + 5x^2 + 15x^4 + 35x^6 + 70x^8 + 126x^{10} + \ldots$

IV. TRIGONOMETRY. 3. $10^0, 80^0, 100^0, 170^0$.

5. $n\pi + (-1)^n \dfrac{\pi}{10}$ and $n\pi - (-1)^n \dfrac{3\pi}{10}$. 6. $15'$.

8. 7, and $\sqrt{129}$. Area $20\sqrt{3}$. 9. $A\ 87^0\ 27'\ 25'' : B\ 32^0\ 32'\ 35''$

10. $\dfrac{2c(\cot\alpha - \cot\beta)}{\sqrt{\{1 + (2\cot\beta - \cot\alpha)^2\}}}$.

V. PURE MATH. (1). 3. $\dfrac{a^3}{3\sqrt{2}}$.

4. Planes must be parallel to edge of cone and at equal distances.

9. $x^2 + y^2 - 2\{2h \pm \sqrt{2(h^2-k^2)}\}x + 3h^2 - k^2 \pm 2h\sqrt{2(h^2-k^2)} = 0$.

12. $\frac{3}{4} : 8y = 6x + 3a$ and $2x - a = 0$.

VI. PURE MATH. (2). 1. $(a^2-b^2)^2 = a^2b^2$. 6. $\frac{1}{2}\left\{\frac{1}{3} - \dfrac{1}{2n+3}\right\} : \frac{1}{6}$.

11. $x^3 - p^2x^2 - p^2r = 0$.

VII. PURE MATH. (3).

1. $\dfrac{1}{\sqrt{x^2+a^2}} : -\dfrac{x^3+3x^2+4x-6}{3x^3(x+1)^{\frac{1}{3}}(x^2+1)^{\frac{1}{2}}} : \sin^x x(\log\sin x + x\cot x)$.

2. $13\sin x + 8x\cos x - x^2\sin x$. 3. ·100167.

4. $\sin a\sin b\left(1 - \dfrac{\alpha^2+\beta^2}{4}\right) + \alpha\beta\cos a\cos b + \alpha\sin b\cos a + \beta\sin a\cos b$.

5. $2, \sin^2\theta$. 6. $\left(\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right)^2 : (a+b)^2$.

8. $y = \pm\left\{x + \dfrac{a-c}{2}\right\} : x + c = 0$.

9. $\frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x : x\sin^{-1}x + \sqrt{1-x^2} : \frac{1}{2}\tan^{-1}x^2$.

11. $\dfrac{\pi}{4} : \pi - 2 : \dfrac{3\pi(a-\beta)^2}{4}$. 12. $\dfrac{3a^2\pi}{8}$.

IX. DYNAMICS. 1. 39:40. 2. $6\frac{11}{15}$ ft. 3. $\dfrac{V^2\cos^2\beta}{2g\sin\alpha}, \dfrac{2V\cos\beta}{g\sin\alpha}$.

4. $4h$. 5. $1', 42\frac{2}{3}''$. 7. $\pi\sqrt{\dfrac{l}{g}}\left(1 - \dfrac{1}{4n}\right)$.

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July, 1885.

II. ARITHMETIC. 1. $18\frac{9}{10}$. 2. ·8. 3. 3927 : 5000. 4. ·1418 : 2·65. 5. 19*s.* $1\frac{1}{2}$*d.* 6. $13\frac{1}{2}$ acres. 7. $5\frac{1}{4}$ lbs. 8. ·790625 : 107·5625. 9. 840. 10. £4. 3*s.* 5*d.* 11. $754\frac{13}{18}$ sq. ft. 12. £1044. 12*s.* 1*d.* 13. 8*s.* 5*d.* 14. 4%. 15. 10%. 16. 50 tons 17 cwt. 3 qrs. 12 lbs. 17. $15\frac{15}{23}$ past 24 : 1 hr. $2\frac{14}{23}$ min. 18. £1234. 4*s.*, £617. 2*s.*, £205. 14*s.* 19. $24\frac{4}{9}$. 20. £350. 21. 7, 3 : ·1902751 : ·4829844. 22. $1\frac{1}{4}$ years.

III. ALGEBRA.

1. $\frac{x^5}{4}-\frac{11x^3}{24}+\frac{77x}{24}-\frac{23}{4} : 16x^{\frac{4}{3}}-8x^{\frac{3}{3}}y^{\frac{1}{2}}+4x^{\frac{2}{3}}y-2x^{\frac{1}{3}}y^{\frac{3}{2}}+y^2.$

2. $x^2+x+1 : 4(9x^2-4)(x^2-9).$ 3. 12.

4. (i) $\frac{3}{x(x^3-1)}$. (ii) $\frac{(x-5)^2}{3x^2-32x+64}$. (iii) $\frac{x^2-4xy+y^2}{x^2+y^2}$.

5. $3\sqrt{3}-2\sqrt{5}$. 6. (1) 3. (2) $\frac{3b}{2}, -\frac{a}{2}$. (3) $\pm 3, \pm 1$.

7. 5, 12. 8. $3x^2-2a^2x-a^4=0$. 9. $2\frac{1}{4}$ ft.

10. $2\frac{1}{15}, 1\frac{2}{3}, 1\frac{4}{11}, 1\frac{2}{15}, 1$. 11. $455 : 2(\underline{|12})^2$. 12. Scale of 5.

13. $\frac{2n(2n-1)\ldots(2n-r+2)}{\underline{|r-1}}a^{2n-r+1}x^{r-1} : 3003.$

IV. TRIGONOMETRY. 1. $135^0=\frac{3\pi}{4}$. 3. $n\pi+\frac{\pi}{4}$ and $n\pi+\frac{\pi}{3}$. 9. $66^0\ 17'\ 16''$. 10. 11·71572.

V. PURE MATH. (1). 1. Plane through line of intersection. 8. $-13\frac{1}{2}, 19\frac{1}{2}$. 9. Circle through pt. of intersection.

11. $\frac{x}{a}(\cos\phi-\sin\phi)+\frac{y}{b}(\cos\phi+\sin\phi)=1.$ 12. $\pm a\sqrt{m^2-1}$.

13. $(x-y)(\cos\theta-\sin\theta)=a(\cos\theta+\sin\theta).$

VI. PURE MATH. (2).

1. $1+\frac{n(n-1)}{1^2}+\frac{n(n-1)(n-2)(n-3)}{1^2\cdot 2^2}+\frac{n(n-1)\ldots(n-5)}{1^2\cdot 2^2\cdot 3^2}+\ldots$

5. $\{x+x^2-(2n+1)x^{n+1}+(2n-1)x^{n+2}\}/(1-x)^2.$

6. $x^2+y^2-b^2=a\sqrt{y^2+(b+x)^2}.$

10. $(pq-r)x^3-(p^2-2pq+3r)x^2+(pq-3r)x+r=0.$ 11. 2, 2, 3.

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VII. PURE MATH. (3). 1. $\frac{-2n(1-x)^{n-1}}{(1+x)^{n+1}}$. 3. $\frac{(-1)^{n-1}\lfloor n-1}{x^n}$.

4. $\log 2+\frac{1}{2}x+\frac{1}{8}x^2+\ldots$ 6. $\frac{b^2c}{a^2}$. 7. Perpendicular to axis of x.

8. ∞, $\sqrt{8}$. 9. $xx'^{\frac{2}{3}}+yy'^{\frac{2}{3}}=a^{\frac{4}{3}}$, $\left(\frac{a}{\alpha}\right)^{\frac{4}{3}}+\left(\frac{b}{\beta}\right)^{\frac{4}{3}}=1$.

10. $\frac{2}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}$, $\frac{1}{2}x+\frac{1}{4}\sin 2x$, $\frac{e^{ax}}{a}$. 11. $x\sin^{-1}x+\sqrt{1-x^2}$.
13. $\log 2$.

VIII. STATICS. 3. Force $4\sqrt{2}$, couple of moment $10\times$ side.

5. $\frac{33}{48}$ of distance between the strings. 6. $\frac{4r}{\sqrt{5}}$.

IX. DYNAMICS. 1. $\frac{15v}{22}$. 2. $5\frac{5}{11}$ secs. 4. 6 secs.

5. $16\sqrt{5}$ f. s., 100 ft. 7. Very nearly $12\frac{1}{2}$. 10. $\frac{3}{8}P$.

November, 1885.

II. ARITHMETIC. 1. 20224372008 : ·0123402.
2. ·00007901234$\dot{5}$6. 3. 294 : 5·04. 4. £8. 3*s*. $6\frac{1}{4}$*d*. 5. 7*s*. $0\frac{3}{4}$*d*.
6. 300. 7. 5 cwt. 1 qr. 8. ·3375 : 1*s*. 2·75208*d*.
9. £10. 16*s*. 3·15*d*. 10. 7. 11. £8. 10*s*. 2·88*d*. 12. 3*s*. 4*d*.
13. £1. 1*s*. 14. 3 per cents., £3. 12*s*. $0\frac{33}{47}$*d*. : $3\frac{1}{2}$ per cents., £3. 10*s*. $9\frac{63}{113}$*d*. : 4 per cents., £3. 14*s*. $5\frac{1}{48}$*d*. 15. £6. 14*s*.
16. £14. 1*s*. 8·8125*d*.
17. Course 1800 yds., speed $9\frac{397}{395}$ and $9\frac{3}{5}$ miles per hour. 18. 420.
19. 270. 20. 2·947519 : ·947519 : $\bar{2}$·947519.
21. ·9542426 : 2·1072100 : ·0030103 : ·3802113. 22. 23456.

III. ALGEBRA. 1. $3a-\frac{7}{2}b+\frac{c}{3}$. 2. $2x^2-3x+1$.

3. (i) $\left(\frac{b}{a+b}\right)^3$. (ii) x^2+y^2. (iii) -1.

4. $\frac{3a^mc}{2b^6}-a^nb^3-\frac{2^8}{3}$: $3\sqrt{7}-2\sqrt{6}$. 5. (i) $\sqrt{3}\cdot ab$. (ii) 7.

6. (i) $\frac{ab}{a+b}$. (ii) 1, $-3\frac{1}{2}$; 1, $-\frac{3}{7}$. (iii) $\pm\frac{2}{\sqrt{35}}$, $\pm\frac{2\sqrt{5}}{\sqrt{7}}$, $\pm\frac{2\sqrt{7}}{\sqrt{5}}$.

7. 5, $3\frac{1}{3}$, $2\frac{2}{3}$. 8. $20\frac{1}{2}$, 16 feet. 10. 10 ft. 3 in. 11. $\frac{\beta^8-a^8}{\beta-a}$.
13. $2\frac{31}{4}x^{12}$.

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IV. TRIGONOMETRY. 3. $\frac{1}{8}\left(n\pi \pm \frac{\pi}{6}\right)$, $\frac{1}{8}\left(\overline{2n+1}\,\pi \pm \frac{\pi}{3}\right)$.
10. 1 mile, 1·219714 miles. 11. 25·7834.

V. PURE MATH. (1). 1. A straight line. 7. $a \cos \frac{\alpha - \beta}{2}$.
8. Straight line through the origin.

VI. PURE MATH. (2).

1. $1 + x + \frac{1}{2}(x^2 + x^3) + \frac{1 \cdot 3}{2^2 \lfloor 2}(x^4 + x^5) + \ldots$

$$+ \frac{1 \cdot 3 \cdot 5 \ldots (2n-3)}{2^{n-1} \lfloor n-1}(x^{2n-2} + x^{2n-1}) + \ldots$$

2. $\frac{n(n+1)(3n^2 - n + 2)}{24}$. 4. $0, -\frac{3 \pm \sqrt{5}}{2\sqrt{5}}, \frac{3 \mp \sqrt{5}}{2\sqrt{5}}$.
6. $-a, \pm\sqrt{-a^2}$. 7. $(a^2 - 2b)(b^2 - 2ac) - c^2$. 11. 2.
12. $\Sigma \cos^2 A + 2 \cos A \cos B \cos C = 1$.

VII. PURE MATH. (3). 1. $\frac{2(1+x^2)}{(1-x^2)^2} : \frac{2}{1+x^2} : \frac{2(1+x^2)}{x(1-x^2)}$.
2. $e^{3x}(3 \cos 4x - 4 \sin 4x) : \frac{5^n x^n}{\lfloor n} \cos(n \cos^{-1} \frac{3}{5})$.
3. $40, 8 : n\pi + \alpha, (2n+1)\frac{\pi}{2}$. 4. $yy_1^2 + xx_1^2 = 2c^2$.
5. The point $2, \frac{11}{10}$. 8. $0, \frac{13\sqrt{13} \cdot a}{6}$.
9. $\log(x+1) : \log \frac{x+1}{x+2} : \log(x + \sqrt{x^2+1})$.
10. $\frac{e^x}{2}(\sin x + \cos x) : \frac{x^2 - 1}{2} \log(x+1) - \frac{1}{4}(x-1)^2$. 11. $\frac{2}{\pi}$.
12. (1) $\frac{4a^2}{3}$. (2) $\frac{\pi a^2}{4}$.

VIII. STATICS. 1. $\sqrt{3} : 1 : 1$. 6. 55 lbs. 8. $\frac{W}{\sigma} \sin 2A$, $\frac{W}{\sigma} \sin 2B$, $\frac{W}{\sigma} \sin 2C$ $(\sigma = \sin 2A + \sin 2B + \sin 2C)$. 10. Weight of a rod.
12. $\frac{1}{3}r$ from centre.

IX. DYNAMICS. 3. $15\frac{3}{5}$ lbs. per ton, 2 m. 572 yds. 4. 80 ft.
6. 9 lbs. 7. 4·52, 3·72 secs. 8. 2160. 9. 28.
11. 75 : 4.

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July, 1886.

II. ARITHMETIC. 1. One hundred thousand and two hundred millions, two hundred thousand, one hundred. 2. 90900, 1. 3. £730. 16*s.* 4. £4. 4*s.* $2\frac{3}{4}$*d.* 5. 19*s.* $1\frac{1}{2}$*d.* 6. 25. 7. ·1418, 289. 8. £37. 7*s.* $7\frac{37}{63}$*d.* 9. £13. 0*s.* 4*d.* 10. £6735. 11. £289. 12. 36 sq. ft. $74\frac{5}{8}$ in. 13. $1234\frac{2}{7}$. 14. 5 hrs. 12′. 15. £1. 19*s.* 4*d.* or $21\frac{83}{277}$ p. c. 16. $176\frac{27}{242}$. 17. 2·5984375, 6296·4... 18. $\cdot\dot{7}1428\dot{5}$, £108. 19. $52\frac{1}{4}$. 20. 2·0897995. 21. 855677.

III. ALGEBRA. 1. (i) $\dfrac{x+y}{x^2-xy+y^2}$. (ii) 0. (iii) $\dfrac{x^2-3}{(x-1)^2}$. 2. $x^2+\sqrt{2}\,.\,xy+y^2$, $x^2+y^2+x+y-xy+1$. 3. $2xy$. 4. (i) $\dfrac{a+b+c}{2}-\dfrac{b^2c^2+a^2c^2+a^2b^2}{4abc}$. (ii) $\frac{1}{2}(-5\pm\sqrt{19})$. 5. $acx^2+(b^2-2ac)\,x+ac=0$. 6. 5, 7. 8. $60\frac{1}{2}$ and 40 yds. 9. 1, 4, 7, &c. 10. $\frac{4}{15}$. 11. 0, $-\frac{1}{2}$. 12. 40320.

IV. TRIGONOMETRY. 3. 180^0+A. 4. $\sqrt{2}$. 7. $65^0\ 59'$, $41^0\ 56'\ 12''$.

VI. PURE MATH. (2). 1. 4. 3. $1+5x+19x^2+65x^3+\ldots+(3^{n+1}-2^{n+1})\,x^n+\ldots$ 4. (1) $\dfrac{n\,(n+1)\,(n+2)}{3}$. (2) $\dfrac{n}{n+1}$, 1. 5. $x=42, 31, 20, 9 : y=7, 16, 25, 34$. 6. 24. 8. (i) 21. (ii) $\dfrac{6\,.\,7\,.\,8\ldots(5+n)}{\lfloor n}$. 11. $2\pm\sqrt{-3}$, -6, 2. 12. ·88.

VII. PURE MATH. (3). 1. $\dfrac{1}{1+x^2}$, cosec x, $x^x\,(\log x+1)$.

4. -10, $\pm\dfrac{1}{16\sqrt{2}\,.\,b}\left\{3a\pm\sqrt{a^2+32b^2}\right\}\sqrt{16b^2-a^2\pm a\sqrt{a^2+32b^2}}$.

5. $x_1x^{-\frac{1}{3}}+y_1y^{-\frac{1}{3}}=a^{\frac{2}{3}}$. 7. $(a^2+b^2)\,\dfrac{\pi}{4}$. 8. $y+x+a=0$, $2y+x=2a$, $x+a=0$. 9. $\frac{1}{8}\cos^8 x-\frac{1}{6}\cos^6 x$, $\dfrac{x^3\,(3\log x-1)}{9}$, $\frac{1}{6}\log\dfrac{x^3-1}{x^3+1}$.

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10. $\dfrac{a}{\sin a}$, $\frac{1}{2}(\sqrt{2}-1)$, $\dfrac{\pi}{2}-1$.

12. (i) $\dfrac{8a}{27}\left[\left(1+\dfrac{9x}{4a}\right)^{\frac{3}{2}}\right]_{x_1}^{x_2}$. (ii) $\left[\frac{2}{3}\sqrt[3]{ax^2}\right]_{x_1}^{x_2}$.

VIII. STATICS. 5. 1 lb. at distance 5 ft. 8. ·2678...in.

IX. DYNAMICS. 1. 15, 9 miles an hr. 2. 5 miles an hr.
4. Acc. 8, Tension $3\frac{3}{4}$ oz., Pressures 3, $2\frac{1}{2}$ oz. 7. 12 f. s.

November, 1886.

II. ARITHMETIC. 1. 7256. 2. £962.
4. 7 tons 6 cwt. 1 qr. 5. 1·34455 : ·0110289890777695...
6. £962. 9*s*. $3\frac{3}{4}$*d*. 7. 5 days. 8. $113\frac{1}{12}$: 483. 9. £194. 12*s*. $7\frac{1}{2}$*d*.
10. £171. 1*s*. $10\frac{1}{2}$*d*. 11. £1333. 6*s*. 8*d*. : $3\frac{7}{15}$ p. c.
12. 5 cu. yds. 25 ft. 1567 in. 13. 25 p. c. 14. 5 days.
15. 152·475 lbs. 16. 31 yards 2 ft. 4 in. : 3*s*. 9*d*.
17. 2825761 : 2560000.
18. £3550, £3124, £2982 : £3536, £3128, £2992. 19. 1*s*. $3\frac{1}{4}$*d*. nearly.
20. ·076923 : £7. 9*s*. 4*d*. 21. (1) ·8450980. (2) ·3010299 : ·4771212.
22. 39·405.

III. ALGEBRA. 1. $2a^2b^2+2ab+1 : x^2+2+x^{-2}$.
2. (i) $\dfrac{1}{x^2+y^2}$. (ii) 0. (iii) $\dfrac{x^2+2ax+3a^2}{x^2-3ax+a^2}$.
4. (i) $\dfrac{24a^2}{25b}$. (ii) $x, \pm 3, \pm 1 : y, \pm 2$. 5. 32 ft., 24 ft., 10 ft.
7. $5\frac{1}{3}$, $6\frac{2}{3}$ hrs. 10. 8, and 2. 11. 68, 888. 12. 60.

IV. TRIGONOMETRY. 1. $65^0\ 27'\ 16\frac{4}{11}''$. 3. $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$, $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$, $-\dfrac{\sqrt{3}+1}{2\sqrt{2}}$, $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$. 5. $\frac{311}{348}$. 7. ·22221...
9. $93^0\ 22'\ 21''$, $47^0\ 41'\ 7''$, $38^0\ 56'\ 32''$. 10. 2·39, 1·366 miles.

V. PURE MATH. (1). 3. The line of centres. 8. $\dfrac{a-A}{A\sqrt{a^2+b^2}}$.
11. $\dfrac{b^2\cos^2\theta-a^2\sin^2\theta}{ab\sin\theta\cos\theta}$.

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VI. PURE MATH. (2). 4. $\underline{|5} : \underline{|n} \,.\, \underline{|n-1}$.
6. $x^2 - 2x\cos 2\theta + 1 = 0$. 7. 3, 3, $\frac{3}{4}$.
8. $1 - x + x^4 - x^5 + \ldots + x^{4r} - x^{4r+1} + \ldots$ 11. $\frac{a}{\sqrt{2}}(\cot\theta + 1)$.

VII. PURE MATH. (3). 2. 71·4 yds.
5. $\pm\frac{a^2}{\sqrt{a^2-b^2}}$, $\pm\frac{b^2}{\sqrt{a^2-b^2}}$. 6. $\frac{y^2}{c} : c$. 7. $p^2 = mr$.
8. $y \pm \left(x + \frac{a}{2}\right) = 0$, $x - 2a = 0$.
9. (i) $\log\frac{(x-4)^4}{(x-3)^3}$. (ii) $\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$. 11. $\frac{a^2}{2}$.

VIII. STATICS. 2. *W*. 3. 3 oz. 5. $\frac{45}{56}$ height.
6. 8 oz. : 1 lb. 10. 1000 ft. 11. Slightly over 19.

IX. DYNAMICS. 1. At angle $\cos^{-1}\frac{3}{5}$ to the bank : perpendicularly.
6. $7\frac{321}{192}$ miles.

July, 1887.

II. ARITHMETIC. 1. 18. 2. £46. 5*s*. 3. $1\frac{2}{15}$.
4. £11. 11*s*. 3*d*. 5. £100. 6. £56. 15*s*. $4\frac{7}{8}$*d*.
7. ·000001645, ·51. 8. 1*s*. $2\frac{3}{4}$·00832*d*., ·900448908730$\dot{1}\dot{5}$.
9. 7693, 40·5. 10. $4\frac{1}{2}$. 11. 1*s*. $7\frac{3}{4}$·08736*d*. 12. Int. : Disc. = 5 : 4.
13. £19049. 3*s*. Diff. of income £253. 8*s*. 9*d*.
14. 12 sq. ft. 81 in., £12. 8*s*. $6\frac{1}{2}\frac{5}{12}$*d*. 15. 2*s*. 11*d*.
16. 1007·67 lbs. (if packed in line 1018·3 lbs.). 17. $\frac{3}{55}$, ·0000790123456.
18. 60 days, 12 hrs. 16′. 19. 15·44.
20. $\bar{1}$·9488474, ·3344539, $\bar{3}$·7781513 : value = 6·240325 ; 19 and 20.
21. 10·08.

III. ALGEBRA. 1. $x^4 + x^3 - a^2x^3 + \frac{x^2}{a^2} + a^2x - x + 1$,
$x^2 + (3+n)ax + 3a^2$. 2. $x^2 - x + 1$. 3. (1) $\frac{1}{x(x-a)(x-y)}$. (2) 2.
4. $\sqrt{\frac{27}{2}} - \sqrt{\frac{7}{2}}$, a.
6. (1) 3, 6. (2) 13 and $2\frac{1}{2}$. (3) $\frac{2a^2 + b^2 \pm b\sqrt{b^4 + 8a^3b - 4ab^3 + 4a^4}}{2(a^2 - b^2)}$.
8. $\frac{nb + (n+1)a}{2n+1}$, $\frac{na + (n+1)b}{2n+1}$.
9. Sum $\frac{1 - 3^{2n}}{4}$ last term $(-3)^{2n-1}$: sum $-2n$, last term $1 - 4n$.

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10. 1221759, 720. 11. 28, 3.
12. 3*s.* 4*d.* in the £. Land : personal property = 2 : 3.

IV. TRIGONOMETRY.

1. $\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$, $\cot A = \frac{4}{3}$, $\sec A = \frac{5}{4}$, $\operatorname{cosec} A = \frac{5}{3}$.
7. $49^0\ 28'\ 32''$. 8. $\bar{1}{\cdot}93753$, $\bar{1}{\cdot}76143$. 9. 120^0.
11. Height $= \frac{25}{2}\sqrt{7} = 33{\cdot}07189$, $CD = 17\frac{1}{2}$ ft.

V. PURE MATH. (1).

2. A circle on line, drawn from pt. perpendicular to given line, as diameter. 4. -14. 6. St. line. 7. $y = a \pm ex$.

VI. PURE MATH. (2). 1. $y^6 - 6y^4 + 9y^2 - 2$.

2. $A = C = 1$, $B = -D = \frac{1}{2\sqrt{2}}$.

3. (1) $\frac{2}{3}n(n+1)(2n-1) - n$. (2) $\dfrac{1 + x - (2n+1)x^n + (2n-1)x^{n+1}}{(1-x)^2}$.

5. A little less than 36 years. 7. $\frac{7}{120}$. 8. $\dfrac{\sin 2na}{2 \sin a}$.

10. $\dfrac{e^a + e^{-a}}{2} \sin\beta$, $\dfrac{e^{-a} - e^a}{2} \cos\beta$: $\dfrac{2(e^a + e^{-a})}{e^{2a} + e^{-2a} + 2\cos 2\beta}$, $\dfrac{2(e^a - e^{-a})}{e^{2a} + e^{-2a} + 2\cos 2\beta}$.

11. $4, \pm\sqrt{3}$. 12. $2, 2, \frac{1}{2}(-1 \pm \sqrt{-3})$.
13. $rx^3 - (qr - 2q)x^2 + (pr^2 - 4pr + 4p)x - r^3 + 6r^2 - 12r + 8 = 0$.

VII. PURE MATH. (3).

1. $\dfrac{x}{x^4 - 1}$, $\dfrac{a^2}{(a^2 - x^2)^{\frac{3}{2}}}$, $(\sin x)^x \{\log \sin x + x \cot x\}$.

2. $2y\dfrac{d^2y}{dx^3} + 6\dfrac{d^2y}{dx^2} \cdot \dfrac{dy}{dx}$. 4. $\frac{1}{2}x^2 + \frac{1}{3}x^3$. 5. $a^a(1 - \log a)$.

6. $\dfrac{a\sqrt{m+n}}{\sqrt{mn}}$. 7. $\dfrac{ny}{y'} + \dfrac{mx}{x'} = m + n$.

8. $x - 4\log(1+x) - \dfrac{4}{1+x}$, $\log(2x + 1 + 2\sqrt{1 + x + x^2})$ or $\sin^{-1}\dfrac{2x-1}{\sqrt{5}}$, $\dfrac{x^r}{r^2}(r\log x - 1)$.

11. $\dfrac{n}{m-n}$ $(m > n)$.

VIII. STATICS. 1. $6\frac{1}{2}$ lbs. 2. $\frac{1}{2}(W_2 + W_1)$, $\frac{1}{2}(W_2 - W_1)$.
3. $2\frac{4}{5}$, $9\frac{3}{5}$ lbs. 4. $7P$. 5. $3\frac{1}{2}$ lbs., $\frac{1}{2}$ in. from centre.
7. $2\sqrt{5}$, 3 inches. 8. 3808 ft.-lbs. 10. $1\frac{1}{3}$ lbs.

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IX. DYNAMICS. 1. 2 f. s. 2. $v \cot \theta$.
3. Accel. $=4f$. $L=32T^2$ (L ft. = unit of length, T secs. = unit of time).
8. A parabola.

November, 1887.

II. ARITHMETIC. 1. 899910. 2. £19. 15*s*. 6*d*. 3. $4\frac{6}{31}$.
5. ·00212, 21200. 6. 2·7765625, 2 qrs. 7 lbs.
7. $\frac{283}{37000}$, $8{\cdot}54\dot{8}54470131697$. 8. £1. 16*s*. 9. £150. 10*s*. $5\frac{17}{28}$*d*.
10. 14·0700007. 11. 8 cub. ft. $656\frac{3}{8}$ in. 12. 12 yrs. 13. £3.
14. 120 days. 15. $16\frac{56}{199}$. 16. $34\frac{4}{9}$.
17. 4, $\frac{1}{4}$: 5·3692159, 1·3692159, $\bar{2}$·3692159. 18. ·0189694.
19. 367182. 20. (1) £6000. (2) £131501. 5*s*. 8*d*.
21. 1 min. $52\frac{1}{2}$ sec. after the dog started. 22. £819. 6*s*. 2·669*d*.

III. ALGEBRA. 1. $x(a+1)-y(b-1)$. 3. $x^2+7xy-y^2$.
4. (1) 2. (2) $-\dfrac{34xy}{16x^2-y^2}$. 6. $2\sqrt[3]{\dfrac{a}{c}}+2\sqrt[3]{\dfrac{c}{a}}-1$.
8. (1) $6a+b$, $2a-b$. (2) $3a$ and $\dfrac{3a}{2}$. (3) $-\frac{5}{4}$. 9. 1020 yds.
10. 180. 11. 210. 12. $(-1)^n \dfrac{\lfloor 2n}{(\lfloor n)^2}$, $\frac{1}{6}(r+1)(r+2)(r+3)$.

IV. TRIGONOMETRY. 4. $\dfrac{\pi}{3}$, $\dfrac{1-\sqrt{3}}{2\sqrt{2}}$.
6. $\dfrac{2\tan A(1-\tan^2 A)}{(1+\tan^2 A)^2}$, $n\pi+\dfrac{\pi}{4}$, $\dfrac{n\pi}{2}+\dfrac{3\pi}{8}$. 8. 20·97615.

V. PURE MATH. (1). 4. $\frac{1}{2}\sqrt{\{b^2c^2+a^2c^2+a^2b^2\}}$. 5. A parabola.
6. $\dfrac{a^2}{\sqrt{a^2+b^2}}$, $\dfrac{b^2}{\sqrt{a^2+b^2}}$. 7. $x^2-y^2=c^2$, Rect. hyperbola.
10. $xx'+yy'=0$. (1) A pt. at the origin. (2) 3 st. lines—axis of y, $x-y=0$ and $x+y=0$. (3) Four points $x=\pm a$, $y=\pm b$.

VI. PURE MATH. (2).
3. $y=1+\frac{1}{2}x^2-\frac{1}{8}x^4+\frac{3}{16}x^5-\ldots=x+\dfrac{1}{2x}-\dfrac{1}{8x^3}+\dfrac{3}{16x^5}-\ldots$
5. $\frac{1}{2}n(n-1)$, $2n(n-2)$. 6. $\frac{169}{425}$. 8. 2, 2, $-\frac{1}{4}$. 9. $(a^{\frac{2}{3}}+b^{\frac{2}{3}})^{\frac{3}{2}}$.
10. $\dfrac{\sin\theta}{1-2x\cos\theta+x^2}$. 11. $\tan\dfrac{\pi}{n}$, $\tan\dfrac{2\pi}{n}$, $\tan\dfrac{3\pi}{n}\ldots\tan\dfrac{(n-1)\pi}{n}$.
12. $\dfrac{d^2+2ed}{(e+d)\cot\theta-e\cot\phi}$.

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VII. PURE MATH. (3). 3. $\frac{ab(1+\tan^2 y)}{a^2+b^2\tan^2 y}$. 6. $\frac{2x(x-a)}{a}$, $\frac{a}{2}$.

7. $r^n = a^n \cos n\theta$ where $n = \frac{m}{m+1}$. 8. $y = \pm x + \frac{a}{2}$.

9. $\frac{1}{2}x - \frac{1}{4}\sin 2x$, $x\log x - x$, $\frac{1}{5}\log\frac{x-2}{x+3}$. 10. 1, 1, $\frac{\pi}{3\sqrt{3}}$. 11. $\frac{a^2\pi}{12}$.

12. $\frac{8a}{27}\left(1+\frac{9x}{4a}\right)^{\frac{3}{2}} - \frac{8a}{27}$.

VIII. STATICS. 2. 14·242. 3. $6\frac{1}{4}$, $11\frac{3}{4}$ lbs. 4. 14·56 lbs.

7. $\frac{arW}{(a+b)\sqrt{r^2-ab}}$, $\frac{brW}{(a+b)\sqrt{r^2-ab}}$. 8. $5\frac{8}{11}$ in.

IX. DYNAMICS. 1. 1 : 20 : 360. 4. 115200 ft.

5. $2\frac{1}{2}$, $3\frac{1}{3}$ lbs. 9. 1·713 oz. 10. $5\frac{29}{37}$ tons.

July, 1888.

II. ARITHMETIC. 1. 4623. 2. $1\frac{7}{18}$. 3. 149, 1317160.

4. ·01405609352, ·00892. 5. £42. 13*s*. 4*d*.

6. $\frac{81}{495}$, $11\frac{611}{4950}$, $\frac{17287}{4995000}$. 7. $\frac{9}{100}$, ·95610416. 8. 14·13 : 31.

9. 8 days. 10. £10. 11. $4\frac{1}{2}$. 12. £5. 19*s*. 2*d*.

13. £1. 17*s*. 4*d*. 14. £13. 18*s*. 16. 6, −6, $\frac{5}{8}$.

17. 4·5529844, 2·5529897. 18. 83570·6. 19. 198·5365.

20. 38·7. 21. 8·82888. 22. 2 miles or 24′ after.

III. ALGEBRA. 1. 0.

2. $2(a+b)(b+c)$, $5xy(x-y)(x^2-xy+y^2)$. 3. $\frac{2x^2+3x-1}{x^2-3x-2}$.

4. (i) $\frac{34xy}{49x^2-y^2}$. (ii) $\frac{c^3a-a^2c+b^2c-bc^2+a^2b-ab^2}{abc(a-b)(b-c)(c-a)}$.

5. $\frac{a^2}{4}+\frac{2c}{a}$, $\frac{c^2}{a^2}$. 7. (*a*) −20, (*b*) 5, (*c*) *a* and *b*. 8. ·538461.

9. $r(\sqrt[3]{2}-1) = ·259921r$. 10. $2a$, $\frac{a}{2}$.

12. For $n+r$ put $n+r-1$.

IV. TRIGONOMETRY. 1. $57^0\ 16'\ 21\frac{9}{11}''$. 3. $\frac{1}{2}n\pi \pm \frac{\pi}{12}$.

4. $\cos a + \cos 2a + \cos 3a + \cos 4a$. 5. $\frac{5}{8}$. 8. 39 : 33 : 25.

10. $A = 105^0\ 27'\ 35''$, $B = 15^0\ 5'\ 20''$, $c = 2·44844$. 11. 45^0.

V. PURE MATH. (1). 9. $\tan^{-1}\frac{19}{11}$.

10. A parabola with same vertex and $\frac{1}{8}$th the parameter.

12. (i) Axis of y and a hyperbola. (ii) 2 st. lines. (iii) A parabola.

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VI. PURE MATH. (2).

1. $\dfrac{(a-b)(a-c)}{a^2+b^2+c^2-ab-ac-bc}$, $\dfrac{(b-a)(b-c)}{a^2+b^2+c^2-ab-ac-bc}$,

$$\frac{(c-a)(c-b)}{a^2+b^2+c^2-ab-ac-bc}.$$

2. (i) $\dfrac{n}{3}(4n^2+6n-1)$. (ii) $\frac{1}{2}$. 3. $-2x^2+2x^3-4x^4+6x^5$.

4. $\dfrac{4}{\sqrt{3}}$. 6. 180. 7. -2, $\frac{1}{2}(1\pm\sqrt{-11})$: $\pm\sqrt{2}$, $\frac{1}{2}(1\pm\sqrt{-11})$.

8. (i) a, $-\dfrac{a}{2}\pm\dfrac{\sqrt{16-3a^2}}{2}$. (ii) 1, 1, $\frac{1}{4}(-1\pm\sqrt{-15})$.

VII. PURE MATH. (3). 1. $\dfrac{1}{\sqrt{1-x^2}}$, $\dfrac{1}{\sqrt{x^2-a^2}}$, $\dfrac{a}{x\sqrt{x^2-a^2}}$.

4. Max. 1, min. $-\dfrac{1}{6}$: max. $\dfrac{1}{e}$. 6. $\dfrac{mx}{x'}+\dfrac{ny}{y'}=m+n$.

7. $\dfrac{(a^4y^2+b^4x^2)^{\frac{3}{2}}}{a^4b^4}$. 9. $\sin^{-1}\dfrac{2x-1}{\sqrt{5}}$, $\frac{1}{3}\cos^3 x-\cos x$, $\dfrac{x^2}{2}\log x-\frac{1}{4}x^2$.

10. $\dfrac{\pi}{4}$, $\pm\pi$, $\pm\dfrac{\pi}{3}$. 11. (i) πa^2. (ii) $\frac{3}{8}\pi ab$.

12. $\sqrt{ax+x^2}+a\log\dfrac{\sqrt{x}+\sqrt{a+x}}{\sqrt{a}}$.

VIII. STATICS. 2. $\frac{1}{2}$lb. 3. $2BC$.

6. $\dfrac{W_2}{W_1+2W_2}\cdot BC$ from the hinge.

IX. DYNAMICS. 2. $\dfrac{W}{32}$. 3. $\dfrac{D-d}{D}\cdot V$. 7. $\mu g\cos\alpha$.

8. 68·033 miles per hour. 9. 80000 ft.

November, 1888.

II. ALGEBRA. 1. 1.

3. $4x^2-6x+9$, $(2x+3)(16x^4+36x^2+81)(6x^2-5x-6)$.

4. $2\dfrac{x}{a}-3+4\dfrac{a}{x}$. 5. $b^2>4ac$.

6. (1) $1\frac{1}{27}$. (2) $a+b$, and $\frac{1}{2}(a+b)$. (3) 9, 6, 4 or 4, 6, 9.

7. $1\frac{2}{5}$, $9\frac{3}{5}$ galls. 9. 15. 10. $\dfrac{\lfloor 2m+1}{\lfloor m\,\lfloor m+1}\left(x+\dfrac{1}{x}\right)$.

12. Slightly over 125 years.

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III. TRIGONOMETRY AND MENSURATION.

1. 21600, 6875·49. 4. $2n\pi - \frac{\pi}{4}$.

8. $A = 22^0\ 37'\ 11\frac{1}{2}''$, $B = 67^0\ 22'\ 48\frac{1}{2}''$, $C = 90^0$. Area = 120 sq. ft.

9. 418·4, 430 ft. 11. 165·748.

IV. STATICS. DYNAMICS. 1. Yes arranged in same line.

2. $2P$. 3. 1 ft. from centre of lever.

6. $\frac{l \sin \alpha}{1 + \sin \alpha}$ from free end (α = inclination of plane to horizon).

9. 128. 10. $2\frac{1}{2}''$, $\frac{3}{4}$ weight. 11. 80 f. s., $80\sqrt{3}$ f. s.

12. $\frac{v\sqrt{h}}{4}$.

V. PURE MATH. (1). 7. $x - y = 0$.

8. $(\beta - ma - b)^2 = k^2(1 + m^2)$. 10. $(c + ak^2)(c + bk^2) = h^2k^4$.

11. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Eccentric angle.

12. (1) Parabola. (2) Hyperbola. (3) Ellipse.

VI. PURE MATH. (2). 1. $\pm \frac{1 \pm \sqrt{5}}{2}$. 3. 12, −60, 60.

4. 2160 lbs. weight. 5. 19, 1, 80.

6. $-\frac{1}{6}x^4 - \frac{11}{36}x^5 - \frac{85}{216}x^6 - \ldots\ldots - \frac{1}{2}\left(1 - \frac{1}{2^{n-4}} + \frac{1}{3^{n-3}}\right)x^n \ldots\ldots$

8. $27^0\ 15'$. 9. 3.

VII. MECHANICS. 1. St. line through pt. of intersection.

2. Taking AD horizontal D and $= 9\frac{5}{8}$. $X = 2{\cdot}05$ lbs. and the tensions are 1·7, 1·02, 1·8 lbs. 3. $54\frac{1}{2}^0$, $47\frac{1}{2}^0$, 78^0.

4. Force is $> W \tan \alpha$: $\frac{W \tan \alpha}{W + W'}$, ($W'$ = weight of plane).

6. μg. A st. line. 7. $\frac{P - Q}{P + Q} . h$. 8. $\frac{\pi}{43200}$.

10. $\frac{1}{2}v$, perpendicular to and $\frac{\sqrt{3}}{2}v$ along the line of centres: st. line before impact at angle of 30^0 to line of centres, after impact at angle of 60^0.

11. $\frac{5}{72}$ ft.-lb.

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